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Why Class Differential in Educational Attainment has not been changed?: A Rational Choice Approach

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Why Class Differential in Educational Attainment has not been changed?:
A Rational Choice Approach
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1. Introduction
Since World War II, higher education has expanded in many industrial societies such as the United States, England, Germany and Japan. Less than 10 percent of students went on to university 50 years ago in Japan, but nowadays, nearly 40 percent of students do. In spite of the educational expansion, a class differential in educational attainment (CDEA) has been little changed in many industrial societies (Goldthorpe 2000). Because the expansion of service-class children going on to university is almost comparable with that of working-class children, the differential between them has been sustained in many industrial societies. Goldthorpe (2000) explains these phenomena, based on rational choice theory. According to him, there have been two factors: one raising CDEA and the other reducing it, and these two factors have offset each other. Therefore, CDEA has not changed. Goldthorpe (2000) argues that the factor reducing CDEA is a decline in educational cost. In England, for example, university fee was reduced after World War II. It raised the percentage of children who go on to university in both service and working classes, but it had also an effect to reduce CDEA, because the decline of educational cost gave an opportunity to more working-class children than service-class children. On the other hand, the factor raising CDEA, according to Goldthorpe (2000), is the increasing importance of education, in other words, increasing difficulty of entering the service class without university education. This gave incentive to go on to university to more service-class children than working-class children, because working-class children don't always want to enter the service class but service-class children do. These two factors offset each other and CDEA has not been changed. We call this Goldthorpe hypothesis.

Goldthorpe hypothesis has several minor flaws. Firstly, he doesn't prove a decline in educational cost reduces CDEA. Secondly, Goldthorpe hypothesis can't explain why CDEA has not been changed in Japan in spite of an increase in the cost of university education. Thirdly it is not clear how much the importance of university education has been increased since WW II. According to Takeuchi (1991), university education was already very important before WW II. Other factors might have had an effect to increase CDEA in Japan. The aim of this paper is to extend Goldthorpe hypothesis and make it
applicable to the Japanese case. Firstly, we will show that an increase in educational cost doesn't always entail an increase in CDEA. It sometimes reduces CDEA, because more service-class children sometimes lose an opportunity for university education than working-class children do owing to the increase in educational cost. Therefore the increase in educational cost might have had an effect to reduce CDEA in Japan. Secondly, we will show that the expansion of the service class has an effect to raise CDEA, based on the rational-choice model into which Goldthorpe hypothesis is slightly modified. These extensions of Goldthorpe hypothesis will help us to understand why CDEA has little been changed in Japan.

2. Effect of Change of Educational Cost

In this section, we will analyze a model in which it is determined only by resource whether a child goes on to university or not. If she/he has enough resource, the child goes on to university in our model. If she/he doesn’t, the child can’t. This assumption might be too simple, because other factors have an effect on her/his decision in reality. Owing to the simplicity, however, our model will help to understand the relationship between educational cost and resource distribution on the one hand, and the class differential in educational attainment, on the other hand.

2.1. Axioms of the First Model

Axiom 1: Class Structure

Class structure consists of three classes: service, working and under classes. The service class is the highest and the under class is the lowest.

Axiom 2: Exclusion of under class children from analysis

We analyze only the choices and actions of service- and working-class children and exclude those of underclass children from our analysis in this paper.

This is for simplicity, and Goldthorpe (2000) is also based on the same axiom.

Axiom 3: Measure of CDEA

The measure of CDEA between the service and working classes is odds ratio between them:
\[ OR = \frac{\frac{q_s}{1-q_s}}{\frac{q_w}{1-q_w}} \]

*OR*: odds ratio of CDEA between service and working classes.

*qs*: the ratio of service-class children who go on to university.

*qw*: the ratio of working-class children who go on to university.

We suppose that *qs* is larger than *qw* for simplification. It means that *OR* is always larger than 1.

From Axioms 2 and 3, it follows that we deal with only the odds ratio between the service and working classes. The reason why we pick up the odds ratio among several measures of a class differential is that Goldthorpe has used it in his books (Goldthorpe 1987, 2000, Erikson and Goldthorpe 1992).

**Axiom 4: Educational Cost**

Suppose that \( C_t \) is the cost of education at a time \( t \). If a child (or his/her family) has more resource than \( C_t \), he/she goes on to university.

