DISASTER PREVENTION RESEARCH INSTITUTE
BULLETIN No. 11 DECEMBER, 1955

FUNDAMENTAL CONSIDERATIONS ON THE EARTHQUAKE RESISTANT PROPERTIES OF THE EARTH DAM

BY

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Contents

Synopsis

The remarkable advance has been achieved in the designing and execution of earth dam in accordance with the development which has been made up to now in the soil engineering. Earth dams as high as 40 m have been constructed also in our country. It is often the case that the earth dams above all other kinds of dams may be constructed on the sites where the foundations of earth ground are comparatively weak. Therefore, the dams should be expected to be subjected to large seimic forces, and once they collapse, the failure will cause unforeseen disaster to the downstream region. Accordingly, it is a matter of course that the study on earthquake resistant properties of earth dam is extremely important.

This paper discusses, in part I as the first step toward the clearer understanding of earthquake resistant properties of earth dam, the elastic vibration of the two-dimensional dam surrounded by the ground foundation of rectangular boundary, and clarifies the limit of the possibility of treating the problem as the one-dimensional by comparing the above mentioned vibration with that of the one-dimensional dam. In part II, discussions are made on the seismic coefficient to be used for the earthquake resistant design of the two-dimensional earth dam, and a seismic coefficient of design is proposed.

Part I On the Vibration of Earth Dam

1. Introduction

The stability of earth dam subjected to seismic forces is used to be computed by the method of calculating the statical stability of the sliding surface under the condition of uniform seismic coefficient. It is, however, considered that there is a great need for us to use the calculation method based on the dynamical standpoint taken into account the deformation of the dam, especially for high earth dam.

The studies by M. Matsumura[1] and by E. E. Esmiol[2] were made on the deformation of dam on the basis of the above mentioned standpoint. M. Matsumura discusses that the shear vibration is far important than the bending one for the structures, the base widths of which are large compared with the heights as seen in the earth dams, and studies each of the free vibration and the forced vibration due to stationary motion of ground, etc. one by one, for the one-dimensional dam infinitely long in the direction of dam axis. He also insists that the dam should be dsigned by using the acceleration taken into account the dynamic behaviour. E. E. Esmiol also proposes the method of designing the earth dam, aimed at the stresses caused in the dam subjected to the stationary vibration.

This paper discusses in the first place the effect of bending moment on the fundamental earth dam sections by using the beam theory, clarifying that the shear vibration may be considered more important than the bending one for the earth dam, and then takes up on the vibration behaviour of the earth dam as the first advance toward the investigation of its earthquake resistant properties. Considering that the actual dam is not of one-dimension but is affected by the foundation grounds of both sides, the discusion is mainly made on the basis of the two-dimensional point of view, presenting the limitation in which the one-dimensional treatment may be acceptable.

2. Vibration Characteristics of Dam with Fundamental Section

The studies on vibration of dam, discussing its stability under the action of seismic forces, have been made by M. Matsumura and by T. Hatano. M. Matsumura's study is made, as described above, on the shear vibration for the earth dam; and T. Hatano's[3] mainly on the bending vibration of the asymmetric fundamental triangular section, aiming at the gravity dam. The difference between their treatments is due to the magnitude of the grade of faces of slope. Accordingly, in this paper, the vibration taken into account the shearing force at the same time the bending moment based on the beam theory, (for simplicity's sake, this vibration is called, for the time being from now, as the shear-bending vibration,) is discussed, and the variation in the vibration character with the grade of faces of slope is studied. In considering the vibration character of gravity dam, it is insufficient to regard the vibration as the bending vibration only, but it is also necessary to take account of the vibration due to shearing force to some extent.

On the other hand, in case of earth dam, the author clarified that it is appropriate to regard the vibration as the shear vibration with satisfactory accuracy judged from the technical point of view.

2.1. Fundamental equations

Determine the coordinate axis as shown in Fig. 1, and we consider the vibration in the direction of z-axis. For simplicity's sake, the following assumptions are made:

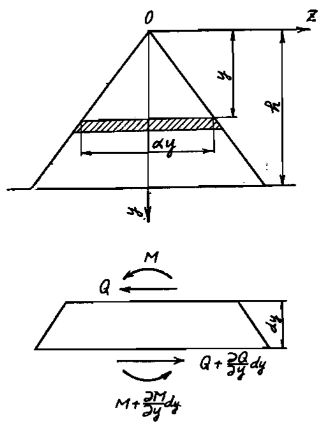


Fig. 1

a) Dam section is symmetrical.

b) Young's modulus E, modulus of rigidity G and density ρ are constant.

c) Bernoulli's assumption is valid, and the distribution of shearing stress is uniform.

Let w_1 be the deflection due to bending, w_2 the deflection due to shear, w the sum of w_1 and w_2, and I the geometrical moment of inertia of section, then the equations of motion are given by

$$\rho a y \frac{\partial^2 w}{\partial t^2}=\frac{\partial Q}{\partial y}, \quad \cdots\cdots\cdots\cdots(1)$$

$$\rho I \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w}{\partial y^2}\right)=-\frac{M}{EI}+Q. \quad \cdots\cdots\cdots\cdots(2)$$

The relations between displacement and stress are expressed by

$$\frac{\partial^2 w_1}{\partial y^2}=-\frac{M}{EI}, \quad \cdots\cdots\cdots\cdots(3)$$

$$\frac{\partial^2 w}{\partial y^2}=-\frac{M}{EI}+\frac{1}{G}\frac{\partial}{\partial y}\left(\frac{Q}{ay}\right) \quad \cdots\cdots\cdots\cdots(4)$$

Eliminating M, Q, and w_1 from the above four equations, we have the following equation representing the shear-bending vibration, considering the inertia of rotation,

$$E\left(\frac{y^3}{12}\frac{\partial^7 w}{\partial y^7}+\frac{5}{4}y^2\frac{\partial^6 w}{\partial y^6}+5y\frac{\partial^5 w}{\partial y^5}+5\frac{\partial^4 w}{\partial y^4}\right.-\rho\frac{\partial^2}{\partial t^2}\left\{\frac{y^3}{12}\frac{\partial^5 w}{\partial y^5}+y^2\frac{\partial^4 w}{\partial y^4}\right.$$

$$\left.+y\left(3-\frac{1}{a^2}\right)\frac{\partial^3 w}{\partial y^3}+\left(2-\frac{3}{a^2}\right)\frac{\partial^2 w}{\partial y^2}\right\}-\frac{\rho E}{G}\frac{\partial^2}{\partial t^2}\left(\frac{y^3}{12}\frac{\partial^5 w}{\partial y^5}+\frac{7}{6}y^2\frac{\partial^4 w}{\partial y^4}\right.$$

$$+\frac{17}{4}y\frac{\partial^3 w}{\partial y^3}+\frac{15}{4}\frac{\partial^2 w}{\partial y^2}\Big)+\frac{\rho^2}{G}\frac{\partial^4}{\partial t^4}\Big(\frac{y^3}{12}\frac{\partial^3 w}{\partial y^3}+\frac{11}{12}y^2\frac{\partial^2 w}{\partial y^2}+\frac{7}{3}y\frac{\partial w}{\partial y}+w\Big)=0. \quad \cdots\cdots\cdots(5)$$

Neglecting the effect of the inertia of rotation, as it is generally so small, we get

$$E\left(\frac{y^2}{12}\frac{\partial^4 w}{\partial y^4}+\frac{y}{2}\frac{\partial^3 w}{\partial y^3}+\frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right)$$

$$-\rho\frac{\partial^2}{\partial t^2}\left\{\frac{E}{12G}y^2\frac{\partial^2 w}{\partial y^2}+\frac{5E}{12G}y\frac{\partial w}{\partial y}+\left(\frac{E}{4G}-\frac{1}{\alpha^2}\right)w\right\}=0. \quad \cdots\cdots\cdots(6)$$

When $G\longrightarrow\infty$ in the above equation, Eq. (6) becomes

$$E\left(\frac{y^2}{12}\frac{\partial^4 w}{\partial y^4}+\frac{y}{2}\frac{\partial^3 w}{\partial y^3}+\frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right)+\frac{\rho}{\alpha^2}\frac{\partial^2 w}{\partial t^2}=0, \quad \cdots\cdots\cdots(7)$$

and when $E\longrightarrow\infty$, Eq. (6) becomes

$$G\left(\frac{y^2}{12}\frac{\partial^4 w}{\partial y^4}+\frac{y}{2}\frac{\partial^3 w}{\partial y^3}+\frac{1}{2}\frac{\partial^2 w}{\partial y^2}\right)$$

$$-\rho\frac{\partial^2}{\partial t^2}\left(\frac{y^2}{12}\frac{\partial^2 w}{\partial y^2}+\frac{5y}{12}\frac{\partial w}{\partial y}+\frac{w}{4}\right)=0. \quad \cdots\cdots\cdots(8)$$

Both Eqs. (7) and (8) are nothing other than the equations of bending and shear vibration, respectively.

