# Observations of Tidal Strain of the Earth by the Extensometer (Part II) 

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1. Horizontal linear strains of the earth caused by the earth-tide and by the crustal deformations are being observed in kyoto University by the use of Sassa-type invar-line extensometers with a sensibility of $10^{-8} \sim 10^{-9} / \mathrm{mm}$. Some results of the observations at Osakayama ${ }^{1)}$ and Ikuno ${ }^{1)}$ have already been reported. In the present paper the tidal strains which were observed at the Makimine mine, Miyasaki Prefecture, and at Osakayama, were discussed more throughly than the first report, especially to the effects due to the oceanic tide.
2. Due to tidal generating potential $W_{2}$ of heavenly bodies the surface displacements $S_{\mathrm{r}}, S_{0}$, and $S_{\varphi}$ along the radius, colatitude and longitude respectively of the earth are given as follows:

$$
\left.\begin{array}{l}
S_{\mathrm{r}}=\frac{h}{g} \cdot W_{2}  \tag{1}\\
S_{\theta}=\frac{l}{g} \cdot \frac{\partial W_{2}}{\partial \theta} \\
S_{\mathrm{\gamma}}=\frac{l}{g \sin \theta} \cdot \frac{\partial W_{2}}{\partial \varphi}
\end{array}\right\}
$$

where $h$ is the Love-number, $l$ the Lambret number, $g$ acceleration of gravity, $\theta$ co-latitude and $\varphi$ longitude. While the value of $h$ is given to $h=0.60^{\circ}$ ) as the most probable value, deduced from Chandler's period of latitude variation and observations of tilting of the earth-tide. The Lambert's number $l$ has been deduced heretofore from the relation

$$
\begin{equation*}
L=1+k-l \tag{2}
\end{equation*}
$$

which is obtained from observation of the tidal change of latitude. According to Dr. Eiichi Nishimura, ${ }^{3)} L=? .20$ and $l=0.07$ when $k$ equals 0.267 , as calculated from Chandler's ${ }^{4}$ ) period. This value is for smaller than $l=\frac{3}{10} h=0.18$ which is a relation obtained when the earth is assumed to be an elastic sphere homogeneous and incompressible, although it was not possible to discuss this point because of insufficient of accurary of observations of latitude. On the other hand, Dr. Hitoshi Takeuchi ${ }^{5}$ ) has obtained theoretically the value of $l=0.080 \sim 0.082$ using a model of the structure of the earth obtainable from the velocity of seismic waves inside the earth.

However from our observations of the change in horizontal linear strain of the earth due to the earth-tide with sufficient accuracy we
obtained a most probable value of $l=0.05$ as reported in the previous paper. Taking the $x$-axis to the south and the $y$-axis to the west along the earth's surface, the components $e_{x x}, e_{y y}$ and $e_{x y}$ of horizontal linear strain will be expressed as follows:

$$
\left.\begin{array}{l}
e_{x x}=\frac{1}{a} \cdot \frac{\partial S_{\theta}}{\partial \theta}  \tag{3}\\
e_{y y}=\frac{1}{a \sin \theta} \cdot \frac{\partial S_{\varphi}}{\partial \varphi} \\
e_{x y}=\frac{1}{a} \cdot \frac{\partial S_{\varphi}}{\partial \theta}+\frac{1}{a \sin \theta} \cdot \frac{\partial S_{\theta}}{\partial \varphi}
\end{array}\right\}
$$

Also, the tidal generating potential $W_{2}$ of the semi-diurnal tide is given by the following expression:

$$
\begin{equation*}
W_{2}=a g A \sin ^{2} \theta \cos 2 \varphi \tag{4}
\end{equation*}
$$

In the case of the $M_{2}$-tide,

$$
\begin{equation*}
A_{M_{2}}=\frac{3}{2} \frac{M}{E}\left(\frac{a}{c}\right)^{3}\left(\frac{1}{2}-\frac{5}{4} e^{a}\right) \cos ^{4} \frac{I}{2} \tag{5}
\end{equation*}
$$

where $M$ is mas of the moon, $E$ the mass of the earth, $c$ the average radious of the moon, $e$ the eccentricity of the moon's orbit, $I$ the inclination of the moon's orbit to the equotor. From Eqs. (1), (3) and (4), the components of strain are given as follows:

$$
\left.\begin{array}{l}
e_{x x}=2 l A \cos 2 \theta \cos 2 \varphi  \tag{6}\\
e_{y y}=-4 l A \cos 2 \varphi \\
e_{x y}=-6 l A \cos \theta \sin 2 \varphi
\end{array}\right\}
$$

When the direction cosines of the direction of observations of linear strain are $\lambda$ and $\mu$, the tidal change of strain $E$ observed will be:

$$
\begin{equation*}
E=e_{x x} \lambda^{2}+e_{y y} \mu^{2}+e_{x y} \lambda \mu \tag{7}
\end{equation*}
$$

Tidal change of strain include, besides the so-called primary term, i.e., lifnear strain caused by tide generating force emanating from heavenly bodies, the so-called secondary term which is an indirect effect produced by the tidal changes of the sea which are caused by tide generating force of heavenly bodies.

