Predicting Synchronization of an Electronic Genetic Network

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1. Introduction

Synchronization is an ubiquitous phenomenon of coupled nonlinear oscillators, which are common in natural science and engineering. One of the greatest theoretical developments of this field is the phase equational modelling of weakly coupled limit cycle oscillators [1]. To apply the theory of phase models for the analysis of real-world data, it is indispensable to extract the phase equations from experimental measurement data [2, 3, 4]. The aim of this Presentation is to demonstrate a parameter estimation technique to the problem. We introduce a multiple shooting method to estimate parameters of the phase equations and apply it to an electronic genetic network.

2. Problem and Method

Problem formulation: Consider a system of weakly coupled N limit cycle oscillators:

$$\dot{x}_i \;=\; F(x_i) + \delta F_i(x_i) + C\,G(x_1, x_2, \cdots, x_N) \tag{1}$$

$(i = 1, \cdots, N)$ (F: system dynamics, δF heterogeneity, G: interaction, C: coupling strength), whose dynamics is well approximated by the phase equations [1]:

$$\dot{\theta}_i \;=\; \omega_i \;+\; C\sum_{j=1}^{N} H(\theta_j - \theta_i). \tag{2}$$

Suppose that simultaneous measurement of all oscillators $\{x_i(t)\}_{i=1,..,N}$ is made at a coupling strength C in a non-synchronous regime. *Under these conditions, predict the regime of synchronization in the parameter space of coupling strength C.*

Method:

1. Extract phases $\theta_i(t)$ from data $x_i(t)$.

2. Fit the phases $\{\theta_i(t)\}$ to the phase equations:

$$\dot{\theta}_i \;=\; \omega_i \;+\; C\sum_{j=1}^{N} \tilde{H}(\theta_j - \theta_i), \tag{3}$$

$$\tilde{H}(\Delta\theta) \;=\; \sum_{j=1}^{m} a_j \sin j\Delta\theta + b_j \cos j\Delta\theta, \tag{4}$$

by estimating the unknown parameters $p = \{\omega_i, a_j, b_j\}$ via multiple-shooting method [5]. Note that the interaction function H, which is in general nonlinear, is approximated by the Fourier expansion up to the order of m. In the multiple-shooting, we denote time evolution of the phase equations (3),(4) with respect to initial condition $\theta(0)$ by $\theta(t) = \phi^t(\theta(0), p)$. Then, the following nonlinear equations:

$$\theta(t_i + \Delta t) = \phi^{\Delta t}(\theta(t_i), \mathbf{p}) \tag{5}$$

($t_1, t_2, \cdots, t_m$: time points, Δt: sampling time interval) are solved by the Newton method. For the computation of the gradients $\partial\phi/\partial(p)$, which are needed to solve the Newton method, variational equations of the phase equations (3),(4) are integrated numerically.

3. Determine the optimal order of the Fourier series m by the cross-validation method [6].

4. Using the estimated parameters p, simulate the phase equations (3),(4) to detect the coupling strength C, which gives rise to synchronous regime.

3. Application to Electronic Genetic Network

We apply our method to electronic genetic network, which is composed of weakly coupled 16 circuit oscillators [7]. The following figures show our estimation results. In particular, Fig. 2 shows synchronization diagram of the experimental system (solid line) and the model prediction (dotted line). We see that the model predicts the synchronization structure of the experimental system in a fairly good precision.