2.2. Numerical calculation

Numerical calculations are made in the following, where we indicate how the period of free vibration and the form of vibration are affected by the magnitude of grade of faces of dam slope, in each of the three cases when the vibration of dam is considered only as the shear, the bending and the shear-bending vibration respectively.

Neglect, for simplicity's sake, the inertia of rotation and we consider firstly the shear-bending vibration. If we put $z=y/h$, $w=X(z)\sin\omega t$, Eq. (6) becomes

$$Az^2\frac{d^4X}{dz^4}+6Az\frac{d^3X}{dz^3}+(6A+Bz^2)\frac{d^2X}{dz^2}+5Bz\frac{dX}{dz}$$

$$+(3B+C)X=0, \quad \cdots\cdots\cdots(9)$$

where $A=E/12h^2$, $B=\rho E\omega^2/12G$, $C=-\rho\omega^2/\alpha^2$.

Expressing $M=\bar{M}\sin\omega t$, $Q=\bar{Q}\sin\omega t$, the boundary conditions may be expressed as follows:

$$\left.\begin{array}{llll} \bar{M}=0, & \bar{Q}=0 & \text{at} & x=0, \\ X=0, & \dfrac{dX}{dz}=\dfrac{\bar{Q}}{G\alpha z} & \text{at} & z=1. \end{array}\right\} \quad \cdots\cdots(10)$$

When we use the following power series as the solution satisfying Eqs. (9) and (10)

$$X=\sum_{n=0}^{\infty} b_n z^n, \quad \cdots\cdots(11)$$

the coefficient b_n in Eq. (11) can be determined from Eqs. (9) and (10), and the form of free vibration may be found. The period of free vibration can be solved from the roots of the following Eq. (12), where $B/A=\rho h^2\omega^2/G=m$ is used,

$$\left(7K_7-5K_6-\frac{5}{14}K_5+\frac{1}{6}K_4+3K_5K_2-K_4K_3-\frac{1}{20}K_3K_2\right)m^3$$
$$+\left(5K_5-3K_4+K_3K_2-\frac{3}{10}K_3+\frac{1}{12}K_2\right)m^2$$
$$+\left(3K_3-K_2-\frac{1}{6}\right)m+1=0, \quad \cdots\cdots(12)$$

where $K_2, K_3, \cdots\cdots$ are functions of $b_2, b_3, \cdots\cdots$ respectively. But in this case we take up the vibration of fundamental mode only, because of its predominance, and neglect the coefficients of the terms of higher order than m^4 as they are so small. Letting m_0 be the minimum positive root of Eq. (12), the period T_{BS} of the fundamental mode may be given by

$$T_{BS}=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{m_0}}\sqrt{\frac{\rho}{G}}\,h. \quad \cdots\cdots(13)$$

Assume that the Poisson's ratio σ is 0.15 for concrete and 0.35 for earth and use the relationship of $G=E/2(1+\sigma)$, then we get

$$\left.\begin{array}{l} T_{BS}=\dfrac{9.528}{\sqrt{m_0}}\sqrt{\dfrac{\rho}{E}}\,h, \ (\sigma=0.15), \\ T_{BS}=\dfrac{10.324}{\sqrt{m_0}}\sqrt{\dfrac{\rho}{E}}\,h, \ (\sigma=0.35). \end{array}\right\} \quad \cdots\cdots(14)$$

As to the bending vibration, also neglecting the inertia of rotation and denoting $z=y/h$, $w=X(z)\sin\omega t$, we have

$$Az^2\frac{d^4X}{dz^4}+6Az\frac{d^3X}{dz^3}+6A\frac{d^2X}{dz^2}-B'X=0, \quad \cdots\cdots\cdots\cdots\cdots(15)$$

where $A=E/12h^2$, $B'=\rho\omega^2/\alpha^2$. The boundary conditions connected with this equation can be expressed as follows:

$$\left.\begin{aligned} &z^3\frac{d^2X}{dz^2}=0, \quad \frac{d}{dz}\left(z^3\frac{d^2X}{dz^2}\right)=0 \quad \text{at } z=0,\\ &X=0, \quad \frac{dX}{dz}=0 \quad \text{at } z=1. \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(16)$$

The solution can also be given by using the power series expressed by Eq. (11), and if we perform the calculation just in the same way as in the case of shear-bending vibration, we get the following equation corresponding to Eq. (12):

$$m^3+672.69\,m^2+121551.92\,m-2915613.40=0. \quad \cdots\cdots\cdots\cdots\cdots(17)$$

In this case the coefficients of the terms of higher order than m^4 are also omitted. Since the minimum root of Eq. (17) is $m_0=28.205$, the period T_B of the fundamental mode is

$$T_B=4.098\sqrt{\frac{\rho}{E}}\frac{h}{\alpha}, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(18)$$

and the form of the fundamental mode can be obtained as follows:

$$X_B=(1+2.3503z^2+0.2762z^4+0.0061z^6)-2.5763(z+0.3914z^3+0.0184z^5+0.0002z^7). \quad \cdots\cdots\cdots\cdots\cdots(19)$$

The period of the fundamental mode of the shear vibration is, as everyone knows, expressed by the following Eq.(20) regardless of the grade magnitude of faces of slope,

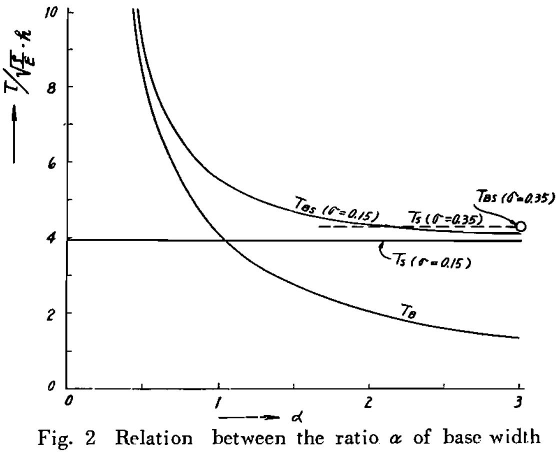


Fig. 2 Relation between the ratio α of base width to height of the dam and period T.

$$\left.\begin{aligned} T_S&=\frac{2\pi}{2.4048}\sqrt{\frac{\rho}{G}}h=3.963\sqrt{\frac{\rho}{E}}h, \quad (\sigma=0.15),\\ T_S&=4.293\sqrt{\frac{\rho}{E}}h, \quad (\sigma=0.35), \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots(20)$$

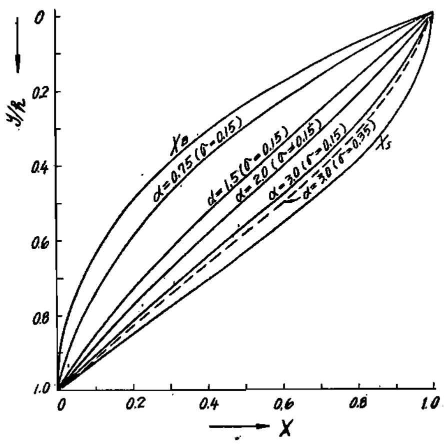


Fig. 3 Relation between α and vibration form.

and the form of vibration is given by

$$X_S = J_0(2.4048\, y/h) \cdots (21)$$

Table 1, Fig. 2 and Fig. 3 show how the period T of free vibration and the form X of vibrations vary with the various values of the ratio α of base width to height of the dam, by using the above-mentioned equations.

Table 1 Relation between the period T of free vibration and the ratio α of base width to height of the dam.

$\alpha,(\sigma=0.15)$	m_0	$T_B/\sqrt{\frac{\rho}{E}}h$	$T_{BS}/\sqrt{\frac{\rho}{E}}h$	$T_S/\sqrt{\frac{\rho}{E}}h$
0	0	∞	∞	
0.50	1.162	8.196	8.839	
0.75	2.080	5.464	6.606	
1.00	2.909	4.098	5.586	
1.25	3.591	3.278	5.028	3.963
1.50	4.066	2.732	4.725	
2.00	4.642	2.049	4.422	
2.50	5.097	1.639	4.220	
3.00	5.279	1.366	4.142	
∞	5.783	0	3.962	
3.00, $(\sigma=0.35)$	5.532	1.366	4.389	4.293

2.3. Variation of the vibration characteristics due to the sectional form of dam

When we calculate the shear-bending and bending vibrations, numerically the terms of higher order than m^4 are omitted for the period, and the terms of higher order than $n=9\sim 11$ are also neglected for the deformation. The former procedure is corresponding to omitting the terms of higher order than $n=8$, but the convergency of the power series is considerably good and so that the solution may have the technically gratifying accuracy. For instance, the period of the bending vibration given by Kirchhoff is as follows:

$$T=1.183\sqrt{\frac{\rho A}{EI}}h^2=4.098\sqrt{\frac{\rho}{E}}\frac{h}{\alpha},$$

which is identical to the solution obtained from Eq. (18). This fact shows that such a degree of accuracy of the calculation is quite adequate.