Now, if the earth-crust in the neighborhood of the place where observations are being carried on is assumed to be an isotropic semiinfinite elastic body, strain of the earth due to the change in load of sea-water is obtained from the solution of Boussineq, as a period of the oceanic tide is long enough. When the surface of a sea-area, surrounded by concentric circles which have the radii $r_{n}$ and $r_{n+1}$, respectively, and whose center is the spot of observations, and the movable radii making polar angles $\theta_{n}$ and $\theta_{n+1}$ with the direction of observations, undergoes
a change of $\Delta h \cos (2 t-T)$, the strain $E_{s}$ in the direction of observations will be expressed as follows:

$$
\begin{equation*}
E_{s}=\frac{\rho g \Delta \cos (2 t-T)}{4 \pi(\lambda+\mu)} \cdot\left(\log \frac{\gamma_{n+1}}{r_{n}}\right) \cdot \frac{1}{2}\left(\sin 2 \phi_{n+1}-\sin 2 \phi_{n}\right) \tag{8}
\end{equation*}
$$

provided the density of sea-water is $\rho$
3. Makimine is a copper mine belonging to Chichibu paleozoic system about 25 km from the east coast of Kyushu. The observation room is situated in a drift about 165 m under the earth's surface. From observations of the tilting of the earth-crust in the same room Prof. Eiichi Nishimura ${ }^{6}$ ) obtained very interresting results. In 1949 this place was equipped with a Sassa-type invar-line extensometer, and ever since observations have been carried on. Results of harmonic analyses of the $M_{2}$-tide observed here are shown in Table I.

Table 1 Makimine.
Location: Long. $131^{\circ} 24^{\prime}$ N.L.
Direction of observation: N $57^{\circ} \mathrm{W}$, Length of Observation Line: 20 m . Period of Harmonic Analyses: Dec. 7, 1949-Dec. 8, 1950. Sensibility of Instrument: $\quad 2.2 \sim 2.6 \times 10^{-9} / \mathrm{mm}$ Observed value of $M_{2}$-tide: $(0.31 \pm 0.15) \times 10^{-8} \cos \left(2 t-261^{\circ} \pm 45^{\circ}\right)$ Theoretical value of $M_{2}$-tide: $12.50 \times 10^{-3} l \cos \left(2 t-155^{\circ}\right)$

Complex disturbances of atmospheric origins other than the tide are seen in same specified months, so it may be best to make harmonic analyses only in months of clam weather. The value presented in this report, however was obtained from analyses made thoughout a year. Results of analyses made only in months of clam weather will be presently. The observing room at Osakayama is located 150 m from earth's surface in the center of the Osakayama tunnel ( 700 m in length) of the former Tokaido Railway Line on the outskirts of Otsu city, Shiga Prefecture. The place is equipped with extensometers three of whose constituent parts are the Sassa-type invar-line strain-meters, and the rest is a vertical component extensometer. Along with it, there is an invar-rod extensometer, for the purpose of comparative observations. The annual variation of temperature in the tunnel is $0.2^{\circ} \mathrm{C}$; the daily variation is less than $10^{-2} \mathrm{C}$. The daily variation of horizontal strain is less than $10^{-9}$. The neighbouring stratum is made up of clay-slate that belongs to Chichibu palaeozoic system

## Table 2. Osakayama.

Location: Long. $135^{\circ} 51^{\prime}$ E. $34^{\circ} 59^{\prime}$ N.L.
I. Direction of Observations: $\mathrm{S} 38^{\circ} \mathrm{W}$. Length of Observations: 20 m . Sensibility of Instrument: $0.41-0.63 \times 10 / \mathrm{mm}$.
Period of Harmonic Analyses: 18mos. from Oct. 24, 1947 to Feb. 24, 1949. Value obtained of $M_{2}$-tide; $(0.33 \pm 0.10) \times 10^{-8} \cos \left(2 t-43^{\circ} \pm 12^{\circ}\right)$.

Theoretical value of $M_{2}$-tide: $9.48 \times 10^{-s} l \cos \left(2 t-220^{\circ}\right)$.
II. Direction of Observations: S $76^{\circ} \mathrm{W}$.

