Domain induced budding in buckling membranes

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この研究において、私たちは２成分からなるやわらかい平面的な膜を想定し、その膜面上での相分離現象とバックリング現象とがカップルしたモデルを提案する。また、数値シミュレーションなどを用いて、そのダイナミクスや平衡状態について調べる。

1 Introduction

In this study, we consider fluid-like membranes and focus on the phase separation on the buckling membranes to understand the budding and the coarsening on membranes.

2 Model equation

We assume that the membrane is initially not deformed, and set this as a reference state and set the z-axis of the Cartesian coordinate (x, y, z) perpendicular to the membrane. A displacement vector $(\mathbf{u}, h) = (u_x, u_y, h)$ is also introduced to describe elastic deformation of the membrane (see Fig. 1).

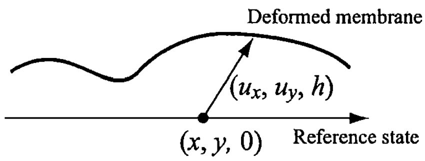

Figure 1: Reference coordinate $(x, y, 0)$ and deviation vector (u_x, u_y, h).

In this situation, the elastic energy $\mathcal{F}_{\mathrm{el}}$ and the free energy of the phase separation $\mathcal{F}_0$ are given by

$$\mathcal{F}_{\mathrm{el}} \approx \int d\mathbf{r} \left[\frac{\lambda}{2} \left(\bar{e} + \frac{1}{2} \left\langle (\nabla h)^2 \right\rangle \right)^2 + \frac{\kappa}{2} \left(\nabla^2 h \right)^2 \right]. \tag{1}$$

$$\mathcal{F}_0 \approx \int d\mathbf{r} \left[\frac{r}{2} \phi^2 + \frac{u}{4} \phi^4 + \frac{C}{2} \left(\nabla_s \phi \right)^2 \right], \tag{2}$$

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where ϕ is the order parameter and r and u are constant parameters. λ and κ mean the surface tension and the bending coefficient. $\bar{e}$ is an applied extension or compression of the membrane. If $\bar{e} < 0$, the membrane is buckled. The third term of eq (2) is the gradient energy evaluated on the deformed surface.

The total free energy is written as

$$\mathcal{F} = \mathcal{F}_{\mathrm{el}} + \mathcal{F}_0. \tag{3}$$

The dynamic equation of h and ϕ are written by

$$\tau_h \frac{\partial h}{\partial t} = -\frac{\delta \mathcal{F}}{\delta h}, \tag{4}$$

$$\tau_\phi \frac{\partial \phi}{\partial t} = \nabla_s^2 \frac{\delta \mathcal{F}}{\delta \phi}. \tag{5}$$

3 Results

We show the results of numerical simulation for $\bar{e} = -0.001$ and $\langle \phi \rangle = -0.3$ in figure 2. In this

(A) $t = 1500$ (B) $t = 9400$ (C) $t = 30000$

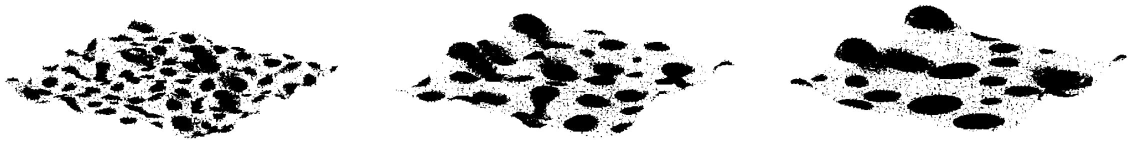

Figure 2: Phase separation on a buckled membrane ($\bar{e} = -0.001$) with average composition $\langle \phi \rangle = -0.3$. Black and white area show $\phi > 0$ and $\phi < 0$ respectively. Height of the membrane is twice exaggerated.

case, the membrane is compressed because $\bar{e}$ is negative. Therefore, the domain budding can be observed at $t = 9400$. The minority domains form caps and the majority domains become flat (see figure 2 (C)).

References

[1] A. Minami and K. Yamada, Eur. Phys. J. E (in press).