We may summarize a conclusion from Fig. 2 indicating the relation between α and the period of the fundamental mode, as follows:

(a) The period T_{BS} of shear-bending vibration approaches to the period T_B of bending vibration when α becomes smaller, and to the period T_S of shear vibration when α becomes larger. Suppose that the errors which might be involved by assuming the shear-bending vibration as the bending or shear vibration are expressed by $(T_{BS}-T_B)/T_{BS}$ or $(T_{BS}-T_S)/T_{BS}$ respectively and confine the errors less than 10 %, then it can be considered that the gravity dam ($\sigma=0.15$) causes the bending vibration for the value of α less than $\alpha \fallingdotseq 0.6$ and the shear vibration for that larger than $\alpha \fallingdotseq 2.0$.

The same consideration leads to the fact that the earth dam ($\sigma=0.35$) causes the shear vibration even in the case when the grade is considerably steeper than $\alpha=2.0$, because of the error involved for $\alpha \fallingdotseq 3.0$ being about 2%.

As to the vibration curve, we can also draw the conclusion from Fig. 3 as follows:

(b) The form of vibration curve approaches to that of bending vibration when α becomes smaller, and to that of shear vibration when α becomes larger, and takes the intermediate deformation curve for $\alpha=1.5\sim 2.0$ ($\sigma=0.15$), approaching to a straight line.

Similarly to the already mentioned example, suppose that the errors which might be involved by assuming the shear-bending vibration as the bending or shear vibration are expressed by the ratio $(X_{BS}-X_B)/X_{BS}$ or $(X_S-X_{BS})/X_{BS}$ respectively of the amplitudes at the level of 1/2 of the dam height, then the errors are expressed as follows:

about 32 % for $\alpha=0.75$ ($\sigma=0.15$),
〃 17 % for $\alpha=3.0$ ($\sigma=$ 〃),
〃 11 % for $\alpha=3.0$ ($\sigma=0.30$).

Comparing (a) with (b), it may be seen that as for the errors which might be introduced by assuming the shear-bending vibration as the bending or shear vibration, those due to the deformation are considerably larger compared with those due to the period for even the same value of α. Thus, as a rule, it is obvious that both the bending and shear should be taken into account for the range of about $0.75<\alpha<3.0$.

Accordingly the period of free vibration may be obtained within the error of less than 10% for the dam section as seen in the gravity dam, but the errors may become considerably larger in the calculated stresses or forms of vibration than the error of the period. In addition, it can be assumed with technically satisfactory accuracy, that the earth dam with the gentle grade of faces of slope causes the shear vibration.

3. Free Vibration of Earth Dam[4)]

3.1. Equation of motion

The sectional form of actual earth dam is trapezoidal, but for simplicity's sake, in the following discussion we may regard it as the fundamental triangular section, because the effect of the upper-cut triangle on the form of dam vibration is small.[1)] In addition, the modulus of rigidity G, the shearing viscosity coefficient γ_t and the density ρ of the dam body material are also assumed constant.

Determine the coordinates as shown in Fig. 4, and consider the vibration in the direction of z-axis. Assume that the distribution of shearing stress is uniform along the z-direction. Then, considering the equilibrium of the forces acting on the infinitesimal body as shown hatched in Fig. 4, we have the

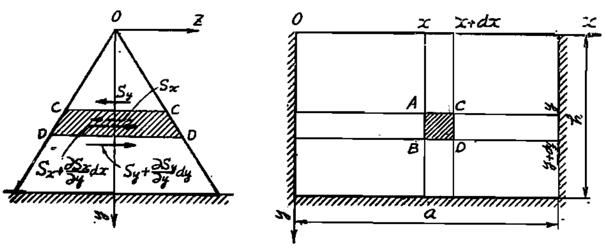


Fig. 4

following differential equation expressing the shear vibration of earth dam.

$$\frac{\partial^2 w}{\partial t^2}=\frac{1}{\rho}\frac{\partial}{\partial x}\left(G\frac{\partial w}{\partial x}+\gamma_t\frac{\partial^2 w}{\partial x\partial t}\right)+\frac{1}{\rho l}\frac{\partial}{\partial y}\left(Gl\frac{\partial w}{\partial y}+\gamma_t l\frac{\partial^2 w}{\partial y\partial t}\right)$$

In the above equation, l can be given by $l=\alpha y$ for the triangular section. If we assume that ρ, G and γ_t are constant along x- axis as well as y- axis and consider that the effect of vibration energy disperting into the ground as an elastic wave is expressed by the damping term $\delta\cdot\partial w/\partial t$ nearly proportional to the velocity, the above equation may be rewritten in the form

$$\frac{\partial^2 w}{\partial t^2}=c_0^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+c_1^2\left(\frac{\partial^3 w}{\partial x^2\partial t}+\frac{\partial^3 w}{\partial y^2\partial t}\right)+c_0\frac{1}{y}\frac{\partial w}{\partial y}$$
$$+c_1^2\frac{1}{y}\frac{\partial^2 w}{\partial y\partial t}-c_2^2\frac{\partial w}{\partial t}. \quad\cdots\cdots\cdots(22)$$

where $\quad c_0^2=G/\rho,\ c_1=\gamma_t/\rho,\ c_2^2=\delta/\rho.$

3.2. Free vibration

The solution of the free vibration of earth dam is obtained as the one which should satisfy Eq. (22) under the following boundary conditions.

$$\left.\begin{array}{l} w=0 \quad \text{at } x=0 \quad \text{and} \quad x=a, \\ \dfrac{\partial w}{\partial y}=0 \text{ at } y=0, \quad w=0 \text{ at } y=h. \end{array}\right\} \cdots\cdots\cdots(23)$$

The initial conditions are, in general, given by

$$w=f_0(x,y), \qquad \dot{w}=F_0(x,y) \qquad \text{at } t=0. \quad\cdots\cdots\cdots(24)$$

The solution which satisfies Eqs. (22) ~ (24) is obtained under the assumption of $n_0>\varepsilon$, as follows:

$$w=\frac{4}{ah^2}\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\frac{1}{J_1^2(\lambda_s)}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)e^{-\varepsilon t}\int_0^a\int_0^h\left[f_0(\lambda,\mu)\cos\omega t\right.$$
$$\left.+\frac{\sin\omega t}{\omega}\left\{\varepsilon f_0(\lambda,\mu)+F_0(\lambda,\mu)\right\}\right]\sin\frac{n\pi}{a}\lambda\,J_0\left(\frac{\lambda_s}{h}\mu\right)\mu\,d\lambda\,d\mu. \quad\cdots\cdots\cdots(25)$$

And the period of free vibration is shown by

$$T=\frac{2\pi}{\sqrt{n_0^2-\varepsilon^2}}, \quad\cdots\cdots\cdots(26)$$

where $\quad n_0^2=c_0^2(n^2\pi^2/a^2+\lambda_s^2/h^2),\quad \varepsilon=(1/2)\{c_1^2(n^2\pi^2/a^2+\lambda_s^2/h^2)+c_2^2\}$.

$c_0^2=G/\rho,\ c_1^2=\gamma_t/\rho,\ c_2^2=\delta/\rho,\quad n=1,2,3,\cdots\cdots,\quad s=1,2,3,\cdots\cdots.$

3.3. Period of free vibration

In order to calculate the period of free vibration, it is necessary to know the values of modulus of rigidity G, the density ρ, or the propagation velocity of transversal wave c_0, the shearing viscosity coefficient γ_t and the factor of resistance δ proportional to the velocity, of the dam soil. And yet, there are little data concerning these values which have been investigated on the actual dams. Generally speaking, these values vary in a very wide range depending upon the kinds of soil and water contents. K. Iida tested more than 100 natural soils and gave $\rho=1.4\sim1.7$, $\gamma_t=10^4\sim10^6$, $c_0=5000\sim25000$ (in C. G. S. Unit) and the Poisson's ratio $\sigma=0.15\sim0.48$[6]. The values, as given above, are the values measured dynamically by the resonance method in the laboratory and not those measured in the actual dams. However, they may be considered more appropriate for the particular problem to be discussed here, than the usual values measured statically.