Length of Observation: 6.8 m .
Sensibility of Instrument: $2.56 \times 10^{-3} / \mathrm{mm}$.
Period of Harmonic Analyses: from Sep. 2, to Oct. 1, 1952.
Observational Value of $M_{2}$-tide: $1.31 \times 10^{-9} \cos \left(2 t-187^{\circ}\right)$.
Theoretical Value of $M_{2}$-tide : $14.42 \times 10^{-3} l \cos \left(2 t-192^{\circ}\right)$.
III. Direction of Observations: $\mathrm{S} 76^{\circ} \mathrm{W}$.

Length of Observation: 3.4 m .
Sensibility of Instrument: $5.70 \times 10^{-9} / \mathrm{mm}$.
Period of Harmonic Analyses: from May 19, to Jun. 3, 1952.
Observational Value of $M_{2}$-tide : $0.06 \times 10^{-s} \cos \left(2 t-178^{\circ}\right)$.
Theoretical Value of $M_{2}$-tide : $2.70 \times 10^{-9} \cos \left(2 t-190^{\circ}\right)$.
Results of analyses of the $M_{2}$-tide of horizontal components in the directions of $\mathrm{S} 38^{\circ} \mathrm{W}, \mathrm{S} 76^{\circ} \mathrm{W}$ and $\mathrm{N} 2^{\circ} \mathrm{E}$ are shown in Table 2.


Fig. 1 Location of observatorys, and directions of observations.
In Fig. 1. are shown locations of Osakayama and Makimine, and the directions of observations. In Fig. 2. are shown observed values of the $M_{2}$-tide analysed monthly at both Osakayama and Makimine. The amplitude of strain of $\boldsymbol{M}_{2}$-tide is expressed by the length of the movable radius, and the phase angle made with the initial line. As clear from those two figures, the values obtained at Osakayama concentrate in a rather small area while the values obtained at Makimine are extensively scattered around. One of the reasons for this will be that the sensibility of the instrument used at Makimine, which is $2.2-2.610^{-8} / \mathrm{mm}$, is only fifth to one forth of that of the instrument used at Osakayama.


Fig. 2 Monthly obtervational value of $M_{2}$-tide.
4. What is being sought in our observations of the earth-tide by the extensometer is firstly the most probable value of the Lambert's number $l$, and secondly how the earth-crust is strained by the load of sea-water.

Assuming that there is no influence of the oceanic tide, it is possible to get $l$ from the observational value and theoretical value shown above Table. 2 as following:
in the direction of $\mathrm{S} 38^{\circ} \mathrm{W}: l=0.035 \pm 0.011$
in the direction of $\mathrm{S} 76^{\circ} \mathrm{W}: \quad l=0.091$
in the direction of $\mathrm{N} 2^{\circ} \mathrm{E}$ : $\quad l=0.021$
at Osakayama
in the direction of $\mathrm{N} 57^{\circ} \mathrm{W}: l=0.025 \pm 0.012 \quad$ at Makimine Assuming $\lambda=\mu$ and using value $\mu=6.0 \times 10^{11} \mathrm{c}$.g.s. of the depth $30-60 \mathrm{~km}$. from the earth's surface, horizontal strains in the direction of observations caused by the oceanic tide are calculated from Boussineq's solution iș following :

$$
\begin{array}{ll}
0.234 \times 10^{-8} \cos \left(2 t-278^{\circ}\right) & \text { at Osakayama } \\
0.265 \times 10^{-8} \cos \left(2 t-138^{\circ}\right) & \text { at Makimine }
\end{array}
$$

Supposing rigidity $\mu=10^{19}$ c.g.s., the strain will be:

$$
\begin{array}{ll}
0.139 \times 10^{-8} \cos \left(2 t-278^{\circ}\right) & \text { at Osakayama } \\
0.158 \times 10^{-3} \cos \left(2 t-138^{\circ}\right) & \text { at Makimine }
\end{array}
$$

The influence of the oceanic tide can not be ascertined independently, and consequently, it is hard to separate the observational value into the primary-term and the secondary term, unless some assumption is made.

However, this question can be solved if either of two terms is given. It is not possible in a small island as Japan to observe tidal strains at a place far distant from the sea-coast. Dr. Takahiro Hagiwara ${ }^{7}$ ) made observations on the contray at a place close to the seashore where the secondary term was so large that the primary term could be disregarded. As a result, it was found out that there were few instances to which the solution of Boussinesq could be applied.

Assuming in the first place various of $l$, values of the secondary term are obtained from the observational values, and shown in Table 3.

Table 3. Secondary term.

|  | Osakayama. |  | Makimine. |  |
| :---: | :---: | :---: | :---: | :---: |
| $l$ | Amplitude. $\times 10^{-8}$ | Phase. | Amplitude. $\times 10^{-8}$ | Phase. |
| 0.04 | 0.71 | $43^{\circ}$ | 0.71 | $313^{\circ}$ |
| 0.06 | 0.90 | $43^{\circ}$ | 0.97 | $318^{\circ}$ |
| 0.08 | 1.09 | $43^{\circ}$ | 1.15 | $322^{\circ}$ |
| 0.18 | 2.07 | $43^{\circ}$ | 2.42 | $328^{\circ}$ |

Relations implicit in the above Table 3 are shown in Fig.


