Nonequilibrium Phenomena in Junction Systems

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電極につながれたメゾ系は無限量子系と考えることができ、その非平衡状態は C*-代数の方法で厳密に扱うことができる。詳細は参考文献に譲り、C*-代数の方法のエッセンスについて説明する。

1 Introduction

Recent progress in mesoscopic systems has opened a new direction in nonequilibrium statistical mechanics since an interplay of dynamical and statistical behaviors might be seen in these systems. One of the essential features of such systems is their openness, namely, the coupling with larger environments, and they can be well modeled by a finite system coupled with infinitely extended systems within the framework of the C*-algebraic approach[1].

The C* algebra was originally introduced to deal with phase transitions in quantum systems [1, 2] and nonequilibrium properties has also been rigorously investigated. Those include analytical studies of nonequilibrium steady states (NESS) of an isotropic XY-chain[3], a one-dimensional quantum conductor[4], systems with asymptotic abelianness[5, 6], an interacting fermion-spin system[7], fermionic junction systems[8], a quasi-spin model of superconductors[9], a bosonic junction system with or without Bose-Einstein condensates[10, 11]. Furthermore, some dynamical property of NESS was discussed[12]. Entropy production has been rigorously studied as well (see [13, 14, 15, 16, 17, 5], and the references therein).

In this article, the essential features of the C*-algebraic approach are briefly explained.

2 C*-Dynamical Systems

Contrary to the conventional quantum mechanics, C*-algebraic approach starts from a set $\mathcal{F}$ of finite observables, which is a Banach *-algebra and whose norm satisfies the C* condition:
(i) $\mathcal{F}$ is a Banach space with norm $\|\cdot\|$.
(ii) Products AB ($\forall A, B \in \mathcal{F}$) are defined and $\|AB\| \leq \|A\|\|B\|$.
(iii) An antilinear involution * is defined and it satisfies the C*-property: $\|A^*A\| = \|A\|^2$.

The time evolution is described by a strongly continuous one-parameter group of *-automorphisms τ_t ($t \in \mathbf{R}$), namely, τ_t is a linear map satisfying $\tau_t(AB) = \tau_t(A)\tau_t(B)$, $\tau_t(A^*) = \tau_t(A)^*$, $\tau_0 = I$ (I is the identity map), $\tau_t\tau_s = \tau_{t+s}$ and, for $\forall A$, $\|\tau_t(A) - A\| \to 0$ as $t \to 0$. Then, the theory of semigroup tells that the generator δ of τ_t is well-defined on a dense domain $D(\delta)$ and

$$\lim_{t\to 0}\left\|\delta(A)-\frac{1}{t}\{\tau_t(A)-A\}\right\|=0\ ,\quad (\forall A\in D(\delta))$$

Then, states are introduced by listing expectation values. Namely, each state is identified with a linear map ω from $A\in\mathcal{F}$ to an expectation value $\omega(A)$. The conditions $\omega(A^*A)\geq 0$ and $\omega(\mathbf{1})=1$ ($\mathbf{1}\in\mathcal{F}$ is the identity) are required. The former asserts the positivity of the expectation value of positive observables and the latter is the normalization condition. From the practical point of view, such a specification is natural since states are determined through measurements of physical quantities.

Canonical states play an important role in dealing with thermal properties. If the system is described by a finite dimensional Hilbert space, one can easily seen that the canonical average $\langle\cdots\rangle_c$ at inverse temperature β satisfies $\langle BA\rangle_c=\langle A\sigma_{i\beta}(B)\rangle_c$ where $\sigma_s(A)=e^{iHs}Ae^{-iHs}$ is a 'time evolution' generated by the Hamiltonian H and $\sigma_{i\beta}$ is the analytic continuation of σ_s to $s=i\beta$. This argument can be generalized to the C*-algebraic approach because the 'time evolution' can be defined as a strongly continuous one-parameter group. One then introduce an analog ω of a canonical state, called a σ-KMS (Kubo-Martin-Schwinger) state, as the one satifying $\omega\left(A\sigma_{i\beta}(B)\right)=\omega(BA)$ where σ_s is a strongly continuous one-parameter group of *-automorphisms and A, B are arbitrary elements of a norm dense *-subalgebra of $\mathcal{F}$.

