Anomalous fluctuations in the annulus billiard : Magnetic-field-induced bifurcations

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２重円ビリヤードに磁場をかけた系を考える。磁場を強くしていくと一部の軌道にタンジェント分岐が発生し、それによって状態密度や磁化に異常揺らぎが発生する。

We investigate energy spectrum and magnetization for the mesoscopic annular quantum dot in the presence of magnetic field. We elucidate anomalous fluctuations of magnetization, which will be explained in terms of magnetic-field-induced orbital bifurcations.

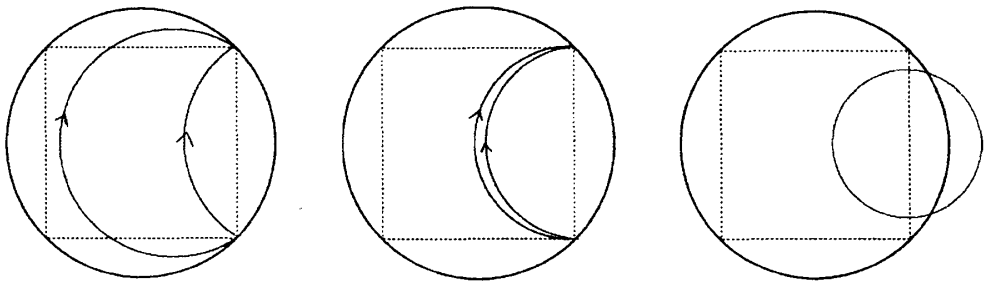
We consider the annulus billiard with R_1 and R_2 as the radii of the outer and inner disks, respectively. The magnetic field B is assumed to be applied perpendicularly to the billiard. The system is described by the Hamiltonian

$$H=\frac{1}{2m}\left(\vec{p}+\frac{e}{c}\vec{A}\right)^2, \qquad \vec{A}=\frac{B}{2}(-y,x,0). \tag{1}$$

For the electron with energy $E=(\hbar k)^2/2m$, its cyclotron radius and frequency are $R_c=\hbar k/eB$ and $\omega_c=eB/m$, respectively.

In the annulus billiard in B field, there are bouncing orbits and the cyclotron orbits. Bouncing periodic orbits are classified into three types 1-3: types 1,2 and 3 consist of periodic orbits colliding with, both inner and outer walls, the inner one only, and the outer wall only, respectively. The cyclotron orbits (type 4) can be classified into two types: one that winds around the inner disk (type 4-a), and one that doesn't (type 4-b). The type 4-a orbit exists when $R_2<R_c<R_1$, and the type 4-b orbit exists when $R_c<(R_1-R_2)/2$.

First we describe orbit bifurcations (FIG.1). With increase of B field, a pair of POs become degenerate and disappear at $R_c=R_1\sin\theta$ $(\theta=\pi/n,\ n=3,4,\ldots)$,which is a phenomenon of tangent bifurcations. Type 2 orbits also bifurcate at $R_c=R_2\sin\theta$. At the bifurcation points, the normal trace formula diverges.



(a) (b) (c)

FIG.1 Bifurcation of type 1 orbit : $R_c>R_1\sin\theta$ (a), $R_c\simeq R_1\sin\theta$ (b), $R_c<R_1\sin\theta$ (c), where $\theta=\pi/4$ in this case. (Here we neglect the inner disk for simplicity.)

At the bifurcations, the trace formula should be improved. In the uniform approximation, the trace formula incorporating the effect of the interference is given as

$$\delta g(E)=\frac{1}{\pi\hbar}\left|\frac{2\pi\Delta S}{3\hbar}\right|^{1/2}\left(\bar{A}\cos\left(\frac{\bar{S}}{\hbar}-\frac{\pi}{2}\bar{\nu}\right)\left(J_{-1/3}\left(\frac{|\Delta S|}{\hbar}\right)+J_{1/3}\left(\frac{|\Delta S|}{\hbar}\right)\right)\right. \tag{2}$$

$$\left.-\mathrm{sign}(\Delta S)\Delta A\sin\left(\frac{\bar{S}}{\hbar}-\frac{\pi}{2}(\bar{\nu}-1)\right)\left(J_{-2/3}\left(\frac{|\Delta S|}{\hbar}\right)+J_{2/3}\left(\frac{|\Delta S|}{\hbar}\right)\right)\right),$$

before the bifurcation .Exactly at the bifurcation point,

$$\delta g(E)=\frac{T_0\Gamma_{1/3}}{\pi v\sqrt{6\pi}\hbar^{5/3}a^{1/3}}\cos\left(\frac{S_0}{\hbar}-\frac{\pi}{2}\nu-\frac{\pi}{2}\sigma\right). \tag{3}$$

This trace fomula has been derived by Schomerus and Sieber (1997). By comparing (3) with the normal trace formula, we see that the amplitude is enhanced by $\hbar^{-1/6}$, which might leads to novel shell structure.

Next we describes the magnetization of this system at finite temperature. FIG.2 shows the magnetization in the B-k plane at $\epsilon\equiv R_2/R_1=0.1$. In this case, the cyclotron orbit is dominant in the strong field regime. At $\epsilon=0.4$ (FIG.3), the Landau-level bunching gets weaker, and there appear novel shell structure corresponding to the bifurcations of the type 2 orbits. (The dotted lines show the Landau levels and the broken lines show the bifurcation lines.)

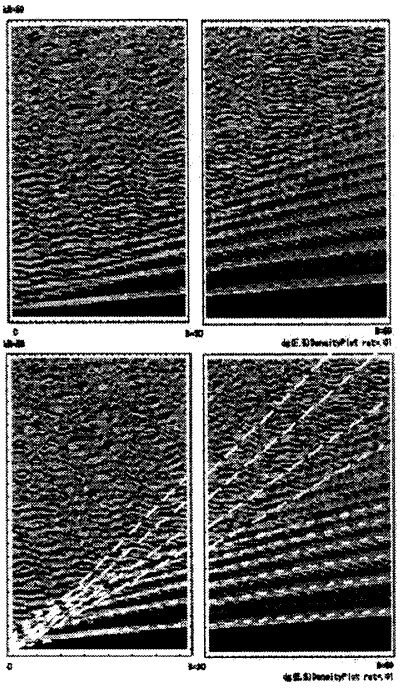


FIG.2 $\epsilon=0.1$

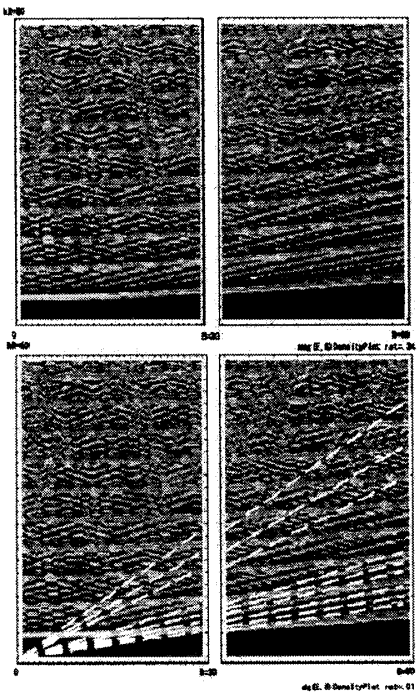


FIG.3 $\epsilon=0.4$

This presentation is based on the work by D.Hotta, T.Okada, A.Sugita, and K.Nakamura.