Fig. 3 Secondary-term for various values of $l$.

As shown in Fig. 3, in the case of Osakayama, all phase angles of the secondary term are found in the first quadrant, while in the case of Makimine, they are found in the fourth quadrant.

Eqation (8) is wirtten to the following form,

$$
\left.\begin{array}{l}
E_{s}=\psi\left(r_{n}, \phi_{n}\right) \cdot f\left(t, r_{n}, \phi_{n}\right) \\
f\left(t, r_{n}, \phi_{n}\right)=\Delta h \cos (2 t-T) \log \frac{\boldsymbol{r}_{n+1}}{\gamma_{n}} \cdot \frac{1}{2}\left(\sin 2 \phi_{n+1}-\sin 2 \phi_{n}\right)  \tag{9}\\
\psi\left(r_{n}, \phi_{n}\right)=\frac{\rho g}{4 \pi(\lambda+\mu)}
\end{array}\right\}
$$

putting

$$
\begin{equation*}
f\left(t, r_{n}, \phi_{n}\right)=a_{n} \cos 2 t+b_{n} \sin 2 t \tag{10}
\end{equation*}
$$

the value of $a_{n}$ and $b_{n}$ are caluculated as to a sea-ares divisible by concentric circles having the radii $25 \mathrm{~km} .-40 \mathrm{~km}$., $40 \mathrm{~km} .-63 \mathrm{~km}$., 63 km .$100 \mathrm{~km} ., 100 \mathrm{~km} .-160 \mathrm{~km} ., 160 \mathrm{~km} .-250 \mathrm{~km} ., 250 \mathrm{~km} .-400 \mathrm{~km}$., $400 \mathrm{~km} .-630 \mathrm{~km}$., $630 \mathrm{~km} .-880 \mathrm{~km}$., $880 \mathrm{~km} .-1,400 \mathrm{~km}$., and $1,400 \mathrm{~km} .-2,200 \mathrm{~km}$., and centering at the place of observations. They are shown in Table 4 and Fig. 4.

Table 4

|  | Sea-area |  | Osakayama |  | Makimine |  |
| :---: | :---: | ---: | ---: | ---: | ---: | ---: |
| $n$ | min. radius <br> km | max. radius <br> km | $\boldsymbol{a}_{\boldsymbol{n}}$ | $\cdot b_{n}$ | $\boldsymbol{a}_{\boldsymbol{n}}$ | $b_{n}$ |
| 1 | 25 | 40 | - | - | -5.24 | - |
| 2 | 40 | 63 | -0.45 | -0.25 | -7.89 | 1.28 |
| 3 | 63 | 100 | 0.54 | -2.76 | -3.98 | 1.84 |
| 4 | 100 | 160 | 1.38 | -4.38 | -6.74 | 1.90 |
| 5 | 160 | 250 | 3.40 | -4.89 | -3.70 | -0.07 |
| 6 | 250 | 400 | 0.34 | -4.99 | -1.81 | -1.56 |
| 7 | 400 | 630 | -0.49 | -3.34 | -0.04 | 1.55 |
| 8 | 630 | 880 | -1.10 | -5.84 | -1.24 | 10.44 |
| 9 | 880 | 1400 | 0.66 | -4.69 | -1.27 | 4.27 |
| 10 | 1400 | 2200 | -1.67 | -4.36 | 1.30 | 3.98 |



Fig. $4 \sqrt{a_{n}^{3}+b_{n_{6}}^{2}}$ and $0=\tan ^{-1} \frac{b_{n}}{a_{n}}$ of divided sea-area by consentric circles.

In the Figure 4, the space between the center of the large concentric circle and the center of the small circle is the distance $r_{n}$ from the place of observations to the fan-shaped sea-area, the polar angle $\theta_{n}$ from the original line showing the position of the small circle is equal to $\tan ^{-1} \frac{b_{n}}{a_{n}}$. and the area of the small circle shows the size of $V \overline{a_{n}{ }^{2}+b_{n}{ }^{2}}$.

