Velocity correlation of a lipid in the lipid-bilayer membrane at the equilibrium

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The velocity correlation function (VCF) of a tracer particle decays away as \( t^{-d/2} \) as the time \( t \) tends to the infinity in a \( d \)-dimensional equilibrium fluid[1]. The Brownian particle – a larger impurity particle – also has this long time tail[2]. A biomembrane is a two-dimensional (2D) fluid surrounded with three-dimensional fluids. A membrane protein can be regarded as a Brownian particle[3]. Sera & Rubi[4] showed theoretically that both \( t^{-1} \) and \( t^{-3/2} \) appear in its velocity correlation[4]. This is reasonable, considering that the momentum on the membrane spread out into the outer fluids. Seki & Komura[5] obtained similar results in a more simple way by introducing a phenomenological momentum relaxation time \( \tau \) which represents coupling strength between the membrane and the outer fluids. The VCF of the membrane protein was found to decay as \( e^{-t/\tau t^{-3/2}} \) or \( e^{-t/\tau t^{-1}} \) in the case of the strong- or the weak-coupling limits, respectively.

We study the VCF of the lipid molecule not a membrane protein. We assume that the lipid-bilayer membrane is a compressive 2D Newtonian fluid with the bending rigidity. It fluctuates about the equilibrium in the fluids on its both sides. Using unsteady Stokes approximation, we obtain the Eulerian VCF, \( \langle v(k, t) \cdot v(k', t') \rangle \), with the aid of the linear response theory[6]. Here, the angular brackets represent the equilibrium ensemble average, while \( v(k, t) \) denotes the Eulerian velocity field with \( k \) representing the wave number vector.

Writing \( V(t) \) for a lipid molecule velocity, apart from the fast decaying term, we have

\[
\langle V(t) \cdot V(0) \rangle \propto D(t) \equiv \int_0^{kU} dk e^{-D_0 k^2 t} \langle v(k, t) \cdot v(k, 0) \rangle
\]

(1)

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in terms of the mode-coupling theory. Here, $k_U$ is a upper cutoff wave number and $D_0$ is a 'bare' diffusion coefficient, which is independent of the hydrodynamics. We can evaluate its value in terms of the vacancy assisted diffusion[7].

Integrating right-hand side of (1) numerically by use of typical values, we obtain results shown as crosses in Fig.1. Two dotted lines with the slope of $-1$ and $-\frac{3}{2}$ are shown as guides. We can find that the VCF shifts from $t^{-1}$ to $t^{-\frac{3}{2}}$ as $t$ increases. We are now studying which factor gives the time around which the transition takes place.

![Figure 1: The behaviour of $D(t)$](image)

**References**