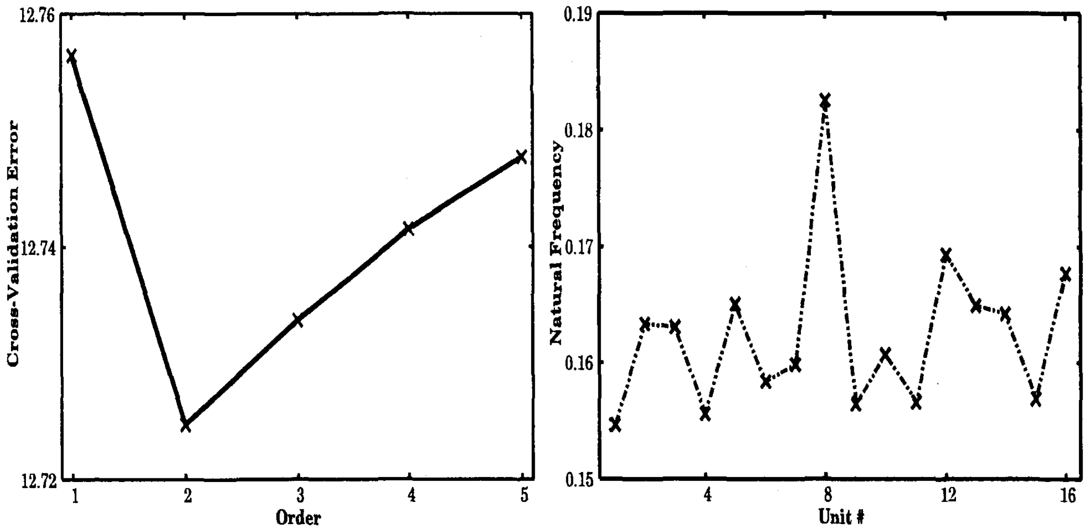

Fig. 1 *Left:* Result of the cross-validation test, pointing the optimal Fourier order at $m = 2$. *Right:* Estimated natural frequencies $\{\omega_i\}_{i=1}^{16}$.

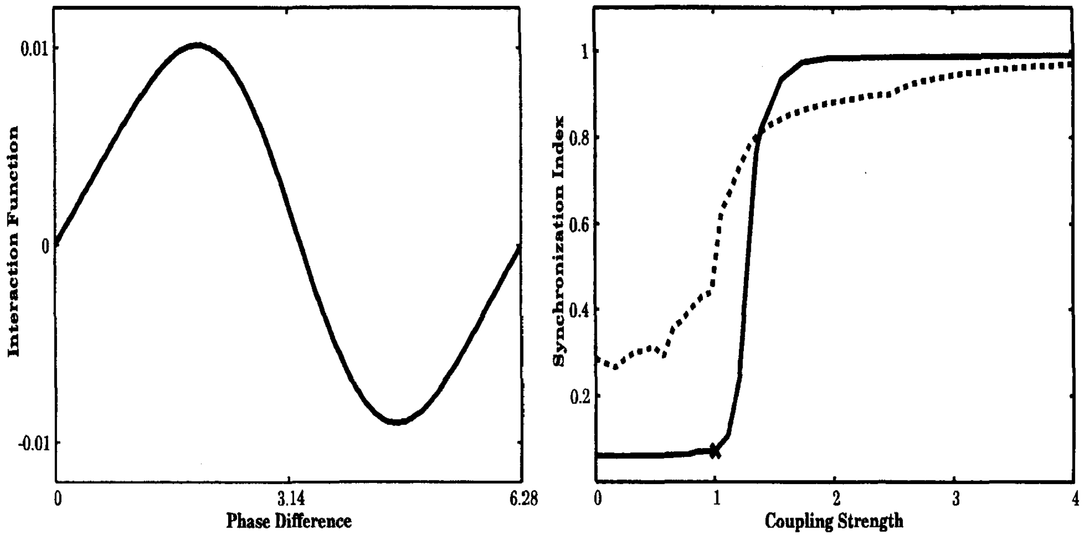

Fig. 2 *Left:* Estimated interaction function $H(\Delta\theta)$. *Right:* Synchronization diagram of the experimental system (solid line) and the model prediction (dotted line). Location of the measurement data used for the parameter estimation is indicated by the cross.

4. Summary

Multiple shooting method has been applied to the parameter estimation of phase equations for experimental data. Application to electronic genetic network has shown that the present approach is good enough to predict the regime of synchronization from a single measurement data. Advantage of this approach is that it can be applied to network systems and that all parameters are estimated simultaneously. Moreover, extension is possible to the case of nonuniform coupling $C_{i,j}$ as well as inhomogeneous interaction function $H_i(\Delta\theta)$. It is important to note that the estimation results depend crucially upon (I) coupling strength C set for the measurement data and (II) network size. At least, C should be taken from non-synchronous regime. The detailed results will be reported elsewhere. A big future challenge is to apply the present approach to a network of real suprachiasmatic nucleus (SCN) neurons.

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