Supposing $\psi(r, \phi)$ in (9) is a function only of the distance from the observatory to the sea-area as

$$
\begin{equation*}
\psi(r)=\frac{1}{\alpha\left(1+\beta r^{\prime \prime \prime}\right)} \tag{11}
\end{equation*}
$$

Next, calculations are made assuming different vslues for $\alpha$ and $\beta$. (In this case, a unit for large of $r$ is 100 km .) In Equation (11), the case where $\beta=0$ corresponds to the case where the earth-crust is an isotropic semi-infinite elastic body. For convenience' sake $\alpha$ is assumed to be $1.03 \times 10^{10}$ so that $\frac{\rho g}{4 \pi(\lambda+\mu)}=\frac{1}{\alpha}$ when the rigidity of the earth crust $\mu=$ $0.4 \times 10^{10}$ c.g.s. and $\lambda=\mu$. Results found by calulation are shown in Table 5.

Table 5 (A)
Osakayama : $\quad \psi=\frac{1}{\alpha(1+\beta \gamma)^{m}}, \quad \alpha=1.03 \times 10^{10}, \quad \beta=0.0$,

| $n$ | max. radius <br> km | $a_{i} \times 10^{-\mathrm{s}}$ | $b_{i} \times 10^{-\mathrm{s}}$ | $\sum_{i=1}^{n} \psi_{i} a_{i} \times 10^{-\mathrm{s}}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-\mathrm{s}}$ | g |
| ---: | :---: | ---: | ---: | ---: | ---: | :---: |
| 1 | 40 | - | - | - |  |  |
| 2 | 63 | -0.004 | -0.002 | -0.004 | -0.002 | $200^{\circ}$ |
| 3 | 100 | 0.005 | -0.027 | -0.001 | -0.029 | $289^{\circ}$ |
| 4 | 160 | 0.013 | -0.043 | 0.014 | -0.072 | $238^{\circ}$ |
| 5 | 250 | 0.033 | -0.047 | 0.047 | -0.119 | $292^{\circ}$ |
| 6 | 400 | 0.003 | -0.048 | 0.051 | -0.167 | $288^{\circ}$ |
| 7 | 630 | -0.005 | -0.032 | 0.046 | -0.200 | $284^{\circ}$ |
| 8 | 880 | -0.011 | -0.057 | 0.035 | -0.256 | $283^{\circ}$ |
| 9 | 1400 | 0.006 | -0.046 | 0.042 | -0.302 | $289^{\circ}$ |
| 10 | 2200 | -0.016 | -0.042 | -0.025 | -0.344 | $278^{\circ}$ |

Table 5 (B)
Makimide : $\quad \psi=\frac{1}{\alpha(1+\beta \gamma)^{m}}, \quad \alpha=1.03 \times 10^{10}, \quad \beta=0.0$

| $n$ | max. radius <br> km | $a_{i} \times 10^{-\mathrm{s}}$ | $b_{i} \times 10^{-\mathrm{s}}$ | $\sum_{i=1}^{n} \psi_{i} a_{i} \times 10^{-\mathrm{s}}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-\mathrm{s}}$ | 0 |
| ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| 1 | 40 | -0.051 | - | -0.051 |  |  |
| 2 | 63 | -0.077 | 0.012 | -0.127 | 0.012 | $180^{\circ}$ |
| 3 | 100 | -0.039 | 0.018 | -0.166 | 0.030 | $175^{\circ}$ |
| 4 | 160 | -0.066 | 0.018 | -0.231 | 0.048 | $167^{\circ}$ |
| 5 | 250 | -0.036 | -0.001 | -0.267 | 0.047 | $170^{\circ}$ |
| 6 | 400 | -0.018 | 0.015 | -0.285 | 0.063 | $167^{\circ}$ |
| 7 | 630 | -0.004 | 0.015 | -0.285 | 0.078 | $164^{\circ}$ |
| 8 | 880 | -0.001 | 0.102 | -0.297 | 0.180 | $147^{\circ}$ |
| 9 | 1400 | -0.001 | 0.041 | -0.310 | 0.221 | $143^{\circ}$ |
| 10 | 2200 | 0.001 | 0.039 | -0.297 | 0.260 | $138^{\circ}$ |

'Table 5 (C)
Osakayama: $\alpha=1.03 \times 10^{10} \quad \beta=1.0 \quad m=1$

| $n$ | $a_{i} \times 10^{-s}$ | $b_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \Psi_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-s}$ | 0 |
| ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | - | - | - | - | - |
| 2 | -0.003 | -0.002 | -0.003 | -0.002 | $205^{\circ}$ |
| 3 | 0.003 | -0.015 | 0.000 | -0.017 | $270^{\circ}$ |
| 4 | 0.006 | -0.019 | 0.006 | -0.036 | $280^{\circ}$ |
| 5 | 0.011 | -0.016 | 0.017 | -0.047 | $290^{\circ}$ |
| 6 | 0.001 | -0.012 | 0.018 | -0.053 | $289^{\circ}$ |
| 7 | -0.001 | -0.005 | 0.017 | -0.060 | $281^{\circ}$ |
| 8 | 0.001 | -0.007 | 0.018 | -0.067 | $280^{\circ}$ |
| 9 | 0.001 | -0.004 | 0.019 | -0.071 | $280^{\circ}$ |
| 10 | -0.001 | -0.002 | 0.018 | -0.073 | $279^{\circ}$ |

