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Failure of the Embankment Foundation

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Abstract

For the stability computation of so-called embankment type structures such as an earth dam, levee and railway embankment built on a soft foundation, it is necessary to have a comprehensive knowledge as to the rupture phenomena of the earth foundation during the consolidation of soft clay. In this paper, taking the standpoint that the pore water pressure, which has the dengerous effect on the stability of the foundation during or just after the fill construction, must be diminished by the adequate control of execution speed, a theoretical treatment is performed to clear the mechanism of the plastic flow of the foundation containing the pore water pressure, whose distribution is determined in the following way.

Starting from the fundamental theoretical equation of the two-dimensional consolidation, the research is performed for the foundation which has infinite depth or under which a perfectly rigid and smooth rock base exists at any depth. First, applying Neuber's theory of elasticity, the stress distribution just after loading is obtained, and using this as the initial condition, the solution of the consolidation equation is represented in the form of Fourier's integral or Fourier's series with regared to each boundary condition. Next, performing the numerical calculation for the case of a uniformly distributed load, the pore water pressure is computed as to the sudden loading and also as to the gradually increasing load on the semi-infinite foundation with an allowable precision. It is also shown that such a solution is obtained for the foundation where the permeability is different in its vertical and horizontal directions, and for the case of parabolic load whose distribution is closer to the actual dam foundation. Comparing the results of these calculations with each other, the change of the distribution of the pore water pressure in respect to situation and time is studied for each loading condition.

1. Fundamental Theoretical Equations of Two-Dimensional Consolidation

Generally in the construction of earth dams, the pore water pressure occurs in the dam foundation at an early period of the construction, since at the instance of loading, the total load is charged by the pore water, and then as the water escapes while time elapses, a part of load is transmitted to soil particles gradually. If the load is constant, the water pressure vanishes with the time, and it depends upon the permeability and the situation in the foundation.

In the first part of this paper, the distribution of the pore water pressure which occurs in the dam foundation is cleared as a two-dimensional problem. The general theory of three-dimensional consolidation which has been introduced by Biot[1] is applied to the two-dimensional consolidation process of the embankment foundation. The following assumptions are made for the development of the theoretical equations; the foundation is of homogeneous, isotropic, perfectly elastic material and the pores between soil particles are fully saturated with water. (Anisotropy of the permeability of the foundation is treated in *Appendix III.*) The former assumption is acceptable for the reason that it is very rare that the stress concentration phenomenon occurs in the soft foundation consisting of a clay layer when it is loaded, and the latter assumption is permitted for the boundary condition of dam foundation. As the velocity of the pore water flow is very slow, it follows the rule of Darcy's law. In the following article, the distribution of the surcharged load is treated as a uniformly distributed load on the surface of the foundation, and the parabolic load is treated in *Appendix I.*

In the two-dimensional consolidation caused under the above assumptions, the following partial differential equation is deduced respecting the pore water pressure w[2]:

$$\dot{w} = c\nabla^2 w , \qquad \cdots\cdots(1)$$

where the coefficient of consolidation $c = k/\gamma v$ is assumed constant.

As the initial condition for solving this equation, it is necessary to find the distribution of the pore pressure when the load is applied. Stresses caused in the foundation whose surface is loaded by a long embankment can be treated as a plane-strain problem in the plane perpendicular to the axis of the embankment. According to Neuber[3] in this two-dimensional problem, let it be assumed that $\boldsymbol{u}$ is strain vector, φ_0 scalar harmonic function of x, y, $\boldsymbol{\Phi}$ vector whose components φ_x, φ_y are harmonic functions, $\boldsymbol{r}$ position vector, Θ divergence of $\boldsymbol{u}$ and ν Poisson's ratio. Then,

$$\left.\begin{aligned} \boldsymbol{u} &= -\operatorname{grad}(\varphi_0+\boldsymbol{r}\boldsymbol{\Phi})+4(1-\nu)\boldsymbol{\Phi} , \\ \Theta &= \operatorname{div}\boldsymbol{u} = -\operatorname{div}\operatorname{grad}\varphi_0-\operatorname{div}\operatorname{grad}\boldsymbol{r}\boldsymbol{\Phi}+4(1-\nu)\operatorname{div}\boldsymbol{\Phi} \\ &= 2(1-2\nu)\operatorname{div}\boldsymbol{\Phi} . \end{aligned}\right\} \cdots(2)$$

The stress components, σ_x, σ_y, τ_{xy} are represented in the following form by Eq. (2), assuming G modulus of rigidity:

$$\left.\begin{aligned}\frac{\sigma_x}{2G}&=\frac{\partial u}{\partial x}+\frac{\Theta\nu}{1-2\nu}=\frac{\partial u}{\partial x}+2\nu\operatorname{div}\boldsymbol{\Phi},\\ \frac{\sigma_y}{2G}&=\frac{\partial v}{\partial y}+\frac{\nu\Theta}{1-2\nu}=\frac{\partial v}{\partial y}+2\nu\operatorname{div}\boldsymbol{\Phi},\\ \frac{\tau_{xy}}{G}&=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots(3)$$

Two cases are treated by using the above equation: for the foundation which has infinite depth (in the following article), and under which a perfectly rigid and smooth rock base exists horizontally at any depth (in *Appendix II*). As the stress distribution is independent to Poisson's ratio ν, $\nu=0.5$ is adopted in the following calculation for simplicity.

