Observation of Tidal Strain of the Earth

(Part I)

By Kenzo Sassa, Izuo Ozawa and Soji Yoshikawa

1. According to E. Nishimura, Diminishing Factor $D$, obtained by observing with tiltmeters, the inclination of the earth surface due to earth-tide, is as follows:

$$D = 1 + k - h = 0.661 \pm 0.024.$$  

The ratio $(L)$ of the change of latitude on the elastic earth caused by earth-tide to that when the earth is considered as perfectly rigid is given as $L = 1.2 \sim 1.6$. $k$ can be obtained from the period of Euler nutation, the chandler period being 1.2 years, according to Pollack. From this, $k = 0.287$. $h$, $k$ and $l$ are Love-numbers and the value of $l$ must be very small when obtained from the above observations of $D$, $L$ and $k$. But if the earth is considered as homogeneous and incompressible, relation $l = 3/10h$ exists theoretically, resulting in not agreeing with the above observations. Considering this inconformity the most serious problem is the fact that the value of $L$ obtained from latitude observation is very inaccurate compared with the other values. Therefore it is necessary to directly measure the true value of $l$ due to earth-tide with the same degree of accuracy as the tiltmeter. $l$ was actually measured by observing the horizontal strain of the earth caused by earth-tide with a horizontal strainmeter which was previously made for the purpose of observing the crustal deformation.

2. One of the authors (Sassa) studied and made the strainmeter in 1941. A superinvar wire, 1.6mm in diameter, is stretched practically horizontally between two fixed bases $L$ appart and a weight of about 350g is suspended at the middle. The sag of the stretched wire is approximately $L/70$ and the suspended weight moves up and down in accordance with change in the distance between the two fixed bases.
The up and down movement of the weight is converted into the revolution of a mirror attached to the weight by means of the principle of the bifilar suspension. Of this suspension, the wire which suspends the weight at the middle point and gives the twisting movement is 0.2mm in diameter, and the bifilar wires are super-invar wires, 0.05mm in diameter. The thermal expansion coefficient of the super-invar is $3 \times 10^{-7}/\text{c}$. The distance from the mirror to the recorder is 2m and the sensitivity is about $10^{-6}/\text{mm}$. The sensitivity is determined from the deflection of the light spot on the recording paper corresponding to a certain displacement to one of the fixed point with a micrometer. Zero displacement is also adjusted by this method of moving the fixed point and the apparatus is used always in a condition of approximately same sag:

3. The horizontal strain of the earth surface caused by earth-tide is as follows:

meridian component; \[ E_s = \frac{1}{ag} \frac{\partial^2 W}{\partial \varphi^2} \]

prime vertical component; \[ E_t = \frac{1}{ag\cos^2 \varphi} \frac{\partial^2 W}{\partial \lambda^2} \]

where, \[ W: \text{ tide generating potential} \]
\[ \lambda: \text{ latitude} \]
\[ \varphi: \text{ longitude} \]
\[ l: \text{ Love-number} \]
\[ g: \text{ gravity-acceleration} \]
\[ a: \text{ mean radius of earth.} \]

4. By applying Boussinesq’s method, horizontal strain \( B \) of the earth caused by the load of oceanic tide can be given as follows in a similar manner as that in the case of the change in tilt of the earth surface.

\[ B = mA; \quad m = \frac{1}{4\pi(\lambda + \mu)} \frac{g^a}{G} \]

where, \[ \lambda, \mu: \text{ Lamé-constants} \]
\[ G: \text{ Gravitational constant} \]
\[ A: \text{ Attraction component of seawater,} \]

where for \( M_2 \) tide is as follows:

\[ A = G\rho h/g\cos(2t - \sigma) \log \frac{r_2}{r_1} (\sin \theta_1 - \sin \theta_2) \]

where, \[ \sigma: \text{ desity,} \]
\[ h: \text{ amplitude of oceanic tide,} \]
\[ t, \sigma: \text{ hour angle and argument of } M_2 \text{ tide} \]
\[ \theta_1, \theta_2: \text{ angles between the point of measurement and the two wings of the acting sea surface.} \]
\[ r_1, r_2: \text{ minimum and maximum distance between the observatory and acting sea surface.} \]
The values given by E. Niseimura were adopted as the rigidity of the earth-crust corresponding to the angular distance in calculating $B$.

5. Observation with the strainmeter placed at a depth of 800m in a shaft of Mitsubishi Ikuno Mine, east longitude $135^\circ 47'$ north latitude $35^\circ 40'$.

Sensitivity of the instrument: $1.04 \times 10^{-8}$/mm in a direction of E3$^\circ$S
Period of the observation; from Sept. 19th to Oct. 5th, 1943
Observed value of $M_2$ tide; $0.79 \times 10^{-8}\cos(2t-30.9^\circ)$
Strain due to oceanic $M_2$ tide calculated; $0.12 \times 10^{-8}\cos(2t-136.3^\circ)$
Value, the influence of oceanic tide subtracted; $0.77 \times 10^{-8}\cos(2t-40.7^\circ)$

Hence

$I = 0.051$

The period of observation being short, there is some doubt as to the accuracy of the result. In remarking that it was reported at the Annual Meeting of Geophysical Society (1946).

6. An observation was opened in the old Osaka-Yama Tunnel, the Tokaido railway line, in Aug. 1948. A horizontal strainmeter of Sassa-type was placed at a point in the tunnel 300m from the entrances and about 150m deep from the earth surface. The annual variation of temperature of the room was $0.2^\circ$C and the daily variation of the earth strain was less than $10^{-9}$ c.g.s. The results of the observation are as follows.

Position of Observatory; east longitude $135^\circ 51'$, north latitude $34^\circ 54'$
Sensitivity of the instrument; $0.63-0.41 \times 10^{-9}$/mm in a direction of S 38$^\circ$W.
Period of observation; Oct. 24th, 1947 to Oct. 29th, 1948
Observed $M_2$ tide of strain; $0.348 \times 10^{-8}\cos(2t-44^\circ)$
Strain due to oceanic $M_2$ tide; $0.174 \times 10^{-8}\cos(2t-278^\circ)$
Value, the influence of oceanic tide subtracted $0.470 \times 10^{-8}\cos(2t-61^\circ)$

Hence

$I = 0.05$

7. The value of $I$ is quite different from the theoretical value obtained, considering the earth as homogenious and incompressible. Love-numbers are $k=0.287$, $h=0.626$, $l=0.050$ and the $L$ calculated from the above Love-number is $L=1.235$, the results agreeing fairly well with those of latitude observation analysis. The observed value of $I$ is somewhat closer to, but still far smaller than that which Takeuchi recently computed theoretically as $I=0.082\sim0.080$.

Kyoto University, Japan.

INTERNATIONAL ASSOCIATION OF GEODESY
BRUSELLS ASSEMBLY, AUGUSP 1951.