Table 5 (D)
Makimine: $\alpha=1.03 \times 10^{10} \quad \beta=1.0 \quad m=1$

| $n$ | $a_{i} \times 10^{-9}$ | $b_{i} \times 10^{-9}$ | $\sum_{i=1}^{n} \psi_{i} a_{l} \times 10^{-s}$ | $\sum_{i=1}^{n} \Psi_{i} b_{i} \times 10^{-s}$ | $\theta$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.039 | - | -0.039 | - | $180^{\circ}$ |
| 2 | -0.051 | 0.008 | -0.090 | 0.008 | $168^{\circ}$ |
| 3 | -0.022 | 0.010 | -0.111 | 0.018 | $170^{\circ}$ |
| 4 | -0.029 | 0.008 | -0.141 | 0.027 | $169^{\circ}$ |
| 5 | -0.012 | 0.000 | -0.153 | 0.026 | $170^{\circ}$ |
| 6 | -0.004 | 0.004 | -0.157 | 0.030 | $169^{\circ}$ |
| 7 | 0.000 | 0.003 | -0.157 | 0.033 | $168{ }^{\circ}$ |
| 8 | -0.002 | 0.012 | -0.158 | 0.045 | $164^{\circ}$ |
| 9 | -0.001 | 0.004 | -0.159 | 0.048 | $163^{\circ}$ |
| 10 | 0.001 | 0.002 | -0.159 | 0.051 | $162^{\circ}$ |

Table 5 (E)
Osakayama: $\alpha=1.03 \times 10^{10} \quad \beta=2.0 \quad m=1$

| $n$ | $a \times 10^{-s}$ | $b \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-s}$ | 0 |
| ---: | ---: | ---: | :---: | :---: | :---: |
| $\mathbf{1}$ | - | -002 | -0.001 | -0.002 | -0.001 |
| 2 | -0.002 | -0.010 | 0.000 | -0.012 | $150^{\circ}$ |
| $\mathbf{3}$ | 0.002 | -0.013 | 0.004 | -0.024 | $280^{\circ}$ |
| 4 | 0.004 | -0.010 | 0.010 | -0.034 | $288^{\circ}$ |
| 5 | 0.007 | -0.007 | 0.011 | -0.041 | $286^{\circ}$ |
| 6 | 0.001 | 0.003 | 0.010 | -0.043 | $284^{\circ}$ |
| 7 | 0.000 | -0.004 | 0.011 | -0.047 | $284^{\circ}$ |
| 8 | 0.001 | -0.002 | 0.011 | -0.049 | $284^{\circ}$ |
| 9 | 0.000 | -0.001 | 0.011 | -0.050 | $283^{\circ}$ |
| 10 | -0.001 |  |  |  |  |

Table 5 (F)
Makimine: $\alpha=1.03 \times 10^{10} \quad \beta=2.0 \quad m=1$.

| $n$ | $a_{i}$ | $b_{i}$ | $\sum_{i=1}^{n} \psi_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-s}$ | 0 |
| ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | -0.031 | - | -0.031 |  |  |
| 2 | -0.038 | 0.006 | -0.069 | 0.006 | $175^{\circ}$ |
| 3 | -0.015 | 0.007 | -0.084 | 0.013 | $166^{\circ}$ |
| 4 | -0.019 | 0.005 | -0.103 | 0.019 | $160^{\circ}$ |
| 5 | -0.007 | 0.003 | -0.110 | 0.019 | $161^{\circ}$ |
| 6 | 0.002 | 0.002 | -0.113 | 0.021 | $160^{\circ}$ |
| 7 | 0.000 | 0.001 | -0.114 | 0.029 | $160^{\circ}$ |
| 8 | -0.001 | 0.007 | -0.114 | 0.030 | $156^{\circ}$ |
| 9 | -0.001 | 0.002 | -0.114 | 0.030 | $155^{\circ}$ |
| 10 | 0.000 | 0.001 | -0.114 | 0.032 | $155^{\circ}$ |