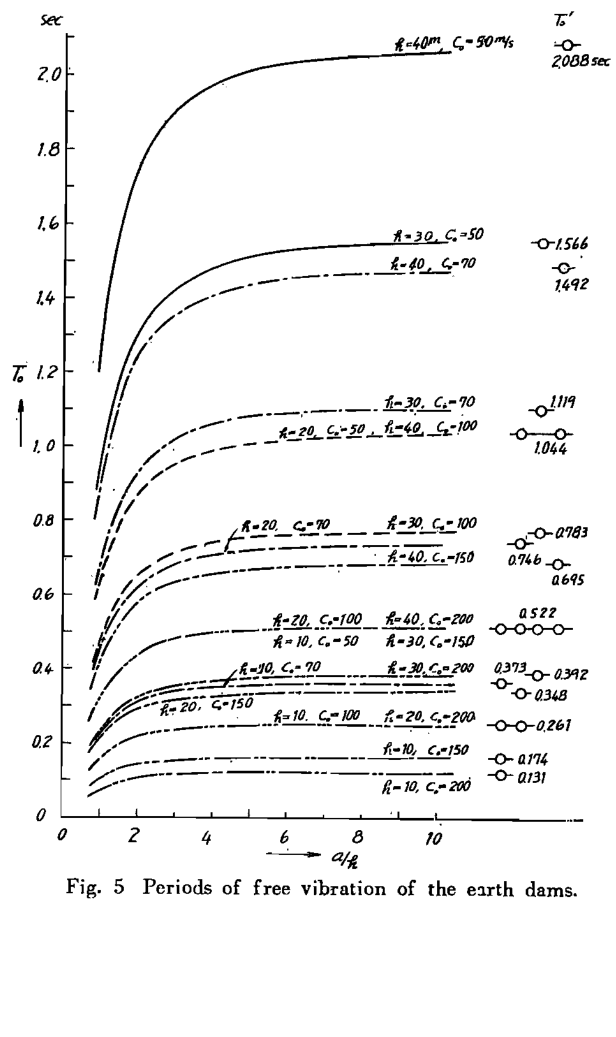


Fig. 5 Periods of free vibration of the earth dams.

Fig. 5 shows, referring to the values described above, the period of the fundamental vibration calculated by Eq. (26) for the various values of height and rigidity of the dam.

When the value of γ_t is as large as given above, it has little effect upon the free vibration, and δ, in general, has also little effect; so we put $\varepsilon=0$ in Fig. 5. As stated before, when the gravity dam of concrete is subjected to the seismic forces, the bending vibration rather than the shear vibration is predominant. Fig. 6, however, shows, for reference, the period of shear vibration, in which the height of dam is taken as 100 m and three values of $c_0=2000$, 2500 and 3000 m/sec are also assumed. According to an example of the measurement of actual dam, c_0 is as large as $c_0=2600$ m/sec.[6]

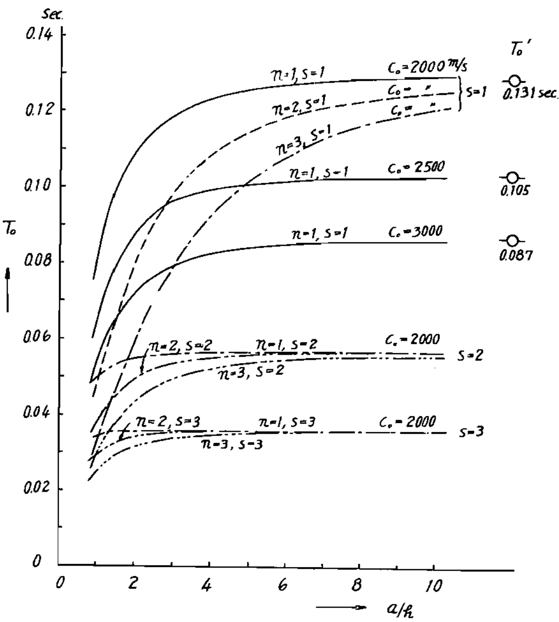


Fig. 6 Periods of shear vibration of the gravity dams.

Comparison between the period of free vibration obtained in case of the two-dimensional treatment and that in case of the one-dimensional treatment shows the following results.

Putting $k=a/h$ and $c_1^2=0$ in Eq. (26), we have

$$T=2\pi h\Big/c_0\sqrt{\frac{n^2\pi^2}{k^2}+\lambda_s^2}\,. \qquad \cdots\cdots(27)$$

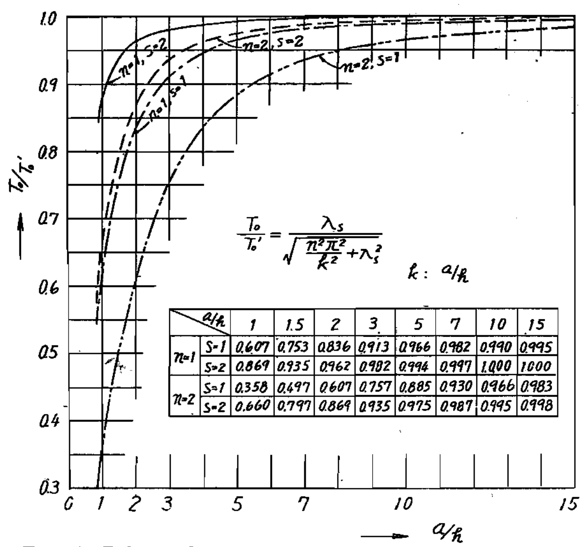
Representing T for the case when the dam length a is extremely large compared with the height h by T', we get

$$T'=2\pi h/c_0\lambda_s\,, \qquad \cdots\cdots(28)$$

which coincides with the period of free vibration of the one-dimensional dam.

Therefore, considering the ratio of both periods, we have

$$\frac{T}{T'}=\frac{\lambda_s}{\sqrt{\dfrac{n^2\pi^2}{k^2}+\lambda_s^2}}\,. \qquad \cdots\cdots(29)$$



a/h		1	1.5	2	3	5	7	10	15
$n=1$	$s=1$	0.607	0.753	0.836	0.913	0.966	0.982	0.990	0.995
	$s=2$	0.869	0.935	0.962	0.982	0.994	0.997	1.000	1.000
$n=2$	$s=1$	0.358	0.497	0.607	0.757	0.885	0.930	0.966	0.983
	$s=2$	0.660	0.797	0.869	0.935	0.975	0.987	0.995	0.998

Fig. 7 Relation between the two- and one-dimensional periods of the earth dam.

Fig. 7 indicates the above-mentioned relationship between the periods of the fundamental and the 1st higher modes. It can be seen also from Fig. 7 that, when the error in the period of free vibration is limited to less than 5 % which might be involved due to the one-dimensional treatment, the length is needed to be 4 times as large as the height for the period of the fundamental mode and more than 8 times as large for that of the 1st higher mode of vibration, and, moreover, when the error is limited to less than 10%, the length must be 3 or 6 times as large as the height for the period of the fundamental or the 1st higher mode of vibration respectively.

3.4. Model experiments

In order to investigate the period of free vibration and the normal mode of vibration of the two-dimensional dam, and to compare them with the theory described above, the vibration experiments were made by using the models made of agar-agar of 3 % concentration. The dimensions of the models are as follows: the grade of surfaces of slope is 2 : 1, the crest width is 1.0 cm, the base width is 33.0 cm, the dam height is 8 cm, and as for the dam length 3 kinds of length 24 cm ($a/h=3$), 40 cm ($a/h=5$) and 56 cm ($a/h=7$) are adopted. In each experiment, the resonance due to the forced vibration, and the vibration which is produced when the shaking table is subjected to a sudden brake by switching the electric motor off, were measured, and the deformations at many points of the dam model were magnified by the optical lever device and recorded simultaneously on the oscillograph papers. The deflections during vibration of the upper end and the center section of the dam model are

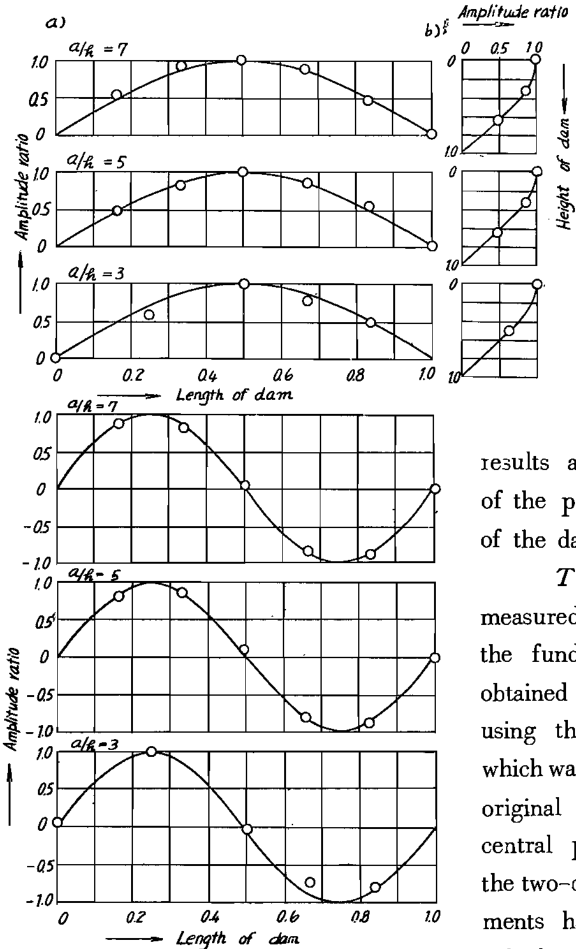


Fig. 8 Deflection curves during vibration of the dam models.

shown in Fig. 8, in which the points as marked o show the observed results and the solid lines represent the calculated ones, and these values show quite consistent with one another.

