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Control Parameter Dependence of the Work in a Process under Stochastic Control

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The second law of thermodynamics tells that the quasistatic process minimizes the macroscopic work done to a system among all the isothermal process starting from one of its equilibrium state to another. This can be reproduced by the stochastic energetics in terms of the classical Langevin dynamics. Mesoscopically, we can control the external force by observing one or some of the fluctuating physical quantities. This control can reduce the minimum of mechanical work, which does not include the work needed for the control. We are interested in how the minimum depends on the kinds of physical quantities observed for the control.

We study a one-dimensional shift of a point-charge, with the charge \( q \) and the mass \( m \), for a given distance \( a \) and during a given time-lapse from \( t = 0 \) to \( t_f \). We control an external electric field \( E \) to minimize the mechanical work done to the point-charge. Its position \( x \) follows

\[
m \frac{d^2 x(t)}{dt^2} = -m \gamma \frac{dx(t)}{dt} + qE(t) + \xi(t)
\]

where the noise \( \xi \) is assumed to be Gaussian white. The control theory requires definition of the evaluation functional \( J[E] \). Although this should be the mechanical work done to the point-charge in our formulation, we push other two terms into the functional. One represents the constraint on the strength of the external field, which is required to convert the functional into a positive quadratic form. The other represents the constraint at the end-point, which is introduced to avoid two-point boundary-value problem. Thus, we define

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\[ J[E] = \left( \int_0^{t_f} qE(t) \frac{dx(t)}{dt} + R_1 E(t)^2 dt + R_2 (x(t_f) - x_f)^2 \right) \]

where \( R_1 \) and \( R_2 \) are control parameters without physical meanings, and \( x(t) \) is stochastically determined by \( E(\tau) \) with \( \tau < t \). Because of (1).

Let us first observe both position \( x \) and velocity \( \dot{x} \) of the point-charge \( \dot{\mathcal{C}} \), and the solution of the variational problem is given by a set of simultaneous equations of (1) and (2).

\[ E_{opt}(t) = -\frac{q}{R_1} \left[ p_1(t)x(t) + \left( p_2(t) + \frac{1}{2} \right) \frac{dx(t)}{dt} \right] \]

where \( p_1(t) \) and \( p_2(t) \) satisfy Ricatti equations, appearing in the standard control theory. Equation (3) represents how we control \( E(t) \) by observing \( x(t) \) and \( \dot{x}(t) \) to minimize \( J[E] \).

The limit of \( R_1 \to 0 \) and \( R_2 \to \infty \), where the strength of \( E \) is not constrained and the boundary condition at \( t = t_f \) is imposed, is physically meaningful. Our numerical results are shown below. As \( R_1 \to 0 \) with a large enough \( R_2 \) value, the time average of \( E^2 \) increases as expected (Fig.1a), while the mechanical work \( W = \int dt qE(t)\dot{x}(t) \) approaches a finite value (Fig.1b). This finite value is found to remain unchanged as \( R_2 \) increases. We thus find physically meaningful minimum of the mechanical work to be well-defined, and to be smaller than the increase of the free energy during this process.

![Figure 1: Cheap-Control Limit](image)

The control above utilizes two observed quantities, \( x(t) \) and \( \dot{x}(t) \); Other control systems can be formulated by changing observed quantities. We find that the minimum increases when only \( x(t) \) is utilized. The difference may reflect that of some information quantity between the two control systems.

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References
