Pulse Dynamics in a Model of Coupled Excitable Fibers

—— *A Variety of Patterns and Spatio-temporal Chaos* ——

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§1. Introduction

It has been identified that there is a rich variety of interactions among spatially localized patterns such as pulses and spots in a reaction-diffusion medium.[1] In addition, interactions among patterns in different reaction-diffusion media[2] should be also of interests from practical viewpoints. In fact, in several nerve systems, it is observed that huge nerve axons are arranged in a densely packed bundle so that pulses traveling in adjacent axons electronically communicate each other.[3] In this paper, we investigate what kinds of pulse dynamics can be emerged when two excitable reaction-diffusion media are coupled with each other, especially focusing on a situation in which parameters of two excitable fibers are not equal.[4] Such a situation is not uncommon because the diameters of fibers are not equivalent in the real nerve systems in general, leading to a difference of diffusion coefficients in mathematical models.

§2. Model

As an illustrative example of coupled excitable fibers, we consider the following mutually coupled one-dimensional FitzHugh-Nagumo (FHN) equations:

$$\begin{cases} \dot{u}_1 = u_1(u_1-\alpha)(1-u_1) - v_1 + \kappa_1\nabla^2 u_1 + \epsilon(u_2 - u_1) \\ \dot{v}_1 = \tau(u_1 - \gamma v_1), \end{cases}$$
$$\begin{cases} \dot{u}_2 = u_2(u_2-\alpha)(1-u_2) - v_2 + \kappa_2\nabla^2 u_2 + \epsilon(u_1 - u_2) \\ \dot{v}_2 = \tau(u_2 - \gamma v_2). \end{cases} \qquad (2\cdot1)$$

Subscripts "1" and "2" denote the first and the second fibers. The state variables $u_{1,2} = u_{1,2}(x,t)$ and $v_{1,2} = v_{1,2}(x,t)$, where $x \in [0, L]$ and $t \in [0, \infty)$ are space and time coordinates, are the activator and the inhibitor, respectively. The parameters κ_1 and κ_2 are diffusion coefficients. The value of κ_1 is fixed as 0.25 throughout this paper. The mutual interaction between two excitable fibers is also introduced as the linear coupling terms $(u_{1,2}(x,t) - u_{2,1}(x,t))$ with the strength ϵ for activators. We take ϵ and κ_2 as the control parameters. The periodic boundary condition is employed.

§3. Simulation Results

3.1. A Variety of Patterns

We investigate pulse dynamics when a right-moving pulse is initiated in fiber 1 and fiber 2 is set to the global resting state, as initial conditions. The parameters of the reaction kinetics is fixed as $\alpha = 10^{-1}, \gamma = 2.5$, and $\tau = 2 \times 10^{-3}$ so that local dynamics exhibits an excitable property.

Soliton-like Pulse Collision

Focus on the case in which diffusion coefficients are different: $\kappa_1 = 0.25$ and $\kappa_2 = 0.09$. When the inter-fiber coupling strength ϵ crosses over a threshold value, a propagating pulse in fiber 1 can induce an excitation in fiber 2. A pulse in fiber 2, however, cannot induce an excitation in fiber 1. This "one-way" excitation is understood as follows. In general, the existence of the diffusion term $\nabla^2 u$ in the equation of the activator suppresses the excitation by an external stimulus. Because we consider the case $\kappa_1 > \kappa_2$ in Eqs. (2·1) now, a more intensive stimulus from fiber 2 is required for inducing an excitation in fiber 1. A typical pattern for $\epsilon = 8 \times 10^{-3}$ caused by this one-way excitation is shown in Fig. 1:

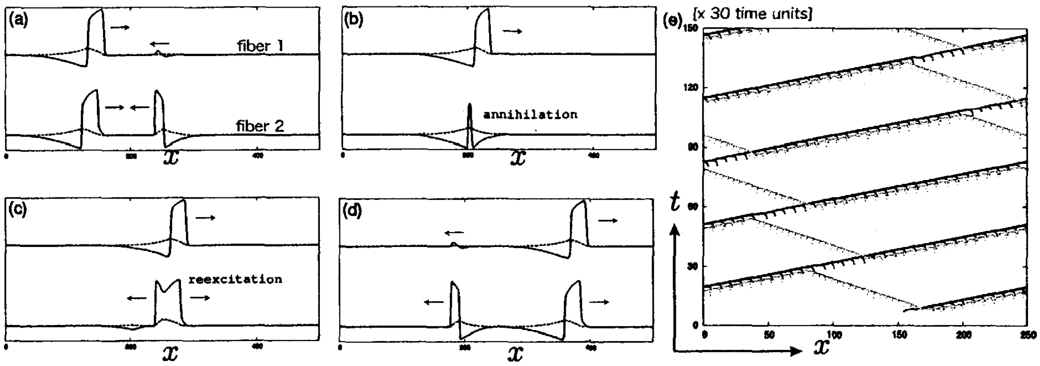

Fig. 1. (a–d) A series of snapshots for the soliton-like pulse collision. (e) A spatio-temporal plot.

(a) Head-on collisions occur in both fibers.
(b) Supra-threshold pulses annihilate each other in fiber 2 through the collision, whereas the sub-threshold pulse in fiber 1 does not significantly affect the propagation of supra-threshold pulse in fiber 1.
(c) A new supra-threshold excitation is induced in fiber 2 by the pulse in fiber 1, and it splits into two pulses propagating in opposite directions.
(d) All profiles are recovered after head-on collisions like *solitons.*

Recombination of Synchronized Pulses

In Fig. 1 (a), two pulses are facing with each other in fiber 2. By changing initial conditions, we can prepare two pulses propagating in the same direction in the fiber 2 as shown in Fig. 2 (a). We find the following dynamical behavior associated with the destruction of synchronized pulses as shown in Fig. 2:

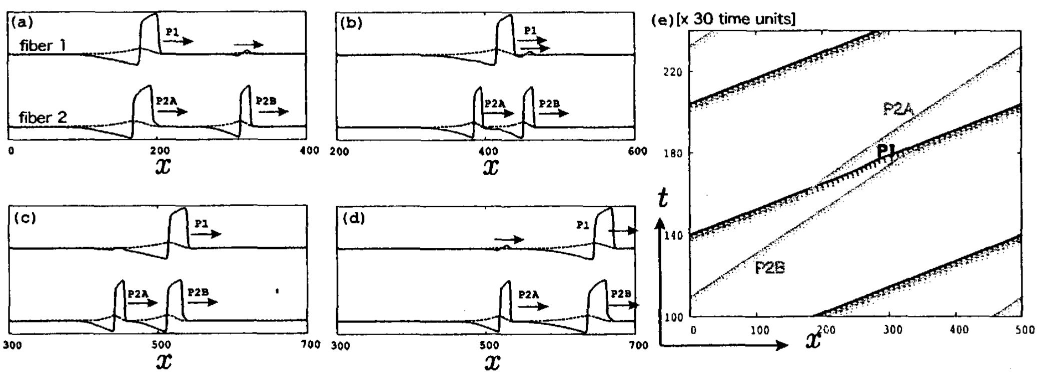

Fig. 2. (a–d) A series of snapshots for the recombination process. (e) A spatio-temporal plot.

(a) A pair of synchronized pulses P1 and P2A becomes close to P2B.
(b) Synchronization is broken by the highly concentrated region of the inihibitor behind P2B.
(c) A new pair of synchronized pulses composed of P1 and P2B forms.
(d) The new synchronized pulses move away from P2A.

We call dynamical processes *recombination* of synchronized pulses.

3.2. Spatio-temporal Chaos

The following parameters $\alpha = 5 \times 10^{-3}, \gamma = 0.5$, and $\tau = 5 \times 10^{-3}$ admit the "oscillating wake" of propagating pulse, since the eigenvalues of the Jacobi matrix at the resting point $(u, v) = (0, 0)$ has an imaginary part. For such parameter values of the reaction kinetics, interesting dynamical behaviors are observed.

