# Part II Considerations of the Earthquake Resistant Properties of Earth Dam

## 1. Introduction

The vibration character of earth dams having various dam lengths is discussed in Part 1, in which the earth dams are assumed visco-elastic. In this paper the author describes the earthquake resistant properties of earth dam from the standpoint of vibration on the basis of the results obtained in Part I. He also describes what size of seismic coefficient is to be adopted for dam design, when the conventional design method of seismic coefficient is used for evaluating the effect of seismic forces upon the structures. The method is based on such a way of thinking that the horizontal force of the magnitude of "the weight of structure multiplied by seismic coefficient" is assumed to act statically on the structures; and finally he makes a proposal concerning the seismic coefficient of design.

#### 2. Seismic Coefficient of One Mass System

Since the forced vibration of solid body is generally expressed by the summation of the free vibrations, each of the vibrations of the normal mode can be treated as the vibration of one mass system having one freedom. Hence, according to M. A. Biot's theory,<sup>1)</sup> at first the seismic coefficient of one mass system shall be discussed.

Letting u be, in Fig. 1, the displacement measured referring to the moving coordinate having the origin O, M be the mass, k the spring constant and  $u_0$  the ground motion, the

vibration of the structure assumed as one mass system may be given by

$$M\ddot{u}+D\dot{u}+ku=-M\ddot{u}_0,$$

or

where

D: coefficient of viscous resistance,

 $n^2 = k/M$ ,  $2\varepsilon = D/M$ ,  $a(t) = -\ddot{u}_0$ .



When the vibration begins at the initial state of stillness, the solution of Eq. (1) will be given as follows under the condition of u = 0,  $\dot{u} = 0$  at t = 0,

where

$$\omega = \sqrt{n^2 - \epsilon^2}$$

When the horizontal force F acts statically on the mass M causing deflection y, the force F which should be applied statically so as to cause M the dynamic deflection u is, using the relation F = ku,

$$F = Mn^2 u = M \frac{n^2}{\omega} \int_0^t e^{-\varepsilon(t-\tau)} \alpha(\tau) \sin \omega(t-\tau) d\tau,$$

and the seismic coefficient K given by K = F/Mg is

$$K = \frac{A}{g}$$
, .....(3)

where

$$A = \frac{\eta^2}{\omega} \int_0^t e^{-\varepsilon(t-\tau)} \alpha(\tau) \sin \omega(t-\tau) d\tau.$$

Therefore, when the external force  $W \cdot K$ —the weight multiplied by the seismic coefficient K given by Eq. (3) — is assumed to act statically, the deflection occured in this case is nothing but the one occured during the vibration expressed by Eq. (2). Thus, if the maximum value of Eq. (3) is used as the design seismic coefficient, the conventional design method may be used quite rationally in designing the structure. When the acceleration of ground motion  $\alpha(t)$  is as complicated as seen in the actual earthquakes, the torsion pendulum<sup>1),2)</sup> or the analyzer using the electrical method<sup>3)</sup> can be used for determining the value of A.

# 3. Determination of Seismic Coefficient on Dam

Expressing the vibration of dam by the moving coordinate having the origin at the crest of dam, and letting u be the displacement and  $\alpha(x, t)$  the acceleration of ground motion, the equation of motion of two-dimensional dam is given by

where  $\alpha(x,t) = -\ddot{u}_0(x,t)$  and  $c_2$  is the coefficient of viscous resistance proportional to the velocity, which is expressed by the moving coordinate. The boundary conditions are given by

The initial conditions are considered to be given by

 $u = 0, \quad \dot{u} = 0$  at t = 0. .....(6)

Assume that u and a(x,t) can be developed respectively by using the "Eigenfunctions" as follows:

$$u = \sum_{n=1}^{\infty} \sum_{s=1}^{\infty} \sin \frac{n\pi}{a} x J_o\left(\frac{\lambda_s}{h} y\right) T_{ns},$$
$$a(x,t) = \sum_{n=1}^{\infty} \sum_{s=1}^{\infty} \sin \frac{n\pi}{a} x J_o\left(\frac{\lambda_s}{h} y\right) \phi_{ns}(t),$$

where

$$\phi_{ns}(t) = \frac{4}{ah^2} \frac{1}{J_1^2(\lambda_s)} \int_0^a \int_0^h \alpha(\lambda, t) \sin \frac{n\pi}{a} \lambda J_0(\frac{\lambda_s}{h}\mu) \mu \, d\lambda \, d\mu,$$