An important aspect of this approach is that one can avoid infinite quantities. To show it explicitly, let us consider the Fano-Anderson model formally described by the Hamiltonian:

$$H=H_L+H_R+\epsilon_g c_\sigma^\dagger c_\sigma+H_T+H_{LR}\ ,\qquad (1)$$

$$H_\lambda=\int dk\omega_{k\lambda}a_{k\sigma\lambda}^\dagger a_{k\sigma\lambda}\quad (\lambda=L\ R)\qquad (2)$$

$$H_T=\int dk\{u_{kL}a_{k\sigma L}^\dagger c_\sigma+u_{kR}a_{k\sigma R}^\dagger c_\sigma+(\text{H.c.})\}\qquad (3)$$

$$H_{LR}=W\int dkdq\{u_{kL}u_{qR}e^{i\varphi}a_{k\sigma L}^\dagger a_{q\sigma R}+(\text{H.c.})\}\qquad (4)$$

where ϵ_g, W, φ are real parameters, $u_{k\lambda}$ ($\lambda=L,R$) is a square integrable function of the wave number k and the energy $\omega_{k\lambda}$ is an increasing function of $|k|$. The Fermionic operators $a_{k\sigma\lambda}$ and c_σ satisfy canonical anticommutation relations (CAR):

$$\{a_{k\sigma\lambda},a_{k'\sigma'\lambda'}^*\}=\delta_{\sigma\sigma'}\delta_{\lambda\lambda'}\delta(k-k')\ ,\quad \{c_\sigma,c_{\sigma'}^*\}=\delta_{\sigma\sigma'}\ ,\quad (\text{Other Anticommutators})=0\ .\quad (5)$$

These formal definitions have apparent difficulties. Firstly, $a_{k\sigma\lambda}$ is not bounded as its anticommutator may involve Dirac's delta function. Secondly, the Hamiltonian H is meaningless as the integrand of (2) is not bounded.

Here we observe two facts: Let f be square integrable $|f|^2\equiv\int dk\,|f(k)|^2<+\infty$, then an operator $a_{\sigma\lambda}(f)\equiv\int dkf(k)a_{k\sigma\lambda}$ has finite norm: $\|a_{\sigma\lambda}(f)\|=|f|$. Also, formal calculation gives

$$\delta\left(a_{\sigma L}(f)\right)\equiv i[H,a_{\sigma L}(f)]=-ia_{\sigma L}(\omega_L f)-i\int dkf(k)u_{kL}\left[c_\sigma+We^{i\varphi}a_{\sigma R}(u_R)\right]\qquad (6)$$

where $(\omega_L f)(k) \equiv \omega_{kL} f(k)$ and $(u_R)(k) \equiv u_{kR}$.

Now we turn to the C*-algebraic approach. The first difficulty mentioned after (5) can be removed by restricting ourselves to observables generated by bounded variables $a_{\sigma\lambda}(f)$ and c_σ, namely observables which can be approximated with arbitrary precision by a finite sum of products of finite number of $a_{\sigma\lambda}(f)$, c_σ and their adjoints. The set of these observables is nothing but $\mathcal{F}$. The second difficulty can be avoided by defining the time evolution τ_t as $\tau_t = e^{\delta t}$, where $\delta : \mathcal{F} \to \mathcal{F}$ is defined by the left-hand side of (6). As seen from (6), δ is well defined for $a_{\sigma\lambda}(f)$ if $\omega_{k\lambda} f(k)$ is square integrable and, actually, it has a dense domain $D(\delta)$. In this way, one can formulate the problem in terms of the finite observables and avoid the use of ill defined quantities such as the Hamiltonian H.