Table 5 (G)
Osakayama: $\alpha=1.03 \times 10^{10} \quad \beta=0.5 \quad m=2$

| $n$ | $a_{i} \times 10^{-s}$ | $b_{i} \times 10^{-\mathrm{s}}$ | $\sum_{i=1}^{n} \Psi_{l} a_{i} \times 10^{-\mathrm{s}}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-s}$ | 0 |
| ---: | ---: | ---: | ---: | ---: | :---: |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 |  |
| 2 | -0.004 | -0.002 | -0.004 | -0.002 | $206^{\circ}$ |
| 3 | 0.004 | -0.020 | 0.000 | -0.022 | $270^{\circ}$ |
| 4 | 0.008 | -0.024 | 0.008 | -0.046 | $280^{\circ}$ |
| 5 | 0.011 | -0.016 | 0.019 | -0.062 | $286^{\circ}$ |
| 6 | 0.001 | -0.008 | 0.020 | -0.070 | $286^{\circ}$ |
| 7 | -0.000 | -0.002 | 0.019 | -0.072 | $285^{\circ}$ |
| 8 | 0.000 | -0.002 | 0.020 | -0.075 | $285^{\circ}$ |
| 9 | 0.000 | -0.001 | 0.020 | -0.075 | $285^{\circ}$ |
| 10 | 0.000 | 0.000 | 0.020 | -0.075 | $285^{\circ}$ |

Table 5 (H)
Makimine: $\alpha=1.03 \times 10^{10} \quad \beta=0.5 \quad m=2$

| $n$ | $a_{i} \times 10^{-8}$ | $b^{2} \times 10^{-8}$ | $\sum_{i=1}^{n} \psi_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-8}$ | $\theta$ |
| ---: | ---: | ---: | :---: | :---: | :---: |
| 1 | -0.050 | - | -0.050 | - | $180^{\circ}$ |
| 2 | -0.066 | 0.011 | -0.114 | 0.011 | $175^{\circ}$ |
| 3 | -0.029 | 0.014 | -0.143 | 0.024 | $170^{\circ}$ |
| 4 | -0.036 | 0.010 | -0.180 | 0.035 | $169^{\circ}$ |
| 5 | -0.012 | -0.000 | -0.192 | 0.035 | $170^{\circ}$ |
| 6 | -0.003 | 0.003 | -0.195 | 0.037 | $170^{\circ}$ |
| 7 | 0.000 | 0.001 | -0.195 | 0.038 | $169^{\circ}$ |
| 8 | -0.000 | 0.004 | -0.195 | 0.042 | $168^{\circ}$ |
| 9 | -0.000 | 0.001 | -0.196 | 0.043 | $168^{\circ}$ |
| 10 | 0.000 | 0.000 | -0.196 | 0.043 | $168^{\circ}$ |

Table 5 (I)
Osakayama: $\alpha=1.03 \times 10^{10}, \quad \mathrm{~g}=1.0 \quad m=2$

| $n$ | $a_{i} \times 10^{-s}$ | $b_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \Psi_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-s}$ | 0 |
| ---: | ---: | :---: | :---: | :---: | :---: |
| 1 | - | - | - |  |  |
| 2 | -0.003 | -0.002 | -0.003 | -0.002 | $213^{\circ}$ |
| 3 | 0.003 | -0.016 | 0.000 | -0.018 | $270^{\circ}$ |
| 4 | 0.005 | -0.016 | 0.005 | -0.035 | $278^{\circ}$ |
| 5 | 0.007 | -0.010 | 0.012 | -0.044 | $286^{\circ}$ |
| 6 | 0.000 | -0.004 | 0.012 | -0.048 | $285^{\circ}$ |
| 7 | 0.000 | -0.001 | 0.012 | -0.050 | $284^{\circ}$ |
| 8 | 0.000 | -0.001 | 0.012 | -0.051 | $284^{\circ}$ |
| 9 | 0.000 | -0.00 | 0.012 | -0.051 | $284^{\circ}$ |
| 10 | -0.000 | -0.000 | 0.012 | -0.051 | $284^{\circ}$ |

Table 4 (J)
Makimine: $\alpha=1.03 \times 10^{10}, \beta=1.0 \quad m=2$

| $n$ | $a_{i} \times 10^{-s}$ | $b_{i} \times 10^{-8}$ | $\sum_{i=1}^{n} 山_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} 山_{i} b_{i} \times 10^{-s}$ | 0 |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.046 | - | -0.046 |  | 0 |
| 2 | -0.058 | 0.010 | -0.105 | 0.010 | $180^{\circ}$ |
| 3 | -0.024 | 0.011 | -0.128 | 0.020 | $174^{\circ}$ |
| 4 | -0.025 | 0.007 | -0.153 | 0.028 | $170^{\circ}$ |
| 5 | -0.007 | 0.000 | -0.161 | 0.028 | $170^{\circ}$ |
| 6 | -0.002 | 0.001 | -0.162 | 0.029 | $170^{\circ}$ |
| 7 | 0.000 | 0.002 | -0.163 | 0.031 | $169^{\circ}$ |
| 8 | 0.000 | 0.000 | -0.163 | 0.031 | $169^{\circ}$ |
| 9 | 0.000 | 0.000 | -0.163 | 0.031 | $169^{\circ}$ |
| 10 | 0.000 | 0.000 | -0.163 | 0.031 | $169^{\circ}$ |