2. Stress Distribution when the Load Is Applied on the Semi-Infinie Foundation

Let the x-axis be taken as perpendicular to the axis of embankment on the ground surface, and the positive y-axis as downwards from the center of the dam base. Assuming

$$\left.\begin{aligned}\varphi_0&=(Ae^{\lambda y}+Be^{-\lambda y})\cos\lambda x,\\ \varphi_y&=(Ce^{\lambda y}+De^{-\lambda y})\cos\lambda x,\\ \varphi_x&=0\quad(A,\ B,\ C,\ D,\ \lambda:\ \text{consts.})\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots(4)$$

and introducing the boundary conditions, and taking the load condition that the load $q_0\cos\lambda x$ is applied on the dam foundation, the stress components are given by

$$\left.\begin{aligned}-\sigma_x&=q_x=q_0e^{-\lambda y}(1-y\lambda)\cos\lambda x,\\ -\sigma_y&=q_y=q_0e^{-\lambda y}(1+y\lambda)\cos\lambda x,\\ -\tau_{xy}&=q_{xy}=q_0e^{-\lambda y}y\lambda\sin\lambda x.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots(5)$$

Now, $p_0(x)$ is considered as an arbitrary surface load symmetric about y-axis and is represented by the following Fourier's integral:

$$p_0(x)=\frac{2}{\pi}\int_0^\infty d\lambda\int_0^\infty d\xi p_0(\xi)\cos\lambda x\cos\lambda\xi,\qquad\cdots\cdots\cdots\cdots\cdots(6)$$

then, the pressure distribution $q(x)$ is represented as follows when $p_0(x)$ is distributed uniformly between $-a<\xi<a$, namely for $p_0(\xi)=q$:

$$\begin{aligned}q_y(x)&=\frac{2}{\pi}\int_0^\infty d\lambda\int_0^\infty d\xi p_0(\xi)e^{-\lambda y}(1+y\lambda)\cos\lambda x\cos\lambda\xi\\ &=\frac{2}{\pi}\int_0^\infty\left\{\int_0^a q\cos\lambda\xi\,d\xi\right\}e^{-\lambda y}(1+y\lambda)\cos\lambda x\,d\lambda\\ &=\frac{2q}{\pi}\int_0^\infty\frac{1}{\lambda}e^{-\lambda y}(1+y\lambda)\sin a\lambda\cos x\lambda\,d\lambda.\end{aligned}$$

Substituting $y\lambda = \alpha$, $d\lambda = d\alpha/y$, then,

$$q_y(x) = \frac{2q}{\pi}\int_0^\infty \frac{1+\alpha}{\alpha} e^{-\alpha} \sin \alpha \left(\frac{a}{y}\right) \cos \alpha \left(\frac{x}{y}\right) d\alpha$$

$$= \frac{q}{\pi}\int_0^\infty \left(1+\frac{1}{\alpha}\right) e^{-\alpha} \left\{\sin\left(\frac{x+a}{y}\right)\alpha - \sin\left(\frac{x-a}{y}\right)\alpha\right\} d\alpha$$

$$= \frac{q}{\pi}\left[\frac{2ay(a^2+y^2-x^2)}{\{(x+a)^2+y^2\}\{(x-a)^2+y^2\}} + \cot^{-1}\frac{x^2+y^2-a^2}{2ay}\right]$$

$$= \frac{q}{\pi}(\sin 2\varepsilon \cos 2\psi + 2\varepsilon)$$

Similarly,

$$q_x(x) = \frac{q}{\pi}(-\sin 2\varepsilon \cos 2\psi + 2\varepsilon), \qquad \cdots\cdots\cdots(7)$$

$$q_{xy}(x) = \frac{q}{\pi}\sin 2\varepsilon \sin 2\psi \,.$$

(see Fig. 1)

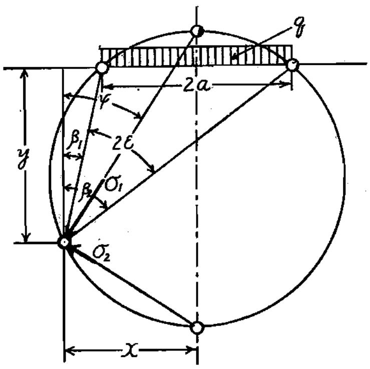


Fig. 1. Principal stresses σ_1, σ_2 caused by the uniformly distributed strip load q on the semi-infinite foundation.

Eq. (7) gives the stress components caused in the foundation by the surface load. As stated above, at the instance of loading on the clay layer which is consolidatable all of the initial stresses are supported by the pore water. Since water cannot resist shear stress, it is charged only by the principal stresses. And the assumption that the average of maximun and minimum principal stresses charges water without regard to the intermediate principal stress has been justified by Biot's discussion. According to this discussion, the initial pore pressure, w_0 in the saturated clay layer when loaded should be satisfied by Laplace's equation $\nabla^2 w_0 = 0$. Then,

$$w_0 = \frac{1}{2}(\sigma_1+\sigma_2) = \frac{1}{2}(\sigma_x+\sigma_y) = \frac{q}{\pi}2\varepsilon = \frac{q}{\pi}\cot^{-1}\frac{x^2+y^2-a^2}{2ay} \qquad \cdots(8)$$

is the initial distribution of pore pressure in the foundation.

3. Distribution of Pore Water Pressure in the Semi-Infinite Foundation

As the initial condition necessary to solve the partial differential equation (1) respecting the pore pressure w in the semi-infinite foundation has been obtained as Eq. (8), then the solution of Eq. (1) should be solved with the boundary condition that the foundation has infinite depth. First, the case in which the overburden uniform load q is suddenly loaded is treated, and then the case of gradually increasing load is studied.