and substitute the above into Eq. (4), we get the differential equation on T as follows:

$$\ddot{T}_{ns}+2\varepsilon\dot{T}_{ns}+n^2T_{ns}=\phi_{ns}(t),$$

where

$$n_0^2 = c_0^2 \{ (n\pi/a)^2 + (\lambda_s/h)^2 \},\$$
  

$$\varepsilon = (1/2) [c_1^2 \{ (n\pi/a)^2 + (\lambda_s/h)^2 \} + c_2^2 ].$$

For the case T=0,  $\check{T}=0$  at t=0, the solution of the above equation becomes

$$T_{ns} = \frac{1}{\omega} \int_{0}^{t} e^{-\varepsilon(t-\tau)} \phi_{ns}(\lambda_{s},\tau) \sin_{\omega}(t-\tau) d\tau,$$

therefore, u is given by

$$u = \frac{4}{a} \sum_{n=1}^{\infty} \sum_{s=1}^{\infty} \frac{1}{\lambda_s J_1(\lambda_s)} \sin \frac{n\pi}{a} x J_0\left(\frac{\lambda_s}{h} y\right)$$

When the distribution of acceleration of ground motion in the longitudinal direction of dam is uniform, we may rewrite Eq. (7) in the following form, by the reason that a(t) must be used instead of a(x, t),

$$u = \frac{8}{\pi} \sum_{n=1,s,\tau}^{\infty} \sum_{s=1}^{\infty} \frac{1}{n\lambda_s J_1(\lambda_s)} \sin \frac{n\pi}{a} x J_0\left(\frac{\lambda_s}{h} y\right) \\ \times \frac{1}{\omega} \int_0^t e^{-\varepsilon(t-\tau)} a(\tau) \sin \omega (t-\tau) d\tau. \quad \dots \dots \dots (8)$$

For the one-dimensional dam, the following solution may be obtained as the solution of the Eqs. (4), (5) and (6), in which all of the terms relating to x are omitted.

$$u = 2 \sum_{s=1}^{\infty} \frac{1}{\lambda_s J_1(\lambda_s)} J_0\left(\frac{\lambda_s}{h} y\right) \frac{1}{\omega_1} \int_0^t e^{-\varepsilon_1(t-\tau)} a(\tau) \sin \omega_1(t-\tau) d\tau, \quad \dots \dots (9)$$

where

$$\omega_1 = \sqrt{n_{01}^2 - \varepsilon_1^2}, \quad n_{01}^2 = c_2^2 (\lambda_s/h)^2,$$
  

$$\varepsilon = (1/2) \{ c_1^2 (\lambda_s/h)^2 + c_2^2 \}, \quad c_0^2 = G/\rho, \quad c_1^2 = \gamma_t/\rho, \quad c_2^2 = \delta'/\rho.$$

Though Eqs. (9), (7) and (8) are the equations which express the deflections during the vibration of the one- and two-dimensional dams, one mass system can be considered per each mode of n or n, s. Considering the relation given by Eq. (3), the seismic coefficient  $K_1$  and  $K_2$  of the one- and twodimensional dams can be shown respectively as follows:

For the one-dimensional dam:

$$K_{1} = \frac{2}{g} \sum_{s=1}^{\infty} \frac{1}{\lambda_{s} J_{1}(\lambda_{s})} J_{0}\left(\frac{\lambda_{s}}{h} y\right) A_{s}, \qquad (10)$$
$$A_{s} = \frac{n_{0}^{2}}{w_{1}} \int_{0}^{t} e^{-\varepsilon(t-\tau)} a(\tau) \sin \omega_{1}(t-\tau) d\tau.$$

For the two-dimensional dam:

for the case where the acceleration of ground motion is not uniformly distributed along the dam length;

for the case where the acceleration of ground motion is uniformly distributed along the dam length;

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Computing the shearing stress for the uniform motion of ground from the conventional seismic coefficient method, by the use of the above-mentioned seismic coefficients,

For the one-dimensinoal dam:

For the two-dimensional dam :

the shearing stress in the direction of dam height;

the shearing stress in the direction of dam length;

where

k = a/h.

When comparing the maximum value of  $S_x$  with that of  $S_y$  for the twodimensional dam,  $S_x$  is smaller than  $S_y$  when k=a/h is about  $2\sim3$  or larger. The value of k found in the actual dam is usually larger than the above obtained, hence  $S_y$  alone may be considered.

