Contact and Quasi-static Impact of a Dissipationless Mechanical Model

Dept. of Chemistry, Kyoto Univ. Hiroto Kuninaka [1]
Dept. of Physics, Kyoto Univ. Hisao Hayakawa [2]

これまでに行った弾性体の衝突シミュレーションにおいて、ばねー質点系のハミルトンモデルが理論や実験から予測される結果を得られることがわかっているが、接触や低速の正面衝突に関してはモデル自体が散逸を含まないために理論的な予測とは整合しないことがわかっている。ここでは新たに固体の格子欠陥を模倣した機構を導入したばねー質点系のハミルトンモデルによるシミュレーションにおいて、弾性接触や低速衝突の理論から予測される結果を得られることを示す。

The contacts between elastic bodies can be described by the Hertzian contact theory[1]. The two-dimensional Hertzian contact theory gives us the relation between the deformation of a disk δ and the compressive force P as

$$\delta \simeq \frac{P}{\pi E}\left\{\ln\left(\frac{4\pi ER}{P}\right) - 1 - \nu\right\}, \tag{1}$$

where R is the radius of the undeformed disk, E is Young's modulus, and ν is Poisson's ratio[2].

On the other hand, in the low speed head-on collisions of macroscopic materials, the relation between the restitution coefficient e and the impact speed v is described by the quasi-static theory of low speed impacts[3]. The quasi-static theory is an extension of the Hertzian contact theory to include the dissipative force. The dissipative force originates from the internal viscosity of materials and is described as a function of the velocity of deformation. By solving the equation of motion for the deformation with adequate initial conditions and calculating the rebound speed, we can obtain the relation between the impact speed v and the restitution coefficient e.

In our previous simulation of oblique impacts between an elastic disk and an elastic wall which are based on the mass-spring model, we have confirmed that the model can reproduce an early experimental result[4, 5] and the prediction from the phenomenological theory of oblique impacts[6]. However, the model cannot reach an equilibrium state described by the Hertzian contact theory because of the lack of the dissipation mechanism. In the simulation of low-speed impact, we also have found a discrepancy between our numerical result and the prediction by the quasi-static theory.

[1] E-mail: kuninaka@kuchem.kyoto-u.ac.jp
[2] E-mail: hisao@yuragi.kyoto-u.ac.jp

In our presentation, we show our refined model can reproduce the Hertzian contact theory and the quasistatic theory of low-speed impact by introducing some defects in our previous model (Fig.1-2)[7]. We also discuss the process to the equilibrium state in the model and the relation between the number of defects and the Shannon entropy calculated from the velocity distribution function of all the mass particles of the disk.

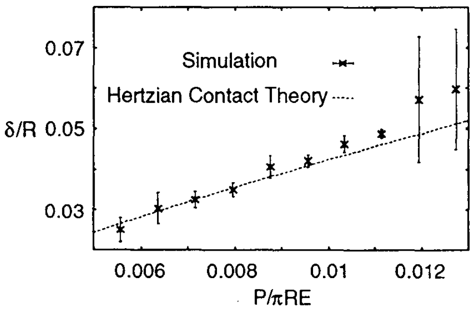

Figure 1: The relation between the deformation and the external force. The dotted line is eq.(1) with $\nu = 0.336$.

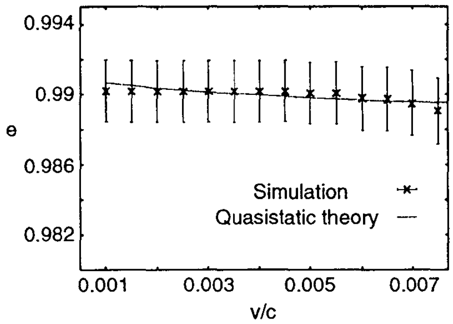

Figure 2: The relation between the colliding speed v and the restitution coefficient e. The dotted line indicates the prediction by the quasistatic theory.

References

[1] H. Hertz, J. Reine Angew. Math. **92** (1882), 156.

[2] F. Gerl and A. Zippelius, Phys. Rev. E **59** (1999), 2361.

[3] G. Kuwabara and K. Kono, Jpn. J. Appl. Phys. **26** (1987), 1230; N.V. Brilliantov, F. Spahn, J.-M. Hertzsch, and T. Pöschel, Phys. Rev. E **53** (1996), 5382.

[4] M. Y. Louge and M. E. Adams, Phys. Rev. E **65** (2002), 021303.

[5] H. Kuninaka and H. Hayakawa, Phys. Rev. Lett. **93** (2004), 154301 (see also Phys. Rev. Focus **14** (2004) Story 14); H. Hayakawa and H. Kuninaka, Phase Trans. **77**(2004), 889.

[6] H. Kuninaka and H. Hayakawa, J. Phys. Soc. Jpn. **72** (2003), 1655.

[7] H. Kuninaka and H. Hayakawa, cond-mat/0504770.