Thermal diode and thermal transistor: Controlling heat flow through nonlinear dynamics

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Abstract

In this paper, we give a brief review of our recent works on the thermal diode and the thermal transistor - two novel thermal devices for the control of the heat flow.

Introduction

The invention of the transistor[1] and other relevant devices that control electric charge flow has led to an impressive technological development. After more than half a century, similar devices for the heat flow are still lacking. However, theoretical studies have been encouraging. A model of thermal rectifier has been recently proposed[2] in which the heat can flow preferentially in one direction. Although this model is far away from a prototype realization, it is based on a mechanism of very general nature and, as such, is suitable of improvement and may eventually lead to real applications. In the following we briefly describe some recent progress in this direction.

Thermal diode

In a recent paper[3], a thermal diode model has been proposed, in which even though the underlying physical mechanism is similar to the one in Ref. [2], there is a new crucial element which allows to improve the efficiency by more than two orders of magnitude.

The system consists of two segments of nonlinear lattices coupled together by a harmonic spring with constant strength k_{int}. Each segment is described by the (dimensionless) Hamiltonian:

$$H = \sum \frac{p_i^2}{2m} + \frac{1}{2}k(x_i - x_{i+1} - a)^2 - \frac{V}{(2\pi)^2}\cos 2\pi x_i. \tag{1}$$

The two ends of the system are put into contact with thermal baths at temperature T_L and T_R for left and right bath, respectively. In fact, Eq. (1) is the Hamiltonian of the Frenkel-Kontorova (FK) model which is known to have normal heat conduction[4]. For simplicity we set the mass of the particles and the lattice constant $m = a = 1$. Thus the adjustable parameters are $(k_L, k_{int}, k_R, V_L, V_R, T_L, T_R)$, where the letter L/R indicates the left/right segment. In order to reduce the number of adjustable parameters, we set $V_R = \lambda V_L$, $k_R = \lambda k_L$, $T_L = T_0(1+\Delta)$, $T_R = T_0(1-\Delta)$ and, unless otherwise stated, we fix $V_L = 5$, $k_L = 1$ so that the adjustable parameters are reduced to four, $(\Delta, \lambda, k_{int}, T_0)$. Notice that when $\Delta > 0$, the left bath is at higher temperature and vice versa when $\Delta < 0$.

In Fig. 1 we plot the heat current J versus Δ for different temperatures T_0. It is clearly seen that when $\Delta > 0$ the heat current (J_+) increases with Δ, while in the region $\Delta < 0$ the heat current (J_-) is almost zero, i.e. the system behaves as a thermal insulator. The results in Fig. 1 show that our model has the rectifying effect in a wide range of temperatures. The rectifying efficiency, defined as $|J_+/J_-|$, could be as high as few hundreds times, depending on temperature as well as on other parameters.

The underlying mechanism of the rectifying effect is due to the match/mismatch of the energy spectra of the two interface particles between the two segments. For more detail discussions see Ref.[3].

Negative Differential Thermal Conductance

Of particular interest is the *negative differential thermal resistance* phenomenon observed in a certain temperature interval (see Fig.1, for $\Delta < -0.2$): for a fixed temperature T_R , a larger value of $T_L (< T_R)$ which corresponds to a smaller temperature difference $T_R - T_L$, can induce a larger heat current since, due to nonlinearity, it can result in a better match in phonon bands (see Fig. 2 and Ref. [5] for more discussions).

Thermal Transistor

In Fig 3(a) we show the configuration of the thermal transistor. It consists of three segments, D, S and G. Segment D (from D to O) has a negative differential thermal resistance in a certain temperature regime while segment S is a normal heat conductor, i.e., heat current inside this segment is positively dependent on temperature difference. Segment G is the control segment, which is connected to the junction particle between segments S and D. Temperature T_G will be used to control temperature T_o (at the junction O) so as to control the heat current from D to S. In analogy to the MOSFET, in which the electronic current in the electrode G is very small, we require here that the heat current J_G through segment G to be as small as possible, (otherwise it is hard to set T_G to a required value in experiment). Moreover the heat resistance of segment G must be small enough in order to well control the temperature T_o by changing T_G so that $T_o \approx T_G$.