(1) Suddenly applied load

Eq. (1) is

$$\frac{\partial w}{\partial t} = c\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \qquad \cdots\cdots\cdots\cdots\cdots (9)$$

Boundary condition is

$$(w)_{y=0} = 0 . \qquad \cdots\cdots\cdots\cdots\cdots (10)$$

Initial condition is

$$(w)_{t=0} = f(x, y) = \frac{q}{\pi} \cot^{-1} \frac{x^2+y^2-a^2}{2ay} \qquad \cdots\cdots\cdots\cdots\cdots (11)$$

In order to obtain the solution of Eq. (9) which satisfies Eq. (10), and Eq. (11), $f(x, y)$ in Eq. (11) is represented by Fourier's double integral as follows:

$$f(x, y) = \frac{2}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty d\lambda\int_0^\infty d\mu f(\lambda,\mu)\cos\alpha(x-\lambda)\sin\beta y\sin\beta\mu .$$

Then, the solution of Eq. (9) is

$$\begin{aligned} w &= \frac{2}{\pi^2}\int_0^\infty d\alpha\int_0^\infty d\beta\int_{-\infty}^\infty d\lambda\int_0^\infty d\mu f(\lambda, \mu)\exp\{-c(\alpha^2+\beta^2)t\}\cos\alpha(x-\lambda)\sin\beta y\sin\beta\mu \\ &= \frac{1}{4\pi ct}\int_{-\infty}^\infty d\lambda\int_0^\infty d\mu f(\lambda, \mu)\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4\,ct}\right\}-\exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4\,ct}\right\}\right] \\ &= \frac{q}{4\pi^2 ct}\int_{-\infty}^\infty d\lambda\int_0^\infty d\mu\cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4\,ct}\right\}\right. \\ &\qquad \left.-\exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4\,ct}\right\}\right]. \end{aligned} \qquad \cdots\cdots\cdots\cdots\cdots (12)$$

For the convenience of the numerical calculation, Eq. (12) is put as follows:

$$\begin{aligned} w = \frac{q}{4\pi^2 ct}\lim_{\substack{\Delta\lambda\to 0\\ \Delta\mu\to 0}}\sum_{\lambda=-\infty}^{\infty}\sum_{\mu=0}^{\infty}\cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4\,ct}\right\}\right. \\ \left.-\exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4\,ct}\right\}\right]\Delta\lambda\Delta\mu . \end{aligned} \qquad \cdots\cdots\cdots\cdots\cdots (13)$$

(2) Gradually increasing load

For the constant load the above equations indicate that the pore pressure decreases with time. In the case of the execution of earth dams, however, all the volume of earth mass is not placed at once, but is filled gradually. In such a case, the following fundamental equation is brought, adding to Eq. (9) $Q(x, y, t)$ which is the pore pressure increased in the unit time:

$$\frac{\partial w}{\partial t} = c\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + Q(x, y, t) \qquad \cdots\cdots\cdots\cdots\cdots(14)$$

Boundary condition is

$$(w)_{y=0} = 0 . \qquad \cdots\cdots\cdots\cdots\cdots(15)$$

Initial condition is

$$(w)_{t=0} = 0 . \qquad \cdots\cdots\cdots\cdots\cdots(16)$$

In order to obtain the solution of Eq. (14), $Q(x, y, t)$ is written in the form of Fourier's double integral:

$$Q(x, y, t) = \frac{2}{\pi^2}\int_0^\infty d\alpha \int_0^\infty d\beta \int_{-\infty}^\infty d\lambda \int_0^\infty d\mu\, Q(\lambda, \mu, t) \cos\alpha(x-\lambda) \sin\beta y \sin\beta\mu .$$

Similarly,

$$w = \int_0^\infty d\alpha \int_0^\infty d\beta \int_{-\infty}^\infty d\lambda \int_0^\infty d\mu\, \chi(\lambda, \mu, t) \cos\alpha(x-\lambda) \sin\beta y \sin\beta\mu$$

is put into Eq. (14), and $\chi(\lambda, \mu, t)$ is determined so as to satisfy the initial condition of Eq. (16), then:

$$\begin{aligned} w &= \frac{2}{\pi^2}\int_0^t dt \int_0^\infty d\alpha \int_0^\infty d\beta \int_{-\infty}^\infty d\lambda \int_0^\infty d\mu \exp\left\{-c(\alpha^2+\beta^2)(t-\tau)\right\} \\ &\qquad \times Q(\lambda, \mu, \tau) \cos\alpha(x-\lambda) \sin\beta y \sin\beta\mu \\ &= \frac{1}{4\pi c}\int_0^t \frac{d\tau}{t-\tau}\int_{-\infty}^\infty d\lambda \int_0^\infty d\mu\, Q(\lambda, \mu, t)\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4c(t-\tau)}\right\}\right. \\ &\qquad \left. - \exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4c(t-\tau)}\right\}\right]. \end{aligned} \qquad \cdots\cdots\cdots\cdots\cdots(17)$$