**Axiom 5: Distribution of resource**

Suppose that the amounts of resource of service- and working-classes children are \( r_s \) and \( r_w \) respectively, and they have log-normal distribution with means \( m(r_s) \) and \( m(r_w) \), \( m(r_s) > m(r_w) \).

We would operationally confine the concept of resource to money, because it is easier to test our hypothesis.

**2.2. Decline of Educational Cost raises CDEA**

Figure 1 shows that the relationship between educational cost and the class differential in educational attainment (CDEA) between the service and working classes.
The dotted curve is resource distribution among service-class children, and the thick curve is that among working-class children. Suppose that educational cost is \( a_0 \) at a time \( t_0 \). Then only children who have more resource than \( a_0 \) go on to university. Suppose that the educational cost is reduced from \( a_0 \) to \( c_1 \) at a time \( t_1 \). Then children who have resource more than \( c_1 \) and less than \( a_0 \) newly obtain the chance of going on to university at a time \( t_1 \). As Figure 1 shows, the increase in the ratio of children who get the new chance at \( t_1 \) is larger among service-class children than among working-class children. In other words, \( q_s \) increases more rapidly than \( q_w \) at \( t_1 \). Therefore, the odds ratio is increased. If the educational cost is reduced furthermore from \( c_1 \) to \( c_2 \) at a time \( t_2 \), then more working-class children newly get the chance of going on to university than service-class children do. In this case, \( q_s \) increases more rapidly than \( q_w \) at \( t_2 \). Therefore, the odds ratio is decreased. In other words, when the price elasticity of the service class is higher than that of the working class, the decline in educational cost raises CDEA. The height of each curve shows price elasticity corresponding to the cost in Figure 1. Roughly speaking, therefore, when educational cost is extraordinarily expensive, few working class children go on to university and CDEA is very high. However, a little decrease in educational cost would raise CDEA, because it gives more service-class children the educational opportunity than working-class children. Only when educational cost is decreased until the price elasticity of the working class exceeds that of the service class, CDEA is reduced.

We will now inquire into what happens when educational cost is increased, as is the
case in Japan. As long as the price elasticity of the working class is higher than that of the service class, the increase in educational cost raises CDEA. Because more working-class children would lose an opportunity to go on to university than service-class children do, owing to the increase in educational cost. On the other hand, when the price elasticity of the service class is higher than that of the working class, increase in educational cost reduces CDEA.

The increase in educational cost sometimes has an effect to reduce CDEA. The cost of university education in Japan has been considerably high since WW II. When the cost is high, the price elasticity of the service class tends to be higher than that of the working class. We, therefore, give an alternative hypothesis to explain the Japanese case: the cost up of university education had an effect to reduce CDEA in post-WW II Japan.

3. Effect of Service-Class Expansion

In this section, we will give another model in which a child maximizes her/his expected utility. Service- and working classes children have utility functions different from each other. Service-class children try to maximize the probability of attaining the service class, but working-class children try to maximize the probability of attaining the service or working class. In other words, they try to minimize the probability of attaining to the under class, although we don't deal with under class children in this paper as mentioned earlier. It means that both classes' children try to minimize the probability of downward mobility. University education helps both service- and working classes children to raise the probability of attaining to the service class, if they succeed in entering and graduating a good university. Investing in education is risky, however, because they might fail in entrance into or graduation from a good university. If service- or working- class children enter into the labor market without investing in university education, the probability of downward mobility is lower than if they invest and fail in university education. This model also might be too simple, because it neglects some important factors including the cost and resource we analyzed in the previous section. It, however, will help us to understand the relationship between the service-class expansion and the class differential in educational attainment (CDEA).

3.1. Axioms of the Second model

We use Axiom 1, 2 and 3 in section 2 again. But, we don't Axiom 4 and 5.

Axiom 6: Choices of Children
Each child has the choice of investing in university education and staying in the educational track, or not, i.e. entering the labor market.

Axiom 7: Outcomes of Investment in Education
Investment in higher education has two outcomes: success or failure. Success means entering and graduating a good university successfully. Failure means failing in the entrance or graduation. We suppose that a child investing in university education can go on to a university whether she/he succeeds or fails, because we assume that even a child who failed can go on to a university that is of little help for upward mobility.

Axiom 8: Probability of Success
The subjective conditional probability of the future success in entering into and graduating from a good university, $\pi$ has a uniform distribution in each class. The assumption of a uniform distribution may be too strong, because it is well known that service-class children get better school records than working class children do on average. It is, therefore, more reasonable to assume that service-class children have the higher probability of success than working-class children do. We will relax this assumption in the next paper.