Notice that, in typical situations, the differential heat resistance, $R_S = \left(\frac{\partial J_S}{\partial T_o}\right)^{-1}_{T_S=\mathrm{const}}$ in segment S, and $R_D = -\left(\frac{\partial J_D}{\partial T_o}\right)^{-1}_{T_D=\mathrm{const}}$ in segment D, are both positive and therefore there exists only one value of T_o for which $J_S = J_D$ so that $J_G = 0$, Since $J_S = J_D + J_G$, the "*current amplification factor*", $\alpha = \left|\frac{\partial J_D}{\partial J_G}\right| = \left|\frac{R_S}{R_S+R_D}\right| < 1$, namely in order to make a change ΔJ_D, the control heat bath has to provide a larger ΔJ_G. This means that the "transistor" can never work!

However, the key point of our transistor model is the "*negative differential heat resistance*" as we observed in our diode model[3]. The latter provides the possibility that when T_o changes both J_S and J_D change simultaneously in the same way. Therefore $J_S = J_D$ (or $J_s \approx J_D$) can be achieved for several different values of T_o or even in a wide region of T_o as shown in Figs.3 and 4. In this situation heat switch and heat modulator/amplifier are possible. In the ideal, limiting case of $R_S = -R_D$ which, in principle, can be obtained by adjusting parameters, the transistor works perfectly.

We first demonstrate the "*switch*" function of our transistor, namely we show that the system can act like a good heat conductor or an insulator depending on the control temperature . This is illustrated in Fig 3(b), where we plot J_G, J_S, and J_D versus T_G. When T_G increases from 0.03 to 0.135, both J_D and J_S increase. In particular, at three points: $T_G \approx 0.04, 0.09$ and $0.135, J_D = J_S$ thus J_G is exactly zero. These three points correspond to "off", "semi-on" and "on" states, at which J_D is $2.4 \times 10^{-6}, 1.2 \times 10^{-4}$ and 2.3×10^{-4}, respectively. The ratio of the heat current at the "on" state and that at the "off" state is about 100, hence our model displays one important function - switch - just like the function of a MOSFET used in a digital circuit.

As demonstrated above, the heat current from D to S can be switched between different values. However, in many cases, like in an analog circuit, we need to continuously adjust the current J_S and/or J_D in a wide range by adjusting the control temperature T_G. In Fig. 4 we demonstrate this "modulator/amplifier" function of our transistor. The basic mechanism of such "modulator/amplifier" is the same as that of the "switch" but we consider here different parameter values. It is seen that in the temperature interval $T_G \in (0.05, 0.135)$, the heat current through the segment G remains very small $((-10^{-5} \sim 10^{-5})$, within the shadow strip in Fig. 4), while the heat currents J_S and J_G continuously increase from 5×10^{-5} to 2×10^{-4}.

Conclusions and discussions

In summary, we have devised a thermal diode by using two coupled nonlinear lattices. Our model exhibits a very significant rectifying effect in a very wide range of system parameters. Moreover, based on the phenomenon of *negative differential thermal resistance* observed in the thermal diode, we have built a theoretical model for a thermal transistor. The model displays two basic functions of a transistor: switch and modulator/amplifier. Although at present it is just a model we believe that, sooner or later, it can be realized in a nanoscale system experiment. After all the Frenkel-Kontorova model used in our simulation is a very popular model in condensed matter physics[6].

It is worth pointing out that it is the temperature dependence of the phonon band that makes the rectifying effect possible. This dependence is due to the nonlinearity of the potential and therefore it should be possible to observe the rectifying effect in different nonlinear lattices.

References

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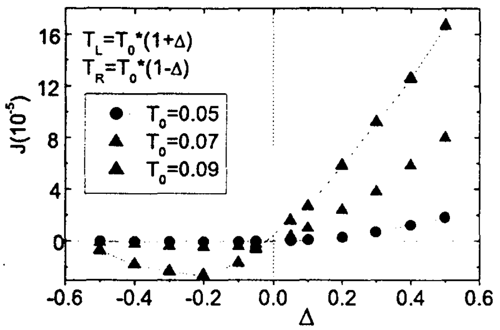

Figure 1: Heat current J versus the dimensionless temperature difference Δ for different values of T_0. Here the total number of particles $N = 100$, $k_{int} = 0.05$, $\lambda = 0.2$. The lines are drawn to guide the eye.