If $Q(x, y, t)$ is the increment of pore pressure caused by the increasing load $\bar{q}$, uniformly increasing with time, then it becomes independent of time, and from Eq. (8):

$$Q(x, y, t) = \frac{\bar{q}}{\pi} \cot^{-1}\frac{x^2+y^2-a^2}{2ay}$$

And Eq. (17) is

$$\begin{aligned} w &= \frac{\bar{q}}{4\pi^2 c}\int_{-\infty}^\infty d\lambda \int_0^\infty d\mu \cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}\int_0^t \frac{d\tau}{t-\tau} \\ &\qquad \times\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4c(t-\tau)}\right\} - \exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4c(t-\tau)}\right\}\right] \end{aligned}$$

$$= \frac{\bar{q}}{4\pi^2 c}\int_{-\infty}^{\infty} d\lambda \int_0^{\infty} d\mu \cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}\left[-Ei\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4ct}\right\}\right.$$
$$\left. + Ei\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4ct}\right\}\right]. \qquad \cdots\cdots\cdots\cdots\cdots (18)$$

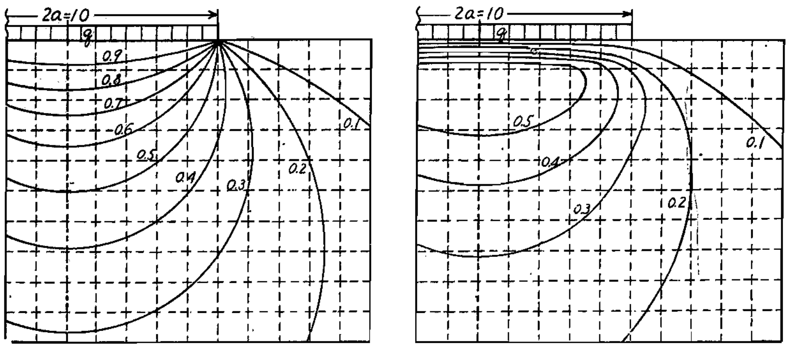
Or, approximately:

$$w = \frac{\bar{q}}{4\pi^2 c}\lim_{\substack{\Delta\lambda\to 0\\ \Delta\mu\to 0}}\sum_{\lambda=-\infty}^{\infty}\sum_{\mu=0}^{\infty}\cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}\left[-Ei\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4ct}\right\}\right.$$
$$\left. + Ei\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4ct}\right\}\right]\Delta\lambda\Delta\mu\,. \qquad \cdots\cdots\cdots\cdots\cdots (19)$$

Although it seems that the first exponential integral in Eqs. (18), (19) is infinite at the coordinate $x = \lambda$, $y = \mu$, it can be proved that the above integral gives a finite value at that point when the integration is finished about λ and μ.

Comparing Eqs. (12), (13) with Eqs. (18), (19), the difference is between the exponential function and the exponential integral. But in the former equations, as the present time t exists in the denominator before the double integral, the pore pressure decreases with the time. On the other hand, by the latter equations, the pore pressure continues to increase as long as the uniformly increasing load is applied on the foundation. This difference is cleared by numerical calculations as follows.

The calculation is performed for the case where the uniformly distributed load q having the width $2a = 10$ is applied on the semi-infinite foundation. Fig. 2 (a) shows the initial distribution of the pore pressure when the load has been placed suddenly. The value of the contour is the ratio of w_0 in Eq. (8) to the load intensity q. Fig. 2 (b) shows the result of the approximate calculation, giving the distribution of the pore pressure



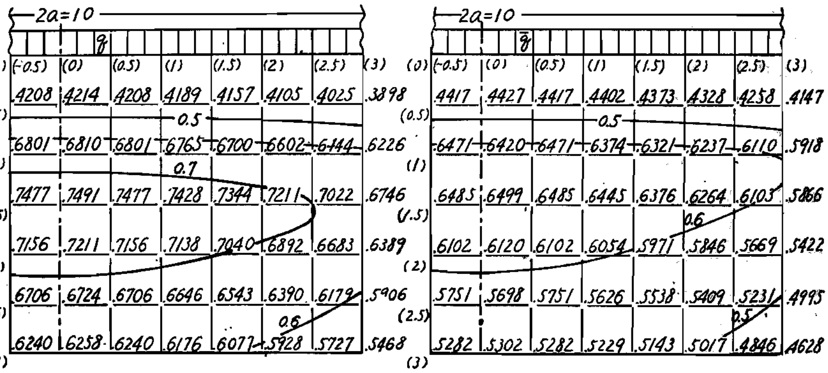
(a) Initial distribution ($t=0$) (b) After the time $t = \frac{1}{4c}$ (first approximation)

Fig. 2. Contour of w/q in the semi-infinite foundation where the uniformly distributed load q is applied on the surface.

when $4ct = 1$ is applied in Eq. (13). In the calculation, the foundation is devided into the square lattice $\Delta\lambda$, $\Delta\mu = 1$, and the contribution to the point under consideration from the self-point (x, y) and from the other nodal point (λ, μ) is accumulated. In this figure, as the first approximation, the contribution from the self-point (x, y) and from the nearest four points is accumulated with regard to each nodal point.

Fig. 3 (a) is the result of the more precise calculation with $\Delta\lambda$, $\Delta\mu = 0.5$, and the contribution from the forty-one points including the self-point is adopted. The value of error is at most 8 % in regard to each point. Fig. 3 (b) shows the ratio of the pore pressure w to the increased load $\bar{q}$ per unit time, which is applicable to the gradually increasing load represented in Eq. (19). The error of calculation is within 3～4 % only.

Next, in order to investigate the manner of change of the pore pressure in respect to time, Fig. 4 is shown at the nodal points $(x = 0,\ y = 1.5)$ and $(x = 2.5,\ y = 2.5)$. According to this figure, upon suddenly loading the pore pressure decreases with the time. On the other hand, it can be shown that



(a) Suddenly applied load ($4ct=1$) (b) Gradually increasing load ($4c=1$, $t=1$)

Fig. 3. Distribution of pore pressure in the semi-infinite foundation.