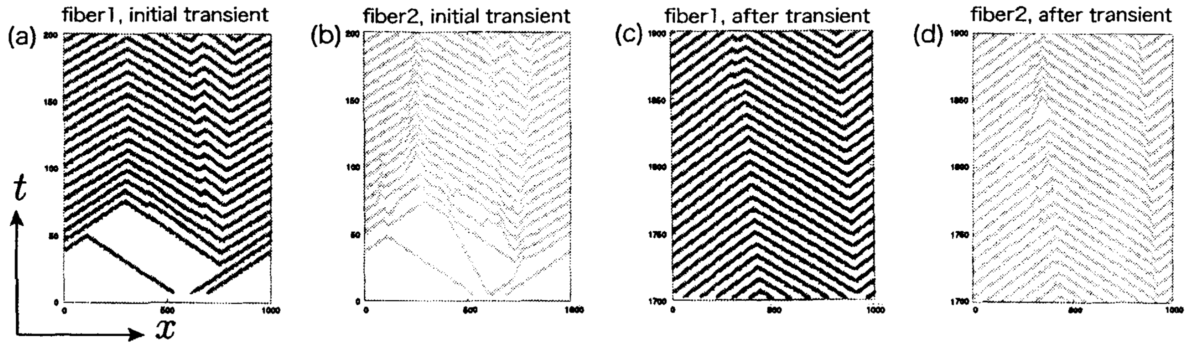

Fig. 3. Spatio-temporal plots at initial stage in (a–b), and after transient in (c–d). $L = 10^3$.

Figures 3 (a) and (b) show spatio-temporal patterns at initial stage of the system of Eqs (2·1) for $\Delta\kappa = 0.218$ and $\epsilon = 5 \times 10^{-3}$. If $\Delta\kappa = 0$, a stable reentrant wave is observed for $\epsilon = 5 \times 10^{-3}$. In Fig. 3 (a), the "V"-shaped structure corresponds to the event of a supra-threshold excitation splitting into two propagating pulses and the "Λ"-shaped structure corresponds to the event of an annihilation between two pulses. It is seen from Fig. 3 (a) that two sources of splitting pulses are generated at initial stage. After transient, however, only one of the two remains as shown in Figs. 3 (c) and (d).

In Fig. 3 (d), some disordered defects are observed near the annihilation locations even after a transient died out. In order to understand the origin of such disordered structures, we investigate the dependence of the return map $\eta(t)$ versus $\eta(t + t_s)$ on $\Delta\kappa$ after a transient, where $\eta(t)$ is a spatially coarse-grained variable: $\eta(t) = \sqrt{(1/L)\int_0^L |u_1(x,t) - u_2(x,t)|^2 dx}$. Results are shown in Figs. 4 (a–c), and corresponding power spectra are also plotted in their insets. Here, we take $t_s = 30$. When $\Delta\kappa$ is small, the return map shows a closed curve, which indicates that the dynamics of $\eta(t)$ is a periodic motion. For larger $\Delta\kappa$, the dynamics of $\eta(t)$ becomes more complicated, and the change of results from Fig. 4 (a) to Fig. 4 (c) suggests that the system shows a quasiperiodicity route to chaos.

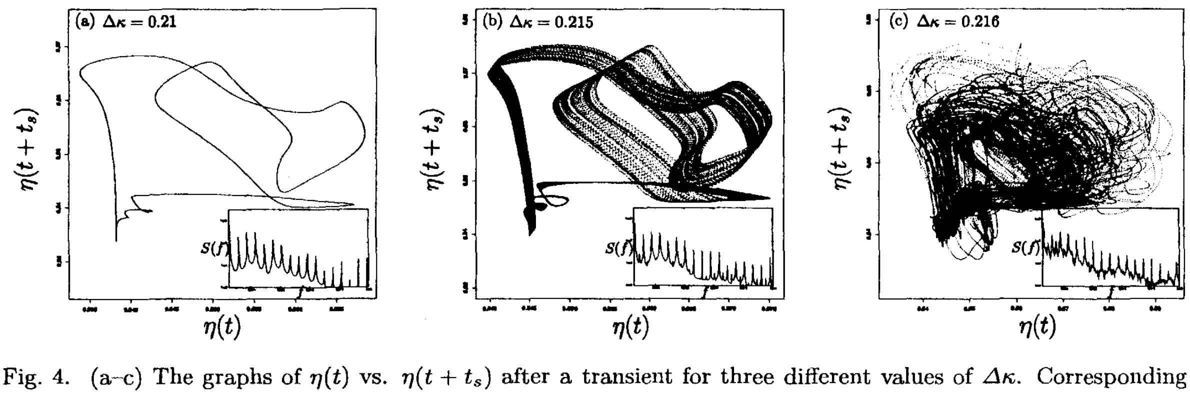

Fig. 4. (a–c) The graphs of $\eta(t)$ vs. $\eta(t + t_s)$ after a transient for three different values of $\Delta\kappa$. Corresponding power spectra are also shown in insets.

References

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