

# Deformation and buckling of polymer crusts formed by droplet evaporation

Dept. of Applied Physics, The University of Tokyo David Head <sup>1</sup>

## 1 Introduction

A partially-wetted droplet of a polymer or colloid solution may, under evaporation, exhibit a glassy *crust* on its surface, which subsequently deforms and possibly even buckles as an elastic solid [1]. This crust is an aggregation of solute that collects as the liquid-vapour interface recedes [2], possibly complicated by radial outward solvent flow [3] or Marangoni effects [4]. These same physical processes also influence deformation of the shell; however, in the quasi-static regime where the shell relaxation is much faster than evaporation, the solvent only appears in one sense: its incompressibility means that the volume between the crust and the substrate is fixed by the evaporation rate. Therefore any deformation or buckling of the crust must evolve under the constraint of a given volume. This is quite unlike typical shell problems, which only consider load-controlling methods of driving. To study this process therefore required the construction of a dedicated numerical procedure.

## 2 Methods and results

### 2.1 Numerical method

A simple finite element method code was constructed in the style of previous spherical shell work [5], only here we chose to adopt a *random* mesh, so as to avoid possible coupling between regular meshing and any buckling modes. For quasi-static deformation, the equilibrium configuration of the shell can be found by energy minimisation (here performed by a non-linear conjugate gradient algorithm), with the constraint of the given volume. Repeatedly re-equilibrating under incrementally smaller volumes generates a sequence of equilibrium states that mimics quasi-static deformation.

### 2.2 Simulation results

Only a single type of buckling has been observed in the simulations, namely a *snap-buckling* (also 'limit point' buckling) to an axisymmetry-breaking configuration with an inverted spherical

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<sup>1</sup>E-mail: david@rheo.t.u-tokyo.ac.jp

surface surrounded by a high strain density circular ‘rim’; see Fig. 1. This inverted region then expands as the volume decreases. Scaling theory in the style of Landau and Lifshitz [6] provides approximate expressions for both the critical buckling volume and the subsequent enlargement, in rough agreement with the simulations; see [7] for more details.

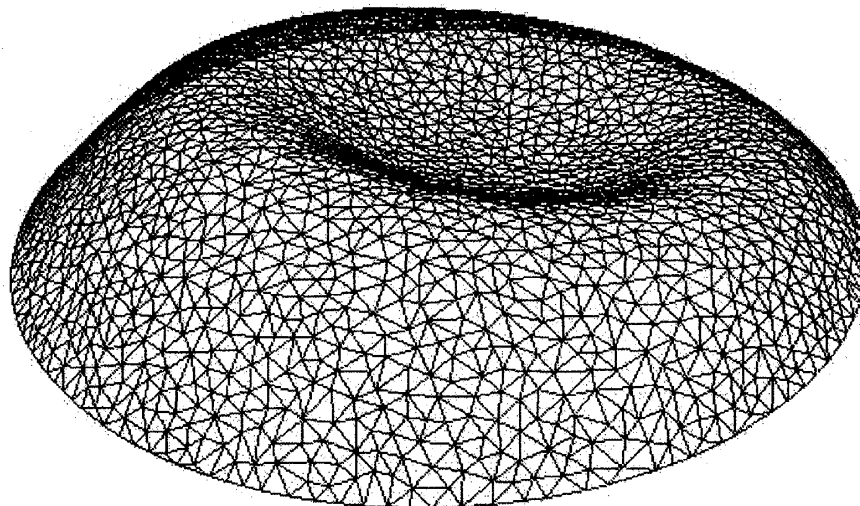


Figure 1: Example of a buckled shell configuration, shown as a wireframe so that the random meshing is also evident. The undeformed surface is spherical.

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## References

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