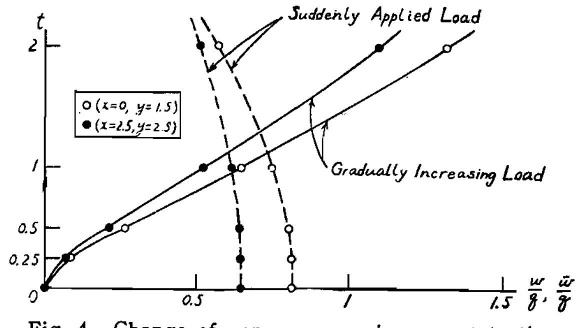


Fig. 4. Change of pore pressure in respect to time.

the pore pressure caused by the gradually increasing load continues to increase as long as the load increases.

4. Plastic Flow Mechanism of the Ground where the Pore Water Pressure Exists

In this article, as the second part of this paper, the theory of the execution control of fill work is investigated with respect to the pore pressure in the foundation.

During the execution of fill work upon the soft foundation, on the basis of piezometer measurement, the pore water pressure must be reduced lest the pore pressure becomes so high that a danger would approach respecting the shear strength of the foundation[4]. In respect to this execution control, Fig. 5 represents the pore pressure-time diagram. The upward part of this wave curve represents the pore pressure in the foundation increasing as the fill work proceeds, and the downward part gives the descrease of the pore pressure after the work stops. The horizontal dotted line in the figure is a critical line giving the maximum allowable pore pressure, and so if the peak of the wave exceeds this line, it should be known that the danger of rupture is approaching in the foundation. The writers wish to give the important key to solve such an execution problem, keeping the standpoint that the plastic flow mechanism of the ground where the pore pressure exists can be cleared by means of using the theoretical equations in respect to the distribution of the pore pressure in the foundation.

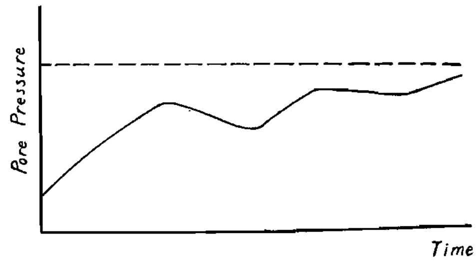


Fig. 5. Pore pressure-time diagram.

It has been cleared that, when the load is applied on ground where no pore pressure has existed and when the load is being increased gradually, the plastic region formed by points where the plasticity condition is satisfied begins to grow at the corner edges of the load and to enlarge as the load increases.[5] But this phenomenon is very complex if the ground has a pore pressure which varies with the time.

Here, it is assumed that Eq. (20) represents the shear strength of the ground where the pore pressure exists:

$$\tau = C + (\sigma - w) \tan \varphi , \qquad \cdots\cdots\cdots\cdots\cdots(20)$$

where σ is the normal stress applied on the shear plane, w the pore pressure at the position under consideration, φ the angle of internal friction of soil, and C cohesion. As the pore pressure w in Eq. (20) is, during the consolidation process, the function of the time as well as of the situation, the shear strength τ varies not only with the situation but with the time.

The representation of the equation giving the plasticity load q, of the ground in such a case becomes the following equation, applying Mohr's flow condition to Eq. (20), and assuming that the coefficient of earth pressure at rest is unity. In the preceding description, the plasticity load means the surcharge load which causes principal stresses σ_1, σ_2 in the foundation whose ratio satisfies Mohr's flow condition.

$$\left.\begin{aligned} q &= \eta_c C + \eta_b \gamma_s B\,; \\ \eta_c &= \frac{2\cos\varphi}{\mu_1 - \mu_2 - (\mu_1 + \mu_2)\sin\varphi + 2w^*\sin\varphi}, \\ \eta_b &= \eta_c \tan\varphi \frac{\cos 2\varepsilon + \cos 2\psi}{2\sin 2\varepsilon}, \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(21)$$

where γ_s is the unit weight of soil, B the width of the load, η_c, η_b the coefficients of plasticity load, μ_1, μ_2 the specific stresses under ground ($\mu_1 = \sigma_1/q$, $\mu_2 = \sigma_2/q$). $w^* = w/q$ is the new term introduced with the existence of the pore pressure, and is a function of the time, with the result that the coefficients of plasticity load η_c, η_b are functions of the time. For the above reason the plasticity load q varies with the time.

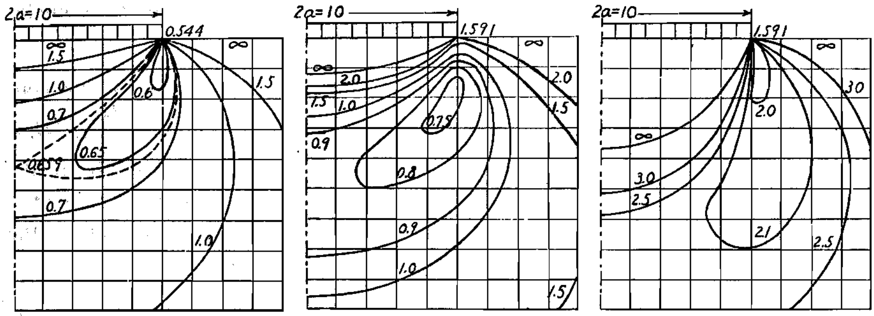
Let such a case be considered where a uniformly distributed load whose width is $B = 2a$ is applied on the surface of the homogeneous foundation. Under this condition the coefficients of plasticity load in Eq. (21) are represented in the following form, using Eq. (12) :

