Quantum Level Statistics of Gaussian Ensembles in One-Dimensional Conservative System

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ガウス不変集団に従う量子エネルギー準位統計を持つ一次元保存系を数値的に研究した。一次元保存系は可積分であることが証明されている。一方、カオス系の固有値を充分多数有限個持つようなポテンシャルを数値的には構成することができることも示されている。ポテンシャルの数値的構成法として最急降下法と dressing 法を採用した。ポテンシャル形状はフラクタル的となり、そのフラクタル次元は数値的に 1.7 と求められた。

One-dimensional (1D) time-independent system with a particle, e.g. an electron, is studied. The system has one degree of freedom and one constant of motion, the Hamiltonian: H=const. It means that the 1D system is integrable [1]. On the other hand it was argued more than a decade ago [2] that a numerical calculation could construct the potential for the Hamiltonian system which has eigen-energies of the finite dimensional Gaussian orthogonal ensemble (GOE) matrices.

1D Schrödinger equation with a potential $V(x)$ is written as

$$\left(-\frac{1}{2}\frac{\partial}{\partial x^2}+V(x)\right)\phi(x)=e\phi(x), \tag{1}$$

where the potential is yet to be constructed. Note that we put the dimensionless unit: $m=\hbar=0$. In this work we numerically construct the 1D potential whose level statistics becomes indistinguishable from that of the GOE-type systems.

The iterative numerical method which was used in Ref.[3] is applied. It is based on the standard gradient method. This is simple and practical enough to create the potential for more than 1000 given eigen-energies [2-4].

Here we also apply new method that is called the dressing method [5]. It is very powerful and it can calculate the potential with more than thousands energy levels. We can add the levels that are to be the eigen-value of the potential one by one. To construct the potential function of the 1D Schrödinger equation with the series of the energy levels $\{e_i(x)\}(i=1,2,\cdots,N)$, we just prepare some initial potential $V(x)$ at first. If we can find the functions $f_i(x)$, which satisfy the relations

$$\frac{df_i}{dx}-f_i^2+2V=2e_i, \tag{2}$$

the new potential can be constructed as

$$W=2e_i+f_i^2-V. \tag{3}$$

Then e_i becomes the eigen-value of the 1D Schrödinger equation with the potential $W(x)$.

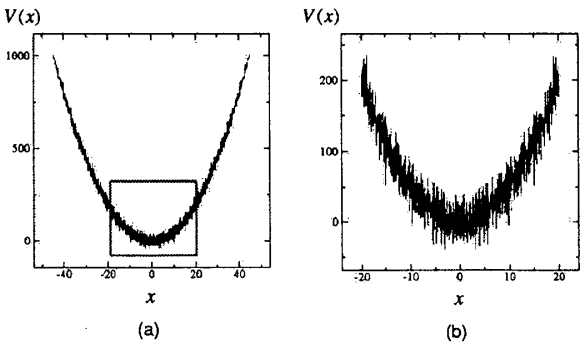

Fig.1: The constructed potential $V(x)$ for reproducing 1000 energy levels of the GOE matrix. (a) $V(x)$ is plotted in whole range |x|<40. (b) $V(x)$ is plotted for |x| <20 to show its details.

We use both methods and check that the resulting potentials are indentical, considering the precision of the calculation. The resulting potential $V(x)$ is shown in Fig.1. The coarse averaged shape is a harmonic oscillator and there are oscillating ripples on it expectedly. We apply the unfolded GOE spectrum whose averaged density becomes unity. The oscillation has larger amplitude and finer structure near the origin: $x \approx 0$. The more target eigen-energies e_i we adopt, the wider range of the potential has the oscillation.

The fractal dimension of the shape of the potential is evaluated, using the box counting method [6]. Fig.2 shows that fractal dimension is 1.7 and in the range where the box size is too small, the dimension reduces to one discontinuously. The bending point moves to the region where the size of counting boxes is smaller. It means that larger number of eigen-energies we use, more finer ripples exists on the shape of the potential as the Fourier analysis [4]. It also implies that the potential of the chaotic eigen-energies at $N \to \infty$ becomes completely fractal.

Note that the method creates symmetric potentials with respect to the origin. We also check the universality of the fractal dimension, changing the GOE spectrum. Thus the fractal structure is the fundamental nature of our 1D system. Its discontinuous derivative ensures smaller and smaller dents in the potential. The wave functions in higher energy regime can be virtually localized and it would weaken the repulsion between energy levels. It allows smaller energy gaps, which never happens in the one-dimensional system, possible. The path integral interpretation of quantum mechanics tells us that there cannot exit the shortest classical periodic orbit if the system follows the RMT predictions for the level statistics [7]. However, finer ripples can create the local minima of the potential here and there, where smaller and even infinitesimal classical orbits can exist. Then our calculation shows that the 1D system which is still mathematically integrable can imitate the chaotic energy levels, if the fractal structure infinitely continues toward smaller scale.

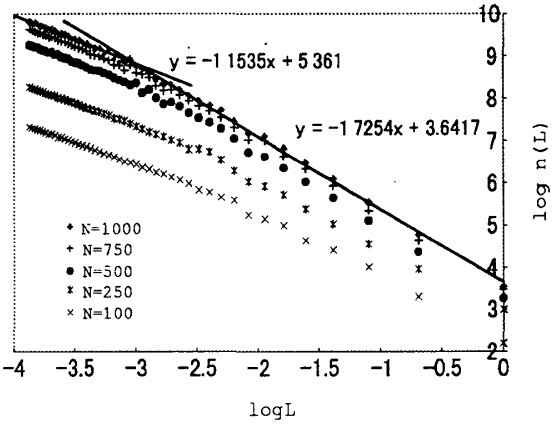

Fig.2: The logarithm of the number of the counting boxes $n(L)$ is plotted against the logarithm of the box size L for N=100, 250, 500, 750, 1000. The absolute values of the gradients of the fitted lines represent the fractal dimension.

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