Table 5 (K)
Osakayama: $\alpha=1.03 \times 10^{10}, \beta=2.0 \quad m=2$

| $n$ | $a_{i} \times 10^{-8}$ | $b_{i} \times 10^{-8}$ | $\sum_{i=1}^{n} \Psi_{i} a_{i} \times 10^{-s}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-8}$ | $\theta$ |
| ---: | ---: | ---: | :---: | :---: | :---: |
| 1 | -0.003 | -0.002 | -0.003 | -0.002 | $213^{\circ}$ |
| 2 | -0.002 | -0.013 | -0.001 | -0.015 | $270^{\circ}$ |
| 3 | 0.003 | -0.013 | 0.002 | -0.028 | $275^{\circ}$ |
| 4 | 0.004 | -0.004 | 0.006 | -0.032 | $281^{\circ}$ |
| 5 | 0.000 | 0.000 | 0.006 | -0.032 | $281^{\circ}$ |
| 6 | 0.000 | -0.001 | 0.006 | -0.033 | $281^{\circ}$ |
| 7 | 0.000 | -0.000 | 0.006 | -0.033 | $281^{\circ}$ |
| 8 | 0.000 | 0.000 | 0.006 | -0.033 | $281^{\circ}$ |
| 9 | 0.000 | 0.000 | 0.006 | -0.033 | $281^{\circ}$ |
| 10 |  |  |  |  |  |

Table $4^{\top}(\mathrm{L})$
Makimine: $\alpha=1.03 \times 10^{10}, \quad \beta=2.0 \quad m=2$

| $n$ | $a_{i} \times 10^{-8}$ | $b_{i} \times 10^{-8}$ | $\sum_{i=1}^{n} \psi_{i} a_{i} \times 10^{-8}$ | $\sum_{i=1}^{n} \psi_{i} b_{i} \times 10^{-8}$ | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.043 | - | -0.043 |  |  |
| 2 | -0.059 | 0.010 | -0.102 | 0.010 | $180^{\circ}$ |
| 3 | -0.017 | 0.008 | -0.119 | 0.018 | $175^{\circ}$ |
| 4 | -0.016 | 0.004 | -0.135 | 0.022 | $170^{\circ}$ |
| 5 | -0.001 | 0.000 | -0.139 | 0.022 | $169^{\circ}$ |
| 6 | -0.001 | 0.001 | -0.140 | 0.023 | $170^{\circ}$ |
| 7 | 0.000 | 0.000 | -0.140 | 0.023 | $170^{\circ}$ |
| 8 | 0.000 | 0.001 | -0.140 | 0.024 | $170^{\circ}$ |
| 9 | 0.000 | 0.000 | -0.140 | 0.024 | $170^{\circ}$ |
| 10 | 0.000 | 0.000 | -0.140 | 0.024 | $170^{\circ}$ |

Fig. 5 (a) $\psi(\gamma), f(\gamma, \phi)$ at Osakayama.


Fig. 5 (b) $\psi(\gamma) \cdot f(\gamma, \phi)$ at Makimine.


Besides the above Table 5, in Fig. 5 some case $m=1,2$, and $\beta=0.5$, 1.0, 2.0, are graphically illustrated. As clear from the above Table and Fig. 5 and also from Fig. 4, it never happens that the value of $\theta_{n}=\tan ^{-1} \frac{\sum_{i=1}^{n} b_{i}}{\sum_{i=1}^{n} d_{i}^{\prime}}$ is found in the first quadrant in the case of Osakayama,
and in the fourth quadrant in the case of Makimine, so long as assume $\phi(r)$ as in a form of (11) moreover amplitude is very much difierent. Therefore, it is not proper for us to suppose that $\psi(r, \phi)$ is a form of (11). It is very interesting that however, is that according to Table, when $\psi(r, \phi)=$ const., the observational value is out of phsse to calculated value in the secondary term by $130^{\circ}$ at Oakayama, and $180^{\circ}$ the case of Makimine. This means probablly that our assumption in this calculation is not satisfied in full. It is necessary to make observation at least in three directions at one place even of horizantal strain alone. Not withstanding in foregoing pages we discussed the results of observations made at Osakayama and Makimine in only one direction. Observations of three hrizontal components are carried on at Osakayama. Therefore more extended discussion will be reported in the next occation waitting accumlation of data of strain observations of three horizontal components which are now carried on Osakayama and others. Our cordinal thanks are extended to Dr. Kenzo Sassa, Professor of Kyoto University who instructed us in these studies, and also to Prof. Eiichi Nishimura whe furnished us with the records of observations at Makimine.

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