$$\left.\begin{aligned} \eta_c &= \frac{\pi\cos\varphi}{\sin 2\varepsilon - 2\varepsilon\sin\varphi + \pi w^*\sin\varphi}, \\ \eta_b &= \eta_c \tan\varphi \frac{\text{cis}\, 2\varepsilon + \cos 2\psi}{2\sin 2\varepsilon}, \\ w^* &= \frac{1}{4\pi^2 ct}\int_{-\infty}^{\infty} d\lambda \int_0^{\infty} d\mu \cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu} \\ &\quad \times\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4ct}\right\} - \exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4ct}\right\}\right]. \end{aligned}\right\} \cdots(22)$$

By means of Eq. (22), the coefficients of plasticity load in respect to situation and time can be determined when the value of φ and the distribution of pore pressure w^* at the arbitrary time t are known.

In a particular case, namely, when the load is just applied at $t = 0$, $w^*(0) = 2\varepsilon/\pi \;\therefore\; \eta_c(0) = \pi\cos\varphi/\sin 2\varepsilon$. And after the consolidation process of the foundation finishes at $t = \infty$, $w^*(\infty) \to 0$ and the above Eq. (22) becomes the customary representation for the foundation where no pore pressure exists.

As the result of the numerical calculation corresponding to the above equations, using the assumptions $\varphi = 30°$, $C = 0.2\,\text{kg/cm}^2$, $\gamma_s = 1.6$, Figs. 6(a), (b) and (c) are obtained for the line of equi-plasticity load with regard to



(a) $t = 0$ (b) $t = 1/4c$ (c) $t = \infty$

Fig. 6. Line of equi-plasticity load in the semi-infinite foundation.

the time $t = 0$, $1/4c$ and ∞, respectively. According to these figures, it can be seen that, at the early period of the consolidation, as the high pore pressure exists in the foundation, the plasticity load at every point is very small compared with the later period.

5. Conclusion

In this paper, starting from the fundamental theoretical equation of the two-dimensional consolidation, the distribution of the pore water pressure in the embankment foundation is studied theoretically. Next, by using the above solutions, the plasticity load in the foundation where the pore pressure exists is obtained through numerical calculation.

In closing, it becomes clear that the above plastic flow mechanism during the consolidation process is a serious factor in the execution control of fill work.

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Appendix I; Distribution of Pore Water Pressure in the Semi-Infinite Foundation Caused by the Parabolic Load

The Pressure distribution on the embankment foundation can be considered such a shape as a parabola whose average intensity q is $\frac{B+B_1}{2B}H\gamma_s$, where H is the height, B, B_1 are the lower width and upper width of the embankment respectively, and γ_s is the unit weight of the embankment material. Under this loading condition, $p_0(\xi)$ in Eq. (6) is written in the form of the following:

$$p_0(\xi) = \frac{3}{2}q\left(1-\frac{\xi^2}{a^2}\right), \quad -a<\xi<a. \qquad \cdots\cdots\cdots\cdots\cdots(23)$$

Putting Eq. (23) into Eq. (5):

$$\begin{aligned}
-\sigma_y = q_y(x) &= \frac{2}{\pi}\int_0^\infty d\lambda\int_0^\infty d\xi p_0(\xi)e^{-\lambda y}(1+y\lambda)\cos\lambda x\cos\lambda\xi \\
&= \frac{3q}{2\pi}\int_0^a d\xi\left(1-\frac{\xi^2}{a^2}\right)\int_0^\infty d\lambda e^{-\lambda y}(1+y\lambda)\{\cos(x+\xi)\lambda+\cos(x-\xi)\lambda\} \\
&= \frac{3q}{2\pi}\int_0^a d\xi\left(1-\frac{\xi^2}{a^2}\right)y\left[\frac{1}{y^2+(x+\xi)^2}+\frac{1}{y+(x-\xi)^2}\right. \\
&\qquad \left.+\frac{y^2-(x+\xi)^2}{\{y^2+(x+\xi)^2\}^2}+\frac{y^2-(x-\xi)^2}{\{y^2+(x-\xi)^2\}^2}\right] \\
&= \frac{3q}{2\pi}\left\{\frac{2y}{a}+\frac{a^2+y^2-x^2}{a^2}\left(\tan^{-1}\frac{x+a}{y}-\tan^{-1}\frac{x-a}{y}\right)\right. \\
&\qquad \left.-\frac{2y^2}{a^2}\left(\tan^{-1}\frac{x+a}{a}-\tan^{-1}\frac{x-a}{a}\right)\right\} \\
&= \frac{3q}{\pi}\frac{\cos 2\varepsilon+\cos 2\psi}{\sin^2 2\varepsilon}(\sin 2\varepsilon-2\varepsilon\cos 2\varepsilon)
\end{aligned}$$

Similarly,

$$-\sigma_x = q_x(x) = \frac{3q}{\pi}\frac{\cos 2\varepsilon+\cos 2\psi}{\sin^2 2\varepsilon}\left\{2\varepsilon(\cos 2\varepsilon+2\cos 2\psi)-3\sin 2\varepsilon +2\sin 2\psi\log\left|\frac{\cos\beta_1}{\cos\beta_2}\right|\right\},$$

$$-\tau_{xy} = q_{xy}(x) = \frac{3q}{\pi}\frac{\cos 2\varepsilon+\cos 2\psi}{\sin^2 2\varepsilon}\left\{2\varepsilon\sin 2\psi -(\cos 2\varepsilon+\cos 2\psi)\log\left|\frac{\cos\beta_1}{\cos\beta_2}\right|\right\} \qquad \cdots(24)$$