3 Nonequilibrium steady states of Fano-Anderson model

Within this framework, nonequilibrium steady states are rigorously constructed as the long-term limit of the state $\omega_0 \circ \tau_t$. The initial state ω_0 is a tensor product of the reservoir states at different temperatures β_λ^{-1} and chemical potentials μ_λ ($\lambda = L, R$) and is defined as a σ-KMS state at $\beta = -1$ with respect to the 'evolution' $\sigma_s = e^{\delta_\omega s}$ where the generator δ_ω corresponds to the commutator $-i[\sum_{\lambda=L,R} \beta_\lambda \{H_\lambda - \mu_\lambda N_\lambda\}, \ \cdot \ ]$ with $N_\lambda = \int dk a^*_{k\sigma\lambda} a_{k\sigma\lambda}$ the reservoir number operator. One has $\delta_\omega \left(a_{\sigma\lambda}(f)\right) = i a_{\sigma\lambda}\left(\beta_\lambda(\omega_\lambda - \mu_\lambda) f\right)$, for example, and $\omega_0 \left(A \sigma_{-i}(B)\right) = \omega_0(BA)$. Then, one can show

Proposition (Takahashi, ST[18])

If the system does not admit bound states, the limits $\lim_{t\to\pm\infty} \omega_0 \circ \tau_t(A) (\equiv \omega_\pm(A))$ exist for any $A \in \mathcal{F}$ and are quasi-free invariant states. The state ω_+ is characterized by the two-point function between the incoming fields $\beta_{k\sigma\lambda}$: $\omega_+(\beta^*_{k'\sigma'\lambda'} \beta_{k\sigma\lambda}) = F_\lambda(\omega_k) \delta_{\sigma'\sigma} \delta_{\lambda'\lambda} \delta(k' - k)$ where $F_\lambda(\omega_k)$ is the Fermi distribution with inverse temperature β_λ and chemical potential μ_λ. Moreover, ω_- is the time reversed state of ω_+: $\omega_- = \omega_+ \circ \iota|_{\varphi\to-\varphi}$ where ι is a time-reversal operation.

Evaluation of the long-term limits as the weak limits $\lim_{t\to\pm\infty} \omega_0 \circ \tau_t(A)$ is crucial for having the unidirectional evolution since $\|\omega_0 \circ \tau_t - \omega_\pm\|$ does not admit long-term limits and states can evolve towards the steady state only in the weak sense. This argument is natural from practical point of view since we can follow the state evolution only through measurements.

The above proposition implies the consistency between the dynamical reversibility and irreversible state evolution. To show it, we reinvestigate the thought-experiment by Loschmidt:

(i) The initial state ω_0 naturally evolves according to τ_t.

(ii) At time $t = t_0$, the time reversal operation ι and the phase inversion $\varphi \to -\varphi$ are performed.

(iii) Again, the state evolves naturally.

When t_0 is sufficiently large, we have $|\omega_0 \circ \tau_{t_0}(A) - \omega_+(A)| < \epsilon$ for any $\epsilon > 0$ and fixed $A \in \mathcal{F}$. Then, just after the time reversal operation $t = t_0 + 0$, $\omega_{t_0+0}(A) = \omega_0 \circ \tau_{-t_0}(A)$ and $|\omega_{t_0+0}(A) - \omega_-(A)| = |\omega_0 \circ \tau_{-t_0}(A) - \omega_-(A)| < \epsilon$. After $t = t_0$, one has $\lim_{t\to+\infty} \omega_{t_0+0+t}(A) = \lim_{t\to+\infty} \omega \circ \tau_{t-t_0}(A) = \omega_+(A)$. Thus, the natural evolution τ_t always derives the system towards ω_+ and the time reversal operation immediately brings back the state $\omega_0 \circ \tau_{t_0}$ near ω_+ to the one ω_{t_0+0} near ω_-. In short, the time reversal operation does not lead the reversed motion, but the sudden jump to the 'opposite' state and the dynamical reversibility is fully consistent with the irreversible state evolution.

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