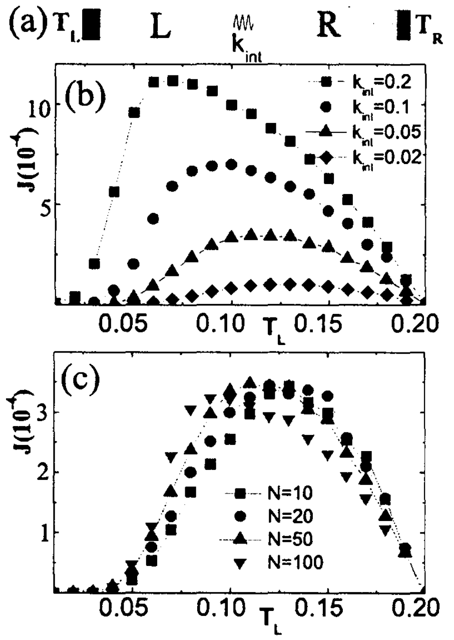

Figure 2: (a) Configuration of the thermal diode with negative differential thermal resistance (see definition in the text). (b) Heat current versus temperature T_L (at fixed $T_R = 0.2$) for different coupling constants, k_{int}, with lattice size $N = 50$. The system parameters are: $V_L = 5$, $V_R = 1$, $k_L = 1$, $k_R = 0.2$. (c) Same as (b) but for different system size N. $k_{int} = 0.05$. Notice that when $T_L \leq 0.1$ the heat current increases with decreasing the external temperature gradient.

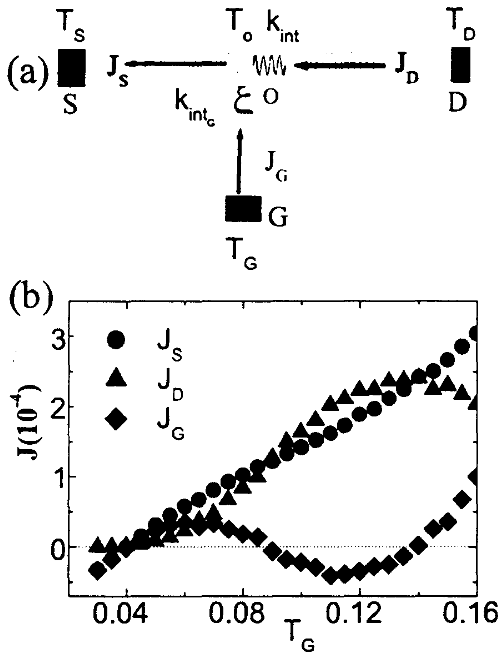

Figure 3: (a) Configuration of the thermal transistor. (b) Heat current versus the control temperature T_G. Parameters are: $T_D = 0.2,, V_D = 1.0, k_D = 0.2, k_{int} = 0.05; T_S = 0.04, V_S = 5, k_S = 0.2, V_G = 5, k_G = 1, k_{int_G} = 1$. Notice that both J_S and J_D inrease when the temperature T_G is increased.

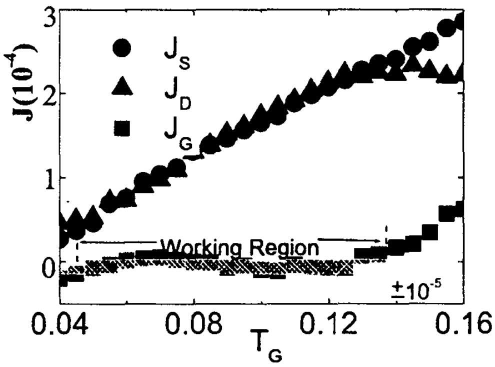

Figure 4: Heat current versus the control temperature T_G. Here: $T_D = 0.2, V_D = 1.0, k_D = 0.2, T_S = 0.04, V_S = 5, k_S = 0.2, k_{int} = 0.05, V_G = 5, k_G = 1, k_{int_G} = 0.1$. The shadow region is the range of variation of J_G in the temperature interval $T_G \in (0.05, 0.135)$.