The initial distribution of pore pressure w_0 is:

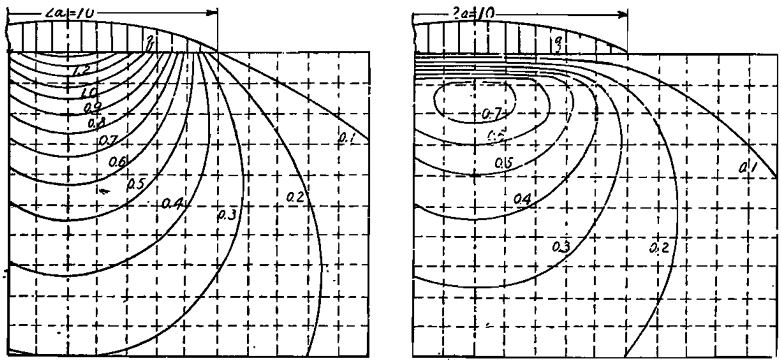
$$\begin{aligned}
w_0 &= \frac{1}{2}(\sigma_1+\sigma_2) = \frac{1}{2}(\sigma_x+\sigma_y) \\
&= \frac{3q}{\pi}\frac{\cos 2\varepsilon+\cos 2\psi}{\sin^2 2\varepsilon}\left\{2\varepsilon\cos 2\psi-\sin 2\varepsilon+\sin 2\psi\log\left|\frac{\cos\beta_1}{\cos\beta_2}\right|\right\}
\end{aligned}$$

$$= \frac{3q}{\pi}\left\{\frac{a^2+y^2-x^2}{2a^2}\cot^{-1}\frac{x^2+y^2-a^2}{2ay}-\frac{y}{a}+\frac{xy}{2a^2}\log\left|\frac{y^2+(x+a)^2}{y^2+(x-a)^2}\right|\right\} \quad (25)$$

The solution of Eq. (9) for the above initial condition is:

$$w = \frac{3q}{4\pi^2 ct}\int_{-\infty}^{\infty} d\lambda \int_0^{\infty} d\mu \left\{\frac{a^2+\mu^2-\lambda^2}{2a^2}\cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}-\frac{\mu}{a}+\frac{\lambda\mu}{2a^2}\log\left|\frac{\mu^2+(\lambda+a)^2}{\mu^2+(\lambda-a)^2}\right|\right\}$$
$$\times\left[\exp\left\{-\frac{(x-\lambda)^2+(y-\mu)^2}{4ct}\right\}-\exp\left\{-\frac{(x-\lambda)^2+(y+\mu)^2}{4ct}\right\}\right]. \quad \cdots\cdots\cdots\cdots(26)$$

The result of the numerical calculation for Eqs. (25), (26) is shown in Figs. 7(a), (b) respectively.



(a) Initial distribution ($t = 0$) (b) After the time $t = \frac{1}{4c}$ (first approximation)

Fig. 7. Contour of w/q in the semi-infinite foundation where the parabolic load q is applied on the surface.

Appendix II; Distribution of Pore Water Pressure in the Foundation of a Finite Depth

Let it be assumed that the embankment foundation has a perfectly rigid and smooth rock base horizontally at the depth h. In this case, the horizontal surface of rock base is taken as the x-axis and the positive y-axis as downwards from the center of embankment base. Assuming the stress functions

$$\left.\begin{aligned} \varphi_0 &= A\cosh\lambda y\cos\lambda x, \\ \varphi_y &= B\sinh\lambda y\cos\lambda x, \\ \varphi_x &= 0 \quad (A, B, \lambda: \text{consts.}) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(27)$$

and introducing the load condition of $q_0\cos\lambda x$ on the embankment foundation, the stress components become as follows:

$$\left.\begin{aligned}
-\sigma_x = q_x &= \frac{2(-\lambda h \cosh \lambda h \cosh \lambda y + \lambda y \sinh \lambda h \sinh \lambda y + \sinh \lambda h \cosh \lambda y)}{\sinh 2\lambda h + 2\lambda h} q_0 \cos\lambda x, \\
-\sigma_y = q_y &= \frac{2(\lambda h \cosh \lambda h \cosh \lambda y - \lambda y \sinh \lambda h \sinh \lambda y + \sinh \lambda h \cosh \lambda y)}{\sinh 2\lambda h + 2\lambda h} q_0 \cos \lambda x , \\
-\tau_{xy} = q_{xy} &= \frac{2(-\lambda h \cosh \lambda h \sinh \lambda y + \lambda y \sinh \lambda h \cosh \lambda y)}{\sinh 2\lambda h + 2\lambda h} q_0 \cos \lambda x .
\end{aligned}\right\} \qquad \cdots\cdots(28)$$

Using Fourier's integral for the uniformly distributed load $p_0(\xi) = q$,

$$\left.\begin{aligned}
& q_y(x) = \frac{q}{\pi}\left\{\int_0^\infty g(\alpha) \sin b_1\alpha d\alpha - \int_0^\infty g(\alpha) \sin b_2\alpha d\alpha\right\}, \\
\text{where}\quad & g(\alpha) = \frac{2(\beta\alpha \cosh \beta\alpha \cosh \alpha - \alpha \sinh \beta\alpha \sinh \alpha + \sinh \beta\alpha \cosh \alpha)}{\alpha(\sinh 2\beta\alpha + 2\beta\alpha)}, \\
& \alpha = y\lambda,\ \beta = \frac{h}{y},\ b_1 = \frac{x+a}{y},\ b_2 = \frac{x-a}{y}
\end{aligned}\right\} \qquad \cdots\cdots(29)$$

