Morphology of Spiral Patterns in Nematic Liquid Crystals Under Rotating Magnetic Fields

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２つのガラス板にはさまれた、ホメオトロピック配向のネマチック液晶の非平衡パターン形成を数値的に考察する。とくに、回転磁場下でのスパイラルパターン形成を系統的に調べた。基礎方程式としては、ディレクターの運動方程式から縮約された時間依存の複素ギンズブルグ＝ランダウ方程式を採用する。

We consider a nonequilibrium pattern formation in homeotropically aligned nematic layer between two glass plates.

A nematic liquid crystal is sandwiched between two glass plates, and subjected to a rotating magnetic field parallel to the plates and also to a static electric field perpendicular to them. The application of a magnetic field on nematic liquid crystal leads to the formation of walls. There exist Bloch(B) and Ising(I) walls. According to their chirality, (B) walls are classified into (B^+) wall and (B^-) walls. Under the rotating field, (B^+) and (B^-) propagate in the opposite directions, but (I) wall remains fixed. Given the initial planar (B^+) and (B^-) walls connected via a defect (i.e.,(I) wall) , the rotating field gives rise to spiral patterns. In this work we shall show the morphology of spiral patterns.

The basic tool we apply is the complex Ginzburg-Landau equation for the transverse component of director and is written in the rotating-coordinate frame by

$$\gamma_1 A_t = (\mu - i\gamma_1\omega)A + \gamma\bar{A} + \frac{1}{2}(K_1 + K_2)\nabla^2 A - a|A|^2 A - \frac{1}{2}b(A^2 + \bar{A}^2)A \quad . \tag{1}$$

where $A(x,y,t) = X(x,y,t) + iY(x,y,t)$ and K_i stand for Frank's elasticity coefficient. γ_1 and ω are the damping constant and frequency of the rotating field, respectively. Eq.(1) oversimplifies the nonlinearity and the improved version is given in terms of a stereographic variable $\hat{\varphi}$ as

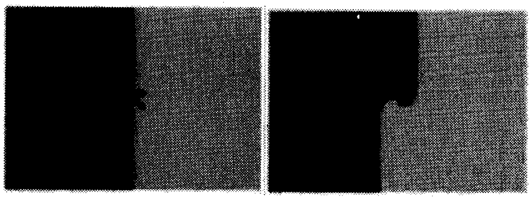
$$\gamma_1\hat{\varphi}_t = (\mu - i\gamma_1\omega)\hat{\varphi} + \gamma\hat{\varphi}^* + K\nabla^2\hat{\varphi} + \frac{3}{2}K\Gamma(\nabla\hat{\varphi})^2\hat{\varphi}^* - a\Gamma|\hat{\varphi}|^2\hat{\varphi} - \frac{b}{2}\Gamma(\hat{\varphi}^{2*} + \hat{\varphi}^2)\hat{\varphi} \quad . \tag{2}$$

We numerically iterate Eq.(2) and systematically investigate the morphology of spiral patterns by changing the initial defects configuration.

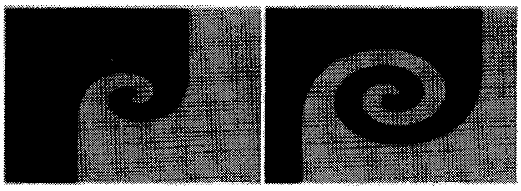
The result of numerical simulations is shown in Figs. 1-3.

Fig.1 The real part of $\hat{\varphi}$ evolving in time . Initial planar wall includes one defect.

(a) (b)



(c) (d)



(e)

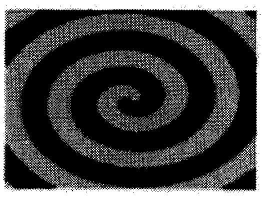
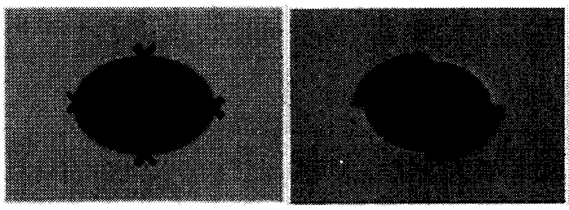


Fig.2 Initial ellipse wall includes four defects.

(a) (b)



(c) (d)

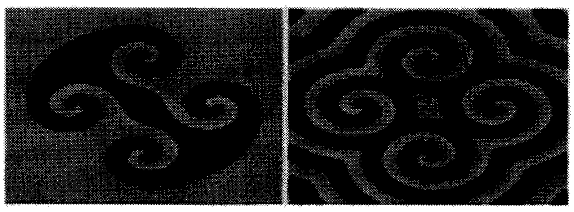
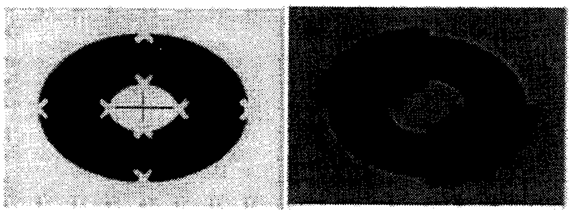


Fig.3 Initial double ellipses include eight defects.

(a) (b)



(c) (d)

We find that , by controlling the initial defect configuration , a variety of regular array of spiral patterns emerges. Taking into account correctly the nonlinearity of the director, we have obtained the equation for stereographic variables, which, in the weakly nonlinear limit , recovers Frisch's complex Ginzburg -Landau equation. Starting from arbitrary defect configurations, we have obtained amazingly rich textures of spirals, which may explain the existing experiments. The analysis of another instability (e.g., Mullins and Sekerka) of the moving planar wall is under way.

Ref. [1] T.Frisch , Physica D 84 (1995) 601-614