Table 2 indicates the comparison between the observed results and the calculated values of the period of free vibration of the dam model.

T' in Table 2 are the measured values of the period of the fundamental free vibration obtained from the experiment using the one-dimenional model which was made by cutting off the original model remainning the central part of 15 cm long after the two-dimensional model experiments had been finished. The calaulated periods T_{11} and T_{21} of the two-dimensional model were obtained from Eq. (29) by using the measured period T' of the one-dimensional model, for the main purpose of clarifying the difference between the one- and the two-dimensional treatments. As seen from this table, the measured result are in good accordance with the computed ones for the fundamental mode, but deviate considerablly from the theoretical ones when the 1st higher mode was produced in the longitudinal direction of the dam, and the shorter the length of dam compared with its height, the greater the deviation. This may be due to the fact that the model made of agar-agar is so elastic compared with that of soil that the bending vibration has much effect upon the higher mode of vibration.

Table 2 Comparison of the mesurements and computed values of the period of free vibration of the dam model.

a/h		3	5	7
Measured periods of one-dimensional free vibration T_0' (sec)		0.0440	0.0482	0.0530
c_0 (cm/sec)		474.0	432.0	394.5
Two-dimensional periods of free vibration T_{11} ($n=1$, $s=1$)	Obs. value (sec)	0.0400	0.0470	0.0520
	Comp. value (sec)	0.0402	0.0466	0.0520
	Error (%)	0.5	−0.9	0
Two-dimensional periods of free vibration T_{21} ($n=2$, $s=1$)	Obs. value (sec)	0.0250	0.0350	0.0420
	Comp. valuc (sec)	0.0334	0.0426	0.0493
	Error (%)	25	18	15
T_{11}/T_0'	Obs. value	0.909	0.975	0.982
	Comp. value	0.913	0.966	0.982
$T_{21}/T'_0,$	Obs. value	0.568	0.726	0.792
	Comp. value	0.757	0.885	0.930

Errors are computed from (Comp. value − Obs. value)/(Comp. value).

4. Forced Vibration

Let $f(x,t)$ be the ground motion, and the boundary condition can be given as follows:

$$\left.\begin{aligned} &w=f(x_0,t) \text{ at } x=0, \quad w=f(x_a,t) \text{ at } x=a, \\ &\frac{\partial w}{\partial y}=0 \quad \text{ at } y=0, \quad w=f(x,t) \quad \text{ at } y=h. \end{aligned}\right\} \qquad (30)$$

And the initial conditions, considering that the ground motion is produced suddenly from the still state, are given by

$$w=0, \qquad \dot{w}=0 \qquad \text{at} \qquad t=0. \qquad\qquad (31)$$

In order to satisfy the differential equation (22), as well as the boundary and initial conditions Eqs. (30), (31), the solution w_0 of $f(x,\tau)$ which corresponds to $f(x,t)$ when the ground motion is independent of time, is to be given in the first place, and then the Duhamel's Theorem may be applied. Thus, the solution is given as follows:

$$w=\frac{4}{ah^2}\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\frac{1}{J_1(\lambda_s)\{(n\pi/a)^2+(\lambda_s/h)^2\}}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)$$
$$\times\frac{n_0^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\left[\frac{h^2}{\lambda_s}\frac{n\pi}{a}\left\{f(x_0,\tau)+(-1)^{n+1}f(x_a,\tau)\right\}\right.$$
$$\left.+\lambda_s\int_0^a f(\lambda,\tau)\sin\frac{n\pi}{a}\lambda\,d\lambda\right]\sin\omega(t-\tau)\,d\tau. \qquad\qquad (32)$$

4.1. Uniform ground motion in the longitudinal direction of dam

When the ground motion is uniform in the longitudinal direction of dam and can be expressed by $f(t)$, Eq. (32) may be rewritten as follows:

$$w=\frac{8}{\pi}\sum_{n=1,3,5,\cdots}^{\infty}\sum_{s=1}^{\infty}\frac{1}{n\lambda_s J_1(\lambda_s)}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)$$
$$\times\frac{n_0^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}f(\tau)\sin\omega(t-\tau)\,d\tau, \qquad\qquad (33)$$

which shows that in this case the vibration can occur only when $n=1,3,5,\cdots\cdots$ — that is — the symmetrical vibration in the longitudinal direction of dam alone can occur and there can be no asymmetric vibration. There is no question, when $f(t)$ can be expressed by the simple function, and consequently Eq. (33) can be easily integrated; but the unit graph method or the analyzer must be required, when $f(t)$ is so complicated as usually found in the actual seismic motion.

Next, consider the central part of the two-dimensional dam in order to compare the two-dimensional vibration considering the effect of the dam length with the one-dimensional one. Putting $x=a/2$ in Eq. (33), we get

$$w=8\sum_{s=1}^{\infty}\frac{1}{\pi}\left(1-\frac{1}{3}+\frac{1}{5}-\cdots\cdots\right)\frac{1}{\lambda_s J_1(\lambda_s)}J_0\left(\frac{\lambda_s}{h}y\right)$$
$$\times\frac{n_0^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}f(\tau)\sin\omega(t-\tau)\,d\tau.$$

Assume that the length a of the dam is extremely long and $a\to\infty$, then w may decrease with the decrement factor $e^{-\varepsilon(n^2)}$ and may be so small for sufficiently large values of n.

Hence, when $a \to \infty$, we have the following relations

$$n_0^2 = c_0^2\{(n\pi/a)^2 + (\lambda_s/h)^2\} \quad \to \quad c_0^2(\lambda_s/h)^2,$$

$$\varepsilon = (1/2)[c_1^2\{(n\pi/a)^2 + (\lambda_s/h)^2\} + c_2^2] \quad \to \quad (1/2)\{c_1^2(\lambda_s/h)^2 + c_2^2\},$$

and also, since

$$(1/\pi)(1 - 1/3 + 1/5 - \cdots\cdots\cdots) \quad \to \quad 1/4,$$

the vibration solution of the central part of the dam may be expected to approach to a great extent to the solution for the one-dimensional dam, in other words, the latter solution is given by omitting the terms relating to x in the differential equation (22), the boundary and the initial conditions (30), (31).

The solution can be written as follows:

$$w = 2\sum_{s=1}^{\infty} \frac{1}{\lambda_s J_1(\lambda_s)} J_0\left(\frac{\lambda_s}{h} y\right) \frac{n_{01}^2}{\omega_1} \int_0^t e^{-\varepsilon_1(t-\tau)} f(\tau) \sin \omega_1(t-\tau)\, d\tau, \qquad \cdots\cdots(34)$$

where

$$\omega_1 = \sqrt{n_{01}^2 - \varepsilon_1^2}, \quad n_{01}^2 = c_0(\lambda_s/h)^2, \quad \varepsilon_1 = (1/2)\{c_1^2(\lambda_s/h)^2 + c_2^2\}.$$

4.2. Comparison between the resonant amplitudes of one- and two-dimensional earth dams

Let the ground motion be expressed simply by $A \cos pt$ and consider the vibration when the sufficiently long time has been elapsed since the ground motion began, then the terms of forced vibration will remain. Expressing such terms for one- and two-dimensional dams by $\overline{w_1}$ and $\overline{w_2}$,

$$\overline{w_1} = 2A\sum_{s=1}^{\infty} \frac{1}{\lambda_s J_1(\lambda_s)} J_0\left(\frac{\lambda_s}{h} y\right) \frac{n_{01}^2}{\sqrt{(n_{01}^2 - p^2)^2 + 4\varepsilon_1^2 p^2}} \sin(pt - \delta_1), \qquad \cdots\cdots(35)$$

where

$$\delta_1 = \tan^{-1} \frac{2\varepsilon_1 p}{n_{01}^2 - p^2}.$$

$$\overline{w_2} = \frac{8A}{\pi} \sum_{n=1,3,5,\cdots}^{\infty} \sum_{s=1}^{\infty} \frac{1}{n\lambda_s J_1(\lambda_s)} \sin\frac{n\pi}{a} x J_0\left(\frac{\lambda_s}{h} y\right) \frac{n_0^2}{\sqrt{(n_0^2 - p^2)^2 + 4\varepsilon^2 p^2}} \times \sin(pt - \delta), \qquad \cdots\cdots(36)$$

where

$$\delta = \tan^{-1} \frac{2\varepsilon p}{n_0^2 - p^2}.$$

Table 3 Values of $1/n\lambda_s J_1(\lambda_s)$

s \ n	1	3	5	7	9	11	13	15
1	0.8011	0.2670	0.1602	0.1144	0.0890	0.0728	0.0618	0.0534
2	−0.5323	−0.1774	−0.1065	−0.0760	−0.0591	−0.0484	−0.0409	−0.0355
3	0.4256	0.1419	0.0851	0.0610	0.0473	0.0387	0.0327	0.0284
4	−0.3647	−0.1216	−0.0729	−0.0521	−0.0405	−0.0332	−0.0281	−0.0243
5	0.3225	0.1075	0.0645	0.0461	0.0358	0.0293	0.0248	0.0215
6	−0.2948	−0.0983	−0.0590	−0.0421	−0.0328	−0.0268	−0.0227	−0.0197
7	0.2720	0.0907	0.0544	0.0389	0.0302	0.0247	0.0209	0.0181
8	−0.2539	−0.0846	−0.0508	−0.0363	−0.0282	−0.0231	−0.0195	−0.0169
9	0.2390	0.0797	0.0478	0.0341	0.0266	0.0217	0.0184	0.0159
10	−0.2264	−0.0755	−0.0453	−0.0323	−0.0252	−0.0206	−0.0174	−0.0151