Eqs. (10), (11) and (12) give the rational value of seismic coefficient of each of the one- and two-dimensional dams, and the vibration of each mode has its particular phase difference  $\delta$ . Considering this point, the vibration of each mode must be added up. The maximum of the above-stated values must be adopted as the seismic coefficient of design. For this purpose, for example,  $A_n$  and  $A_{ns}$  must be determined to be the maximum obtained by means of synchronizing and summing up the amplitudes which can be got by recording the shakes of the torsion pendulum as to each mode of vibrations; so that the

procedure requires much time and labour. However, in general, the nearest mode of vibration to the period of ground motion is most influential and the other modes are considerably small, so that the most dangerous seismic coefficient — the seismic coefficient of design — can be obtained by comparing several terms of n or n, s with each other by using the acceleration spectrum measured as the one mass system. The distribution of seismic coefficient, when the dam is subjected to one or two ground motions, will be considered in the following space.

#### 3.1. One example of the actual earthquake

Fig. 2 shows the acceleration spectrum, obtained by means of above-men-

tioned torsion pendulum by using the ground acceleration of Tanabe Bay Earthquake which was recorded<sup>4</sup>) at the Kobe Marine Meteorological Observatory on Nov. 6, 1950. The figure is drawn for various



Fig. 2 Acceeleration spectrum.

values of the damping coefficient h.



Fig. 3 Record of acceleration of Tanabe Bay Earthquake, Nov. 6, 1950, recorded at the Kobe Marine Meteorological Observatory.

Now, consider, as one example, the case\* where  $c_0 = 95 \text{ m/s}$ , the height h = 40 m and the length a = 200 m. Determining the acceleration  $(A_s, A_{ns}, A_{ns})$ 

<sup>\*</sup> The period of the lst higher mode of free vibration of the one-dimensional dam in the case as shown here, is corresponding to the peak of  $0.47 \sim 0.48$  sec in the acceleration spectrum.

where  $\varepsilon = 0$  for the period of each mode of the free vibration of the dam from the acceleration spectrum shown in Fig. 2, and computing the maximum seismic coefficient of each mode on the crest by Eqs. (10) and (12), and then expressing them as the ratios to the seismic coefficient of ground  $K_0$ , we get following Table. Figs. 4 and 5 show the distributions of the seismic coefficient and of the shearing stress of the one-dimensional dam, respectively. As seen from the figures, in the shearing stress, if there is no damping, the lst higher mode is larger than the fundamental mode. This means the resonance of the lst higher order. It can be seen that there may occur the resonance of the lst higher order for the ground motion having such periods as expected usually, provided the dam is as high as  $30 \sim 40$  m. However, when there is a damping force due to internal viscosity ( $h_1 = 0.05$ ,  $h_2 = 0.12$ ), the shearing stress is smaller

Table Ratio  $K/K_0$  between maximum seismic coefficients of the earth dam and the ground.

s	One dimensio- nal dam	Two-dimensional dam. $a/h=3$				Two-dimensional dam. $a/h=5$			
		<b>n</b> =1	<b>n</b> =3	<b>n</b> =5	<b>n</b> =7	<b>n</b> =1	<b>n</b> =3	<b>n</b> =5	<b>n</b> =7
1	4.60	7.15	-3.64	1.15	-1.10	6.11	-2.40	2.54	-1.31
2	-8.70	-11. 15	1.44	-1.10	0.71	-10.95	2.04	-1.15	0. 59
3	3.40	4. 28	-1.45	0.82	-0.31	4.36	-1.45	0.82	-0.56
4	-1.46	-1.46	0.71	0.46	_	-1.86	0.71	-0:43	0. 38



Fig. 4 Distributions of the seismic coecifient.

Fig. 5 Distributions of the shearing stress.

than that of the fundamental mode all over the dam height. Therefore, if the dam of such size as stated above is subjected to such a ground motion, it is in safety side to design the dam by using the distribution of seismic coefficient for the fundamental mode of vibration (the curve corresponding to s=1 in Fig. 4), notwithstanding that the resonance of the lst higher mode occurs at this time. By the way, the location where the maximum shearing stress occurs in the above case is not at the base, but is nearly at the level of one-fourth of the height above the base.

The fact stated above is one example of the case where the dam is subjected to the actual seismic motion. For the case of a peculiar ground motion, at the present when little data of the seismic motion of the peculiar region are available for us, the appropriate ground motion must be presumed. From such a standpoint, let us consider the initial state of vibration of the dam, when it is subjected to the seismic acceleration of sinusoidal form having the same period as that of free vibration of the dam, in the following space.