Boundary conditions for Eq. (9) are:

$$\left(\frac{\partial w}{\partial y}\right)_{y=0} = 0 , \qquad \cdots\cdots(30)$$

$$(w)_{y=h} = 0 . \qquad \cdots\cdots(31)$$

Initial condition is:

$$(w)_{t=0} = f(x, y) . \qquad \cdots\cdots(32)$$

Under these conditions, putting $w(x, y, t) = T(t)X(x)Y(y)$ into Eq. (9), the following solution is obtained.

$$\begin{aligned}
w &= \frac{2}{\pi h}\sum_{n=0}^{\infty} \exp\left\{-\frac{(2n+1)^2\pi^2 ct}{4h^2}\right\} \cos\frac{(2n+1)\pi}{2h} y \\
&\quad \times \int_0^\infty d\alpha \int_{-\infty}^{\infty} d\lambda \int_0^h d\mu f(\lambda, \mu) \exp\{-c\alpha^2 t\} \cos\alpha(x-\lambda) \cos\frac{(2n+1)\pi}{2h}\mu \\
&= \frac{1}{h\sqrt{\pi ct}}\sum_{n=0}^{\infty} \exp\left\{-\frac{(2n+1)^2\pi^2 ct}{4h^2}\right\} \cos\frac{(2n+1)\pi}{2h} y \\
&\quad \times \int_{-\infty}^{\infty} d\lambda \int_0^\infty d\mu f(\lambda, \mu) \exp\left\{-\frac{(x-\lambda)^2}{4ct}\right\} \cos\frac{(2n+1)\pi}{2h}\mu .
\end{aligned} \qquad \cdots\cdots(33)$$

Appendix III; Distribution of Pore Water Pressure in the Semi-Infinite Foundation of Anisotropic Permeability

The average permeability of a natural soil deposit in a horizontal direction (coefficient of permeability, k_x) is from 2 to 10 or more times that in the vertical direction k_y. The fundamental equation of two-dimensional consolidation for this embankment foundation of anisotropic permeability is:

$$\left.\begin{aligned}\frac{\partial w}{\partial t} &= c_x\frac{\partial^2 w}{\partial x^2}+c_y\frac{\partial^2 w}{\partial y^2}\,;\\ c_x &= \frac{k_x}{\gamma v},\quad c_y=\frac{k_y}{\gamma v}\,.\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots(34)$$

The coefficient of consolidation c_x, c_y defined in Eq. (34) is assumed constant in each axis direction. Let it be assumed that a uniformly distributed load having the width $B=2a$ and the intensity q is surcharged on the surface of a semi-infinite foundation.

Boundary condition for Eq. (34) is :

$$(w)_{y=0}=0\,.\qquad\cdots\cdots\cdots\cdots\cdots(35)$$

Initial condition is same with Eq. (11), namely:

$$(w)_{t=0}=f(x,\,y)=\frac{q}{\pi}\cot^{-1}\frac{x^2+y^2-a^2}{2ay}\,.\qquad\cdots\cdots\cdots\cdots\cdots(36)$$

Putting $w(x, y, t) = T(t)X(x)Y(y)$ into Eq. (34), and applying Eqs. (35), (36), the solution of the fundamental equation becomes as follows:

$$w=\frac{q}{4\pi^2\sqrt{c_xc_yt}}\int_{-\infty}^{\infty}d\lambda\int_0^{\infty}d\mu\cot^{-1}\frac{\lambda^2+\mu^2-a^2}{2a\mu}\left[\exp\left\{-\frac{(x-\lambda)^2}{4c_xt}-\frac{(y-\mu)^2}{4c_yt}\right\}\right.$$
$$\left.-\exp\left\{-\frac{(x-\lambda)^2}{4c_xt}-\frac{(y+\mu)^2}{4c_yt}\right\}\right].\qquad\cdots\cdots\cdots\cdots\cdots(37)$$

If the permeability of the foundation is isotropic, putting $c_x=c_y=c$ in Eq. (37) gives the same equation as Eq. (12).

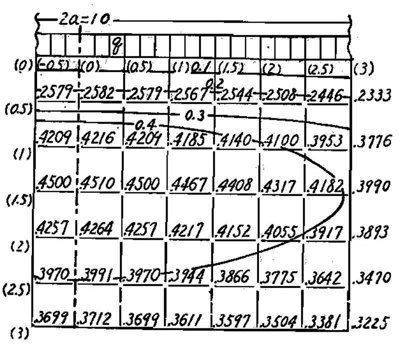


Fig. 8. Distribution of pore pressure in the semi-infinite foundation of anisotropic permeability.

As the numerical calculation of Eq. (37) under the conditions $2a=10$, $k_x=9k_y$, and $4c_yt=1$, Fig. 8 is shown, giving the distribution of pore pressure w/q. Comparing Fig. 8 with Fig. 3 (a), which gives w/q for the foundation of isotropic permeability, it may be concluded that the intensity of pore pressure in the former type of foundation is about as small as 60% of the latter, and that the type of the pressure distribution is similar in both Fig. 3 (a) and Fig. 8.