Figure 2 Decision Tree of Educational Attainment
Axiom 9: Probability of Class Attainment

If a child invests and succeeds in university education, the conditional probabilities of attaining to the service, working and under classes are $\alpha$, $1-\alpha$ and 0 respectively. If she/he fails, the conditional probabilities of attaining to the service, working and under classes are $\beta_1$, $\beta_2$ and $1-\beta_1-\beta_2$. If a child enters into the labor market, they are $\chi_1$, $\chi_2$ and $1-\chi_1-\chi_2$ in the same way. We suppose that $\beta_1$ is equal or smaller than $\chi_1$, and $\beta_2$ is smaller than $\chi_2$, because it seems that if a child fails in the investment of university education, it wouldn’t improve the probability of attaining the service class. Additionally, the probability of attaining to the working class seems larger when a child enters in the labor market directly, than when she/he tries to go on to university and fails. They are constants among all children at a given time $t$ for example, $\alpha$ has the same value for both service- and working- classes children. However, these probabilities vary as time passes because of structural or forced mobility.

Axiom 10: Utility Functions

A service-class child’s utility, $U_s$ is equal to the probability of his attaining to the service class. A working-class child’s utility, $U_w$ is equal to the probability of his attaining to the service or working class.

3.2. Conditions for Investing in University Education

We can now figure out the utilities of each class’s child when she/he invests in university education and when she/he enters into the labor market. If a service-class child invests, her/his expected utility is derived from Axioms as below;

$$U_s = \pi \alpha + (1-\alpha) \beta_1.$$  

Because the probability of attaining to the service class is $\alpha$ when she/he succeeds, and it is $\beta_1$ when she/he fails. If a service-class child enters into the labor market, the expected utility is:

$$U_s = \chi_1.$$  

From these two equations, we can derive the condition for investing in education:

$$\pi \alpha + (1-\alpha) \beta_1 > \chi_1.$$  

$$\pi > (\chi_1-\beta_1) / (\alpha-\beta_1).$$  

We suppose that $p_s = (\chi_1-\beta_1) / (\alpha-\beta_1)$. Then, the above equation is rewritten as below:

$$\pi > p_s \quad (1).$$
$p_s$ is the threshold value of investment in university education for service-class children. If $\pi$ is larger than $p_s$ then it is rational for them to invest and go on to university.

In the same way, we can calculate the utilities of a working class child and the condition for investing in university education. If she/he invests in higher education,

$$U_w = \pi + (1-\pi)(\beta_1 + \beta_2).$$

If he/she leaves from the educational track,

$$U_w = \chi_1 + \chi_2.$$

From the above two equations, we can derive the condition for investing education:

$$\pi + (1-\pi)(\beta_1 + \beta_2) > \chi_1 + \chi_2.
\pi > (\chi_1 + \chi_2 - \chi_1 - \chi_2) / (1 - \beta_1 - \beta_2).$$

We suppose that $p_w = (\chi_1 + \chi_2 - \chi_1 - \chi_2) / (1 - \beta_1 - \beta_2)$. Then, the above equation is rewritten as below:

$$\pi > p_w \quad (2)$$

### 3.3. Ratio of Going on to University

Equation (1) means that a service-class student who has a prospect of success in university education with a probability $\pi$ more than $p_s$ go on to university. The ratio of service-class children with $\pi$ more than $p_s$ is $1 - p_s$, because we have assumed $\pi$ has a uniform distribution in Axiom 8. Figure 3 shows the relationship between the ratio of children going on to university and $p_s$. The shadowed area shows the ratio of service class children who invest in education, because they have $\pi$ larger than $p_s$. In the same way, the ratio of working children going on to university is $1 - p_w$. In other words,

$q_s = 1 - p_s$, and

$q_w = 1 - p_w$, therefore

$$\frac{(1 - p_s)}{p_s} OR = \frac{(1 - p_w)}{p_w} \quad (3)$$

We can derive Equation (3) from Axiom 3.
3.4. Expansion of Service Class Raises Class Differential in Educational Attainment

We now will prove that the expansion of service class has an effect to raise the class differential in educational attainment (CDEA). Before proving it, we must add another axiom.