# 3.2. Distribution of seismic coefficient in the initial state of vibration

Consider, for simplicity's sake, the case where such a ground motion as stated before acts on the dam under the condition of no damping. Since the mode having the same period as that of the ground motion becomes remarkably larger compared with the other modes, the vibration of whole dam may be approximately expressed by the resonant mode. Therefore, the accelerations of the ground motion  $a_0(s)$ ,  $a_0(n, s)$  having the same period as that of each mode of the dam will act on the dam, and the seismic coefficients of the dam, after one period passes, are given by

$$K_{1} = \frac{2}{g} \frac{1}{\lambda_{s} J_{1}(\lambda_{s})} J_{0}\left(\frac{\lambda_{s}}{h} y\right) a_{0}(s) \pi,$$

$$K_{2} = \frac{8}{g\pi} \frac{1}{n\lambda_{s} J_{1}(\lambda_{s})} \sin \frac{n\pi}{a} x J_{0}\left(\frac{\lambda_{s}}{h} y\right) a_{0}(n, s) \pi.$$

$$(16)$$

When the ground accelerations  $a_0(s)$  and  $a_0(n, s)$  are constant, the coefficients in the above Eqs. show the ratios of the seismic coefficients in the resonance of s th and n,s th order for the constant acceleration, respectively. Similarly, for the case of constant velocities the coefficients are given by

$$K_{1} = \frac{2c_{0}}{gh} \frac{1}{J_{1}(\lambda_{s})} J_{0}\left(\frac{\lambda_{s}}{h}y\right) v_{0}(s) \pi,$$

$$K_{2} = \frac{8c_{0}}{g\pi h} \frac{1}{n\lambda_{s}J_{1}(\lambda_{s})} \left\{ \left(\frac{n\pi}{a}\right)^{2} + \lambda_{s}^{2} \right\}^{1/2} \sin\frac{n\pi}{a} x J_{0}\left(\frac{\lambda_{s}}{h}y\right) v_{0}(n,s) \pi,$$

$$(17)$$

where  $v_0(s)$ ,  $v_0(n, s)$  mean the constant velocities. Moreover, the shearing stresses can be computed from Eqs.  $(13)\sim(15)$  as follows:

For the case of constant acceleration

$$S_{1y} = -2h\rho \frac{1}{\lambda_s^2 J_1(\lambda_s)} J_1(\frac{\lambda_s}{h} y) a_0(s) \pi,$$

$$S_{2y} = -\frac{8h\rho}{\pi} \frac{1}{n J_1(\lambda_s)} \frac{1}{(n\pi/k)^2 + \lambda_s^2} \sin \frac{n\pi}{a} x J_1(\frac{\lambda_s}{h} y) a_0(n,s) \pi,$$

$$S_{2x} = 8\rho h \frac{1}{\lambda_s J_1(\lambda_s)} \frac{1}{k \{(n\pi/k)^2 + \lambda_s^2\}} \cos \frac{n\pi}{a} x J_0(\frac{\lambda_s}{h} y) a_0(n,s) \pi.$$
For the case of constant velocity:

For the case of constant velocity;

$$S_{1y} = -2 c_{0\rho} \frac{1}{\lambda_{s} J_{1}(\lambda_{s})} J_{1}\left(\frac{\lambda_{s}}{h}y\right) v_{0}(s) \pi,$$

$$S_{2y} = -\frac{2c_{0\rho}}{\pi} \frac{1}{n J_{1}(\lambda_{s})} \frac{1}{\sqrt{(n\pi/k)^{2} + \lambda_{s}^{2}}} \sin \frac{n\pi}{a} x J_{1}\left(\frac{\lambda_{s}}{h}y\right) v_{0}(n,s) \pi,$$

$$S_{2x} = 8 c_{0\rho} \frac{1}{\lambda_{s} J_{1}(\lambda_{s})} \frac{1}{k \sqrt{(n\pi/k)^{2} + \lambda_{s}^{2}}} \cos \frac{n\pi}{a} x J_{0}\left(\frac{\lambda_{s}}{h}y\right) v_{0}(n,s) \pi.$$
(19)

Fig. 6 and 7 illustrate the distributions of seismic coefficient and of shearing stress for the one-dimensional case. Since there hardly occur the resonances of higher order for the sake of the correlative relation with the period of the seismic motion, the resonances of the higher order less than the 2 nd are shown in the Figs.

