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Domain induced budding in buckling membranes

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1 Introduction

In this study, we consider fluid-like membranes and focus on the phase separation on the buckling membranes to understand the budding and the coarsening on membranes.

2 Model equation

We assume that the membrane is initially not deformed, and set this as a reference state and set the z-axis of the Cartesian coordinate (x, y, z) perpendicular to the membrane. A displacement vector \((u, h) = (u_x, u_y, h)\) is also introduced to describe elastic deformation of the membrane (see Fig. 1).

![Deformed membrane](image)

Figure 1: Reference coordinate \((x, y, 0)\) and deviation vector \((u_x, u_y, h)\).

In this situation, the elastic energy \(\mathcal{F}_{\text{el}}\) and the free energy of the phase separation \(\mathcal{F}_0\) are given by

\[
\mathcal{F}_{\text{el}} \approx \int dr \left[ \frac{\lambda}{2} \left( \tilde{e} + \frac{1}{2} \langle (\nabla h)^2 \rangle \right)^2 + \frac{\kappa}{2} (\nabla^2 h)^2 \right]. \tag{1}
\]

\[
\mathcal{F}_0 \approx \int dr \left[ \frac{r}{2} \phi^2 + \frac{u}{4} \phi^4 + \frac{C}{2} (\nabla \phi)^2 \right]. \tag{2}
\]

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where \( \phi \) is the order parameter and \( r \) and \( u \) are constant parameters. \( \lambda \) and \( \kappa \) mean the surface tension and the bending coefficient. \( \bar{e} \) is an applied extension or compression of the membrane. If \( \bar{e} < 0 \), the membrane is buckled. The third term of eq (2) is the gradient energy evaluated on the deformed surface.

The total free energy is written as

\[
\mathcal{F} = \mathcal{F}_\phi + \mathcal{F}_0.
\]

The dynamic equation of \( h \) and \( \phi \) are written by

\[
\tau_h \frac{\partial h}{\partial t} = -\frac{\delta \mathcal{F}}{\delta h},
\]
\[
\tau_\phi \frac{\partial \phi}{\partial t} = \nabla^2 \frac{\delta \mathcal{F}}{\delta \phi}.
\]

3 Results

We show the results of numerical simulation for \( \bar{e} = -0.001 \) and \( \langle \phi \rangle = -0.3 \) in figure 2. In this case, the membrane is compressed because \( \bar{e} \) is negative. Therefore, the domain budding can be observed at \( t = 9400 \). The membrane is deformed at the domain boundary. The minority domains form caps and the majority domains become flat (see figure 2 (C)).

References