Table 3 shows the coefficient $1/n\lambda_s J_1(\lambda_s)$ in Eq. (36), in which $n=1$ corresponds to the case of one-dimensional dam. As seen from the table, the vibration of higher mode in the longitudinal direction decreases with the ratio of $1/n$, but the one in the direction of dam height decreases with the ratio of 1/1.5, 1/1.9, 1/2.2,……, and the rate of decrement in the latter is considerably lower than that in the former. This fact shows that the vibrations of higher mode have a comparatively large effect upon the direction of dam height.

Fig. 9 indicates $\{1/\lambda_s J_1(\lambda_s)\}\cdot J_0(\lambda_s y/h)$, in which the dotted lines show the similar coefficient concerning the column of rectangular section with uniform modulus of rigidity and density. It can be seen that the rate of decrement of the vibration of higher mode

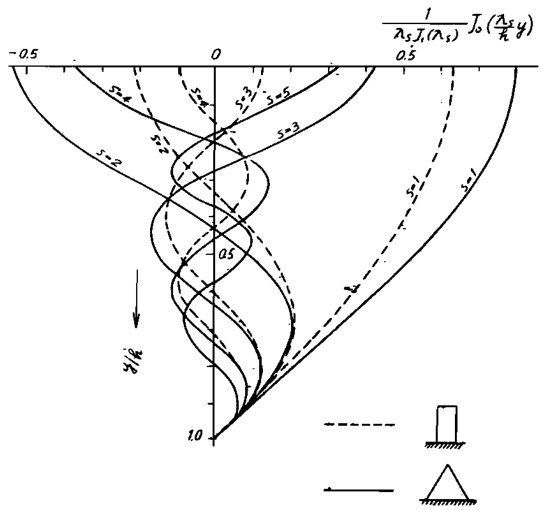
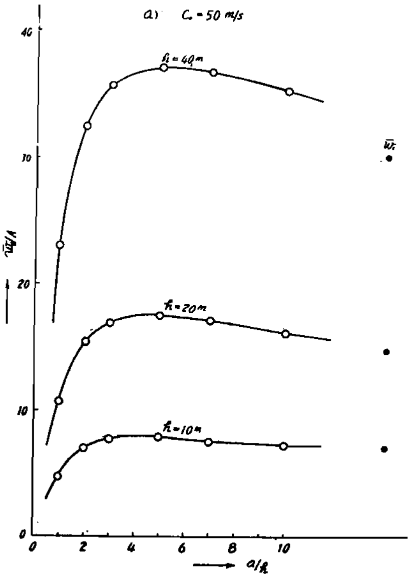


Fig. 9 Normal modes of the earth dam and the homogeneous solid building.

is small for the triangular section as seen in the earth dam, as compared with that for the general building, and that the higher mode is so remarkable in the crest part that the resonances of higher mode may be of serious problem.

Fig. 10 indicates that how the resonance amplitude of the fundamental mode in the central point of the crest varies with the dam length, by plotting the calculated results obtained from Eq. (36) for the various values of dam height h and rigidity c_0. In the process of calculation the author takes the value of $\sin(pt-\delta)$ as unit 1, so that the curves in Fig. 10 does not represent the very resonance amplitude in a strict sense, but gives the general features of the resonance amplitude. The mark • found at the right edge in the figure shows the similar value for the one-dimensional dam, computed from Eq. (35). Shearing viscosity γ_t was determined in order



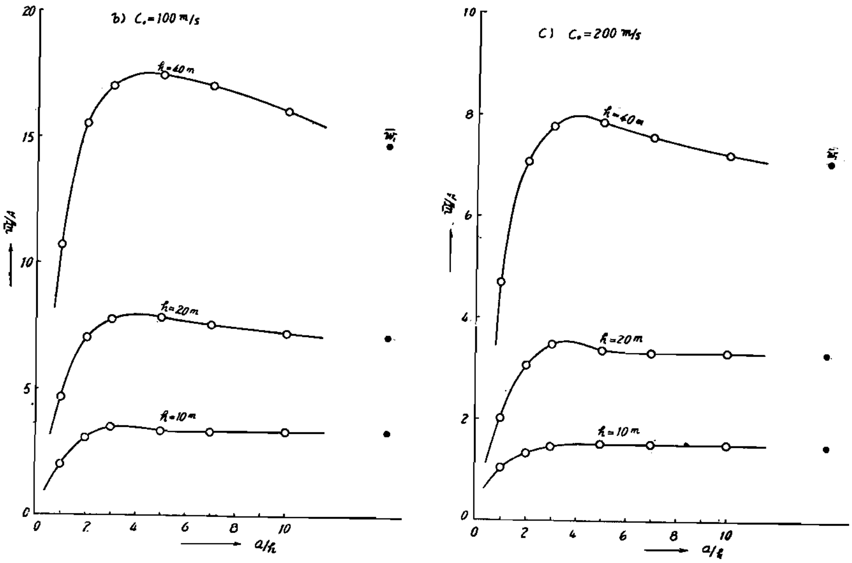


Fig. 10 Relation between the resonance amplitude and the ratio of lengh to height of the earth dam.

that the following condition was satisfied, i.e., $c_0=100$ m/s, $h_1=\varepsilon_1/n_{01}=0.1$ for $h=20$ m in the one-dimensional dam. As known from Fig. 10, the one-dimensional treatment may be acceptable for the larger values over about 3~5 of a/h. The critical limiting value for the above-mentioned period of the free vibration, is also of such a magnitude. Such a consideration leads to a conclusion that the one-dimensional treatment may be carried out without involving serious error for the larger values over 3~5 of a/h.

5. Conclusion

First of all, this paper discusses the vibration character of earth dam as the basis of establishing the rational method of earthquake resistant dam design. All of the discussions are derived from the theory for the elastic dam model having the particular sectional form, but there may be some problems left in abeyance and expected to be solved in future. The summaries of the studies reported in this paper are concluded as follows:

(1) The vibration of such structures having gentle grade of surfaces of slope as earth dams can be considered as the shear vibration. For the gravity dam, however, the bending vibration is more predominant and the effect of shear vibration must be considered to a certain degree.

(2) For higher dams, the value of vibration period may be of such magnitude that the resonance of higher order presents a problem, when we deal with the period of the principal motion of the serious earthquake which have been occurred up to now.

(3) For the dam having the length larger than 3~5 times as large as the height, the vibration period and the amplitude of central part are of the same magnitude as found in the one-dimensional dam; therefore, the one-dimensional treatment may be allowable. This limiting value is very significant, when the distribution of seismic coefficient in the direction of the length must be taken into account.

The author, in addition, has been studying on the seismic coefficient to be used in designing the earth dam, from the dynamic standpoint based on the theory described in this paper, and is intending to make public the above-mentioned research results in the next part.

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Part II Considerations of the Earthquake Resistant Properties of Earth Dam

1. Introduction

The vibration character of earth dams having various dam lengths is discussed in Part 1, in which the earth dams are assumed visco-elastic. In this paper the author describes the earthquake resistant properties of earth dam from the standpoint of vibration on the basis of the results obtained in Part I. He also describes what size of seismic coefficient is to be adopted for dam design, when the conventional design method of seismic coefficient is used for evaluating the effect of seismic forces upon the structures. The method is based on such a way of thinking that the horizontal force of the magnitude of "the weight of structure multiplied by seismic coefficient" is assumed to act statically on the structures; and finally he makes a proposal concerning the seismic coefficient of design.

2. Seismic Coefficient of One Mass System

Since the forced vibration of solid body is generally expressed by the summation of the free vibrations, each of the vibrations of the normal mode can be treated as the vibration of one mass system having one freedom. Hence, according to M. A. Biot's theory,[1] at first the seismic coefficient of one mass system shall be discussed.