Axiom 10
When the ratio of service class increases $a$ times, $\alpha$, $\beta$, and $\chi$ also increase $a$ times proportionally.

Axiom 10 assumes that when the ratio of service class increases—in other words, structural or forced mobility happens—the subjective probabilities of attaining to the service class would increase proportionally.

Theorem: Expansion of Service Class Raises CDEA

If the ratio of the service class expands, that of working class doesn't change and that of the under class reduces, CDEA between the service and working classes increases. In other words, if $\alpha(t_1) = a \cdot \alpha(t_0)$, $\beta_1(t_1) = a \cdot \beta_1(t_0)$, $\gamma_1(t_1) = a \cdot \gamma_1(t_0)$, $\beta_2(t_1) = a \cdot \beta_2(t_0)$, $\gamma_2(t_1) = a \cdot \gamma_2(t_0)$, and $a > 1$, then $OR(t_1) > OR(t_0)$. $\alpha(t_0)$ is $\alpha$ at a time $t_0$, and similarly for $\beta_1(t_0)$, $\beta_2(t_0)$, $\gamma_1(t_0)$, $\gamma_2(t_0)$, $pS(t_0)$, $pW(t_0)$, and $OR(t_0)$.
Thus, the denominator of \( \frac{\partial}{\partial t} f(t_0) \) is larger than that of \( \frac{\partial}{\partial t} g(t_0) \).

The numerator of \( f(t_0) - g(t_0) \) equals \( (a-1)(\gamma_1(t_0) - \beta_1(t_0)) \geq 0 \), because Axiom 9 assumes \( \beta_1 \leq \gamma_1 \)

Thus, the numerator of \( f(t_0) \) is equal to or larger than that of \( g(t_0) \). \( \cdots(5) \)

The numerator of \( p_u(t_0) \) – that of \( p_u(t) \) equals \( \gamma_1(t_0) - \beta_1(t_0) \).

Thus, the numerator of \( p_u(t_0) \) is equal to or larger than that of \( p_u(t) \). \( \cdots(6) \)

From (5) and (6), we derive \( p_u(t_0) > p_u(t) \) \( \cdots(7) \).

From Equation (3),

\[
\begin{align*}
OR(t_0) &= \frac{1 - p_u(t_0)}{1 - p_u(t_0)} \quad \text{and} \quad OR(t) = \frac{1 - p_u(t)}{1 - p_u(t)}
\end{align*}
\]

Equation (4) shows that the numerator of \( OR(t_0) \) is equal to that of \( OR(t) \). Inequality (6) shows that the denominator of \( OR(t_0) \) is larger than that of \( OR(t) \). Therefore, \( OR(t_0) \) is larger than \( OR(t) \).

End of Proof.

This proof argues that the expansion of the service class doesn’t change the ratio of the service-class children who go on to university. Because the expansion raises both \( \alpha, \beta_1, \) and \( \chi_1 \) proportionally, the threshold value doesn’t change in the service class. On the other hand, the expansion of the service class has an effect to reduce the ratio of working-class children going on to university. Because of the service-class expansion, the risk of attaining to the underclass decreases, whether a working-class child goes on to university or not. However, the risk decreases more markedly when she/he doesn’t go on to university than when she/he does. If working class children go on to university and
success, there is no risk of attaining to under class. Their utility is 1, the maximum value in this model. Thus, the expansion of service class doesn't improve the condition of the working-class children who go on to university and success. It makes the decrease in the risk limited when working class children go on to university. This difference raises the threshold value $p_s$ and reduce the ratio of working-class children going on to university.

In fact, the service class has been expanded in Japan since WW II. Therefore, we can argue that the service-class expansion had an effect to raise CDEA in post-WW II Japan.

4. Conclusion

Goldthorpe hypothesis could explain the class differential in educational attainment (CDEA) in several European societies, but couldn't be simply applicable to Japan, because Japanese educational cost has been raised since WW II and university education was already very important before WW II. To explain CDEA in Japan, we extended and developed Goldthorpe hypothesis. Firstly we showed the increase in educational cost has an effect to reduce CDEA under certain conditions. Secondly, we demonstrated the expansion of the service class has an effect to raise CDEA. With this extension, Goldthorpe hypothesis is applicable to CDEA in Japan. It is needless to say that we need further empirical tests to refine or confirm the hypothesis.

5. References


Ojima, F., 1997, "Inequality of Educational Opportunity in Japan: How Gender and