As seen from these Figs., for the case of the constant acceleration without damping, the shearing stress shows the maximum value in the resonance of the fundamental mode; accordingly, the dam is safe for the resonance of higher order, only when the distribution of seismic coefficient of the fundamental mode is taken into account in the design. For the case of constant velocity, however, the stress in the resonance of higher order is larger in the upper part than that in the resonance of the fundamental mode. The dotted lines in Fig. 7 b) show the distribution of the shearing stress, when the internal viscous damping  $(h_1$ =0.05,  $h_2$ =0.12,  $h_3$ =0.18) acts. It also shows that there is no remarkable difference between the stress distributions in the resonance of the fundamental mode and in that of higher order. Therefore, it may be safe as well for the resonance of higher order to take into account the distribution of seismic coefficient



crest length of the two-dimensional dam subjected to ground motion having constant velocity (damping coefficient h=0), showing that the vibration characters of the fundamental mode in the direction of the height and of the mode of higher order in the direction of the length  $(n \ge 2, s=1)$  give the larger stress in the central part of dam than that caused by the resonance of the fundamental mode (n=1, s=1).

# 4. One Proposition for the Seismic Coefficient of Design

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The seismic coefficient as described above is obtained by presuming the constant acceleration or constant velocity. According to the latest studies, it is clarified that the constant velocity occurs in general. For example, the seismic spectrum proposed as the seismic coefficient of design of the upper part of building by R. Tanabashi and others<sup>5</sup>), on the basis of the data recorded in Kanto Earthquake (1923), is of the constant velocity type much the same as the standard acceleration spectrum<sup>6</sup>) proposed by the Joint Committee of the San Francisco, U.S.A., and it is practised in general that small seismic coefficient will be adopted for the structures having long period of vibration. T. Hatano<sup>7)</sup> also points out that the stability during the earthquake is more widely affected by the velocity than by the acceleration.



Fig. 8 Distributions of the shearing stress  $S_x$  in the direction of dam length.

As to the absolute value of the seismic coefficient, the discussion made on building is not always the same as that on dam. Nowadays, many points still remains unexplained on the character of ground motion. Hence, with reference to the value of seismic coefficient conventionally used in designing dam, the author dare propose a seismic coefficient of design described in the following space.

For the top of dam:

usually 0.3/T instead of 0.2/T proposed by R. Tanabashi and others, the upper limit 0.5 and the lower limit 0.15 (corresponding to the upper limit 0.3 and the lower limit 0.09 for the uniform distribution of seismic coefficient).

Along the height:

the Bessel's distribution



b) Distribution of the seismic coefficient in the direction of dam height



c) Distribution, of the seismic coefficient in the direction of dam length



Fig. 9 Seismc coefficient of design.

The seismic spectrum the distribution of and seismic coefficient proposed above are illustrated in Fig. 9 a) and b). Since the earth dam, in general, has a constant grade of faces of slope in the length direction, it may be built uniformly of the same material and by the same construction method. Thereif the height is fore, also constant, the dam may have the one-dimensionally uniform safety over its whole length, but, judging two-dimensionally, seismic coefficient is too large in Accordingly, both sides. the smaller seismic coefficient should be adopted

in both sides, and the distribution of the coefficient may be determined, in entirely the same way as that was done in order to determine the distribution in the direction of height, examining the stresses in two-dimensional dam by Eqs. (18) and (19). As stated before, however, the restraint of both sides has little effect upon the central part of dam when the ratio of length to height is larger than  $3\sim5$ ; thus, the distribution of seismic coefficient may be determined in such a way that it decreases in a sinusoidal form from the onedimensional value at the central part to the zero value at both ends, at each of the parts having the length two times as large as the dam height from each end, as shown Fig. 9, c). Fig. 9, c) corresponds to the case of constant height, but the dams usually built decrease generally in heights from the central parts to both sides. It is necessary, considering one-dimensionally, to assume the larger seismic coefficient at the part of smaller height. In such dams, the distribution of seismic coefficient in the direction of the dam length may be assumed to be nearly constant.

# 5. Conclusion

This paper discusses mainly the distribution of seismic coefficient of design, and presents its absolute value, only as a provisional standard. Of course, the absolute value of seismic coefficient should be determined by taking into account the regional distribution of earthquake frequency<sup>8</sup>, and the character of subbase soil.<sup>9</sup>

The studies reported in this paper are based on the theory that the dam is visco-elastic, but such a theory may have something to be questioned, as to considering the soil visco-elastic. However, consideration that the dam deforms due to seismic forces leads to the conclusion that the seismic coefficient to be used in dam design cannot be uniform in every part of the dam. Further investigations are needed in solving such problems.

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