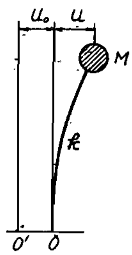

Fig. 1

Letting u be, in Fig. 1, the displacement measured referring to the moving coordinate having the origin O, M be the mass, k the spring constant and u_0 the ground motion, the vibration of the structure assumed as one mass system may be given by

$$M\ddot{u}+D\dot{u}+ku=-M\ddot{u}_0,$$

or

$$\ddot{u}+2\varepsilon\dot{u}+n^2u=a(t), \qquad \cdots\cdots(1)$$

where $\qquad D$: coefficient of viscous resistance,

$$n^2=k/M, \qquad 2\varepsilon=D/M, \qquad a(t)=-\ddot{u}_0.$$

When the vibration begins at the initial state of stillness, the solution of Eq. (1) will be given as follows under the condition of $u = 0$, $\dot{u} = 0$ at $t=0$,

$$u=\frac{1}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\alpha(\tau)\sin\omega(t-\tau)\,d\tau, \quad \cdots\cdots\cdots\cdots(2)$$

where

$$\omega=\sqrt{n^2-\varepsilon^2}$$

When the horizontal force F acts statically on the mass M causing deflection y, the force F which should be applied statically so as to cause M the dynamic deflection u is, using the relation $F=ku$,

$$F=Mn^2u=M\frac{n^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\alpha(\tau)\sin\omega(t-\tau)\,d\tau,$$

and the seismic coefficient K given by $K=F/Mg$ is

$$K=\frac{A}{g}, \quad \cdots\cdots\cdots\cdots(3)$$

where

$$A=\frac{n^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\alpha(\tau)\sin\omega(t-\tau)\,d\tau.$$

Therefore, when the external force $W\cdot K$—the weight multiplied by the seismic coefficient K given by Eq. (3) — is assumed to act statically, the deflection occured in this case is nothing but the one occured during the vibration expressed by Eq. (2). Thus, if the maximum value of Eq. (3) is used as the design seismic coefficient, the conventional design methed may be used quite rationally in designing the structure. When the acceleration of ground motion $\alpha(t)$ is as complicated as seen in the actual earthquakes, the torsion pendulum[1),2)] or the analyzer using the electrical method[3)] can be used for determining the value of A.

3. Determination of Seismic Coefficient on Dam

Expressing the vibration of dam by the moving coordinate having the origin at the crest of dam, and letting u be the displacement and $\alpha(x, t)$ the acceleration of ground motion, the equation of motion of two-dimensional dam is given by

$$\frac{\partial^2 u}{\partial t^2}=c_0{}^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+c_1{}^2\left(\frac{\partial^3 u}{\partial x^2\partial t}+\frac{\partial^3 u}{\partial y^2\partial t}\right)+c_0{}^2\frac{1}{y}\frac{\partial u}{\partial y}$$
$$+c_1{}^2\frac{1}{y}\frac{\partial^2 u}{\partial y\partial t}-c_2{}^2\frac{\partial u}{\partial t}+\alpha(x,t), \quad \cdots\cdots(4)$$

where $\alpha(x,t)=-\ddot{u}_0(x,t)$ and c_2 is the coefficient of viscous resistance proportional to the velocity, which is expressed by the moving coordinate. The boundary conditions are given by

$$\left.\begin{array}{llll} u=0 \ \text{at} \ x=0, & u=0 \ \text{at} \ x=a, \\ \dfrac{\partial u}{\partial y}=0 \ \text{at} \ y=0, & u=0 \ \text{at} \ y=h. \end{array}\right\} \quad \cdots\cdots(5)$$

The initial conditions are considered to be given by

$$u=0, \quad \dot{u}=0 \quad \text{at} \quad t=0. \quad \cdots\cdots(6)$$

Assume that u and $\alpha\ (x,t)$ can be developed respectively by using the "Eigenfunctions" as follows :

$$u=\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\sin\frac{n\pi}{a}xJ_0\left(\frac{\lambda_s}{h}y\right)T_{ns},$$
$$\alpha\ (x,t)=\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\sin\frac{n\pi}{a}xJ_0\left(\frac{\lambda_s}{h}y\right)\phi_{ns}(t),$$

where

$$\phi_{ns}(t)=\frac{4}{ah^2}\frac{1}{J_1{}^2(\lambda_s)}\int_0^a\int_0^h\alpha\ (\lambda,t)\sin\frac{n\pi}{a}\lambda\ J_0\left(\frac{\lambda_s}{h}\mu\right)\mu\ d\lambda\ d\mu,$$

and substitute the above into Eq. (4), we get the differential equation on T as follows:

$$\ddot{T}_{ns}+2\varepsilon\dot{T}_{ns}+n^2T_{ns}=\phi_{ns}(t),$$

where

$$n_0{}^2=c_0{}^2\{(n\pi/a)^2+(\lambda_s/h)^2\},$$
$$\varepsilon=(1/2)\left[c_1{}^2\{(n\pi/a)^2+(\lambda_s/h)^2\}+c_2{}^2\right].$$

For the case $T=0$, $\dot{T}=0$ at $t=0$, the solution of the above equation becomes

$$T_{ns}=\frac{1}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\phi_{ns}(\lambda_s,\tau)\sin\omega(t-\tau)\,d\tau,$$

therefore, u is given by

$$u=\frac{4}{a}\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\frac{1}{\lambda_sJ_1(\lambda_s)}\sin\frac{n\pi}{a}xJ_0\left(\frac{\lambda_s}{h}y\right)$$

$$\times\frac{1}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\left\{\int_0^a a(\lambda_s,\tau)\sin\frac{n\pi}{a}\lambda\,d\lambda\right\}\sin\omega(t-\tau)\,d\tau. \quad\cdots\cdots\cdots\cdots(7)$$

When the distribution of acceleration of ground motion in the longitudinal direction of dam is uniform, we may rewrite Eq. (7) in the following form, by the reason that $a(t)$ must be used instead of $a(x, t)$,

$$u=\frac{8}{\pi}\sum_{n=1,3,5\cdots}^{\infty}\sum_{s=1}^{\infty}\frac{1}{n\lambda_s J_1(\lambda_s)}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)$$

$$\times\frac{1}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}a(\tau)\sin\omega(t-\tau)\,d\tau. \quad\cdots\cdots\cdots\cdots(8)$$

For the one-dimensional dam, the following solution may be obtained as the solution of the Eqs. (4), (5) and (6), in which all of the terms relating to x are omitted.

$$u=2\sum_{s=1}^{\infty}\frac{1}{\lambda_s J_1(\lambda_s)}J_0\left(\frac{\lambda_s}{h}y\right)\frac{1}{\omega_1}\int_0^t e^{-\varepsilon_1(t-\tau)}a(\tau)\sin\omega_1(t-\tau)\,d\tau, \quad\cdots\cdots(9)$$

where

$$\omega_1=\sqrt{n_{01}^2-\varepsilon_1^2},\quad n_{01}^2=c_2^2(\lambda_s/h)^2,$$

$$\varepsilon=(1/2)\{c_1^2(\lambda_s/h)^2+c_2^2\},\quad c_0^2=G/\rho,\quad c_1^2=\gamma_t/\rho,\quad c_2^2=\delta'/\rho.$$

Though Eqs. (9), (7) and (8) are the equations which express the deflections during the vibration of the one- and two-dimensional dams, one mass system can be considered per each mode of n or n, s. Considering the relation given by Eq. (3), the seismic coefficient K_1 and K_2 of the one- and two-dimensional dams can be shown respectively as follows:

For the one-dimensional dam:

$$K_1=\frac{2}{g}\sum_{s=1}^{\infty}\frac{1}{\lambda_s J_1(\lambda_s)}J_0\left(\frac{\lambda_s}{h}y\right)A_s, \quad\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

$$A_s=\frac{n_0^2}{w_1}\int_0^t e^{-\varepsilon(t-\tau)}a(\tau)\sin\omega_1(t-\tau)\,d\tau.$$

For the two- dimensional dam:

for the case where the acceleration of ground motion is not uniformly distributed along the dam length;

$$K_2'=\frac{4}{ga}\sum_{n=1}^{\infty}\sum_{s=1}^{\infty}\frac{1}{\lambda_s J_1(\lambda_s)}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)C\cdot A_{ns}, \quad\cdots\cdots\cdots\cdots(11)$$

$$C\cdot A_{ns}=\frac{n_0^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\left\{\int_0^a a(\lambda,\tau)\sin\frac{n\pi}{a}\lambda\,d\lambda\right\}\sin\omega(t-\tau)\,d\tau.$$

for the case where the acceleration of ground motion is uniformly distributed along the dam length;

$$K_2=\frac{8}{g\pi}\sum_{n=1,3,5,\cdots}^{\infty}\sum_{s=1}^{\infty}\frac{1}{n\lambda_s J_1(\lambda_s)}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)A_{ns}, \quad\cdots\cdots\cdots\cdots(12)$$

$$A_{ns}=\frac{n_0^2}{\omega}\int_0^t e^{-\varepsilon(t-\tau)}\,a(\tau)\sin\omega(t-\tau)d\tau.$$

Computing the shearing stress for the uniform motion of ground from the conventional seismic coefficient method, by the use of the above-mentioned seismic coefficients,

For the one-dimensinoal dam :

$$S_y=-2h\rho\sum_{s=1}^{\infty}\frac{1}{\lambda_s^2 J_1(\lambda_s)}J_1\left(\frac{\lambda_s}{h}y\right)A_s. \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(13)$$

For the two-dimensional dam :

the shearing stress in the direction of dam height ;

$$S_y=-\frac{8\rho h}{\pi}\sum_{n=1,3,5,\cdots}^{\infty}\sum_{s=1}^{\infty}\frac{1}{nJ_1(\lambda_s)}\frac{1}{(n\pi/k)^2+\lambda_s^2}$$

$$\times\sin\frac{n\pi}{a}x\;J_1\left(\frac{\lambda_s}{h}y\right)A_{ns}, \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(14)$$

the shearing stress in the direction of dam length ;

$$S_x=8\rho h\sum_{n=1,3,5,\cdots}^{\infty}\sum_{s=1}^{\infty}\frac{1}{\lambda_s J_1(\lambda_s)}\frac{1}{k\{(n\pi/k)^2+\lambda_s^2\}}$$

$$\times\cos\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)A_{ns}, \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(15)$$

where

$$k=a/h.$$

When comparing the maximum value of S_x with that of S_y for the two-dimensional dam, S_x is smaller than S_y when $k=a/h$ is about $2\sim3$ or larger. The value of k found in the actual dam is usually larger than the above obtained, hence S_y alone may be considered.

Eqs. (10), (11) and (12) give the rational value of seismic coefficient of each of the one- and two-dimensional dams, and the vibration of each mode has its particular phase difference δ. Considering this point, the vibration of each mode must be added up. The maximum of the above-stated values must be adopted as the seismic coeffcient of design. For this purpose, for example, A_n and A_{ns} must be determined to be the maximum obtained by means of synchronizing and summing up the amplitudes which can be got by recording the shakes of the torsion pendulum as to each mode of vibrations ; so that the procedure requires much time and labour. However, in general, the nearest mode of vibration to the period of ground motion is most influential and the other modes are considerably small, so that the most dangerous seismic coefficient — the seismic coefficent of design — can be obtained by comparing several terms of n or n, s with each other by using the acceleration spectrum measured as the one mass system. The distribution of seismic coefficient, when the dam is subjected to one or two ground motions, will be considered in the following space.

3.1. One example of the actual earthquake

Fig. 2 shows the acceleration spectrum, obtained by means of above-mentioned torsion pendulum by using the ground acceleration of Tanabe Bay Earthquake which was recorded[4] at the Kobe Marine Meteorological Observatory on Nov. 6, 1950. The figure is drawn for various values of the damping coefficient h.

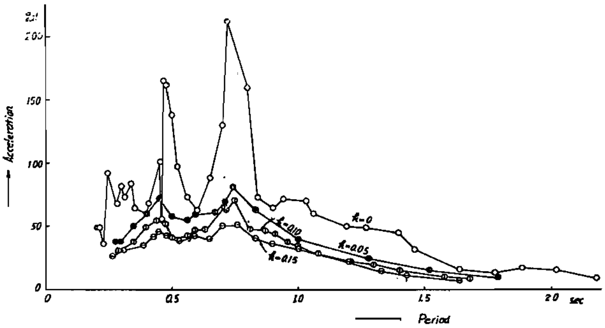

Fig. 2 Acceeleration spectrum.

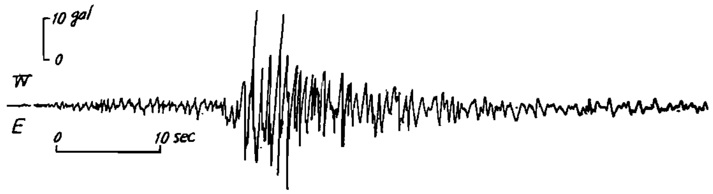

Fig. 3 Record of acceleration of Tanabe Bay Earthquake, Nov. 6, 1950, recorded at the Kobe Marine Meteorological Observatory.

Now, consider, as one example, the case* where $c_0 = 95$ m/s, the height $h = 40$ m and the length $a = 200$ m. Determining the acceleration (A_s, A_{ns},

* The period of the 1st higher mode of free vibration of the one-dimensional dam in the case as shown here, is corresponding to the peak of 0.47~0.48 sec in the acceleration spectrum.

where $\varepsilon=0$) for the period of each mode of the free vibration of the dam from the acceleration spectrum shown in Fig. 2, and computing the maximum seismic coefficient of each mode on the crest by Eqs. (10) and (12), and then expressing them as the ratios to the seismic coefficient of ground K_0, we get following Table. Figs. 4 and 5 show the distributions of the seismic coefficient and of the shearing stress of the one-dimensional dam, respectively. As seen from the figures, in the shearing stress, if there is no damping, the lst higher mode is larger than the fundamental mode. This means the resonance of the lst higher order. It can be seen that there may occur the resonance of the lst higher order for the ground motion having such periods as expected usually, provided the dam is as high as 30~40 m. However, when there is a damping force due to internal viscosity ($h_1=0.05$, $h_2=0.12$), the shearing stress is smaller

Table Ratio K/K_0 between maximum seismic coefficients of the earth dam and the ground.

s	One-dimensional dam	Two-dimensional dam. $a/h=3$				Two-dimensional dam. $a/h=5$			
		$n=1$	$n=3$	$n=5$	$n=7$	$n=1$	$n=3$	$n=5$	$n=7$
1	4.60	7.15	-3.64	1.15	-1.10	6.11	-2.40	2.54	-1.31
2	-8.70	-11.15	1.44	-1.10	0.71	-10.95	2.04	-1.15	0.59
3	3.40	4.28	-1.45	0.82	-0.31	4.36	-1.45	0.82	-0.56
4	-1.46	-1.46	0.71	-0.46	—	-1.86	0.71	-0.43	0.38

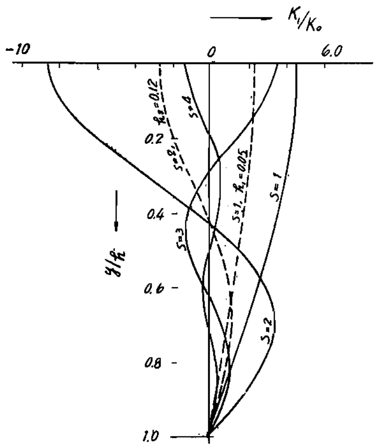

Fig. 4 Distributions of the seismic coeciffient.

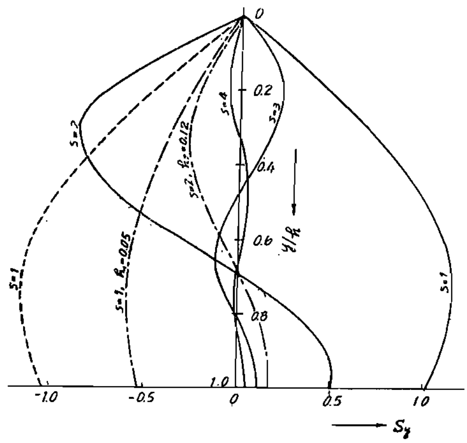

Fig. 5 Distributions of the shearing stress.

than that of the fundamental mode all over the dam height. Therefore, if the dam of such size as stated above is subjected to such a ground motion, it is in safety side to design the dam by using the distribution of seismic coefficient for the fundamental mode of vibration (the curve corresponding to $s=1$ in Fig. 4), notwithstanding that the resonance of the lst higher mode occurs at this time. By the way, the location where the maximum shearing stress occurs in the above case is not at the base, but is nearly at the level of one-fourth of the height above the base.

The fact stated above is one example of the case where the dam is subjected to the actual seismic motion. For the case of a peculiar ground motion, at the present when little data of the seismic motion of the peculiar region are available for us, the appropriate ground motion must be presumed. From such a standpoint, let us consider the initial state of vibration of the dam, when it is subjected to the seismic acceleration of sinusoidal form having the same period as that of free vibration of the dam, in the following space.

3.2. Distribution of seismic coefficient in the initial state of vibration

Consider, for simplicity's sake, the case where such a ground motion as stated before acts on the dam under the condition of no damping. Since the mode having the same period as that of the ground motion becomes remarkably larger compared with the other modes, the vibration of whole dam may be approximately expressed by the resonant mode. Therefore, the accelerations of the ground motion $a_0(s)$, $a_0(n, s)$ having the same period as that of each mode of the dam will act on the dam, and the seismic coefficients of the dam, after one period passes, are given by

$$\left.\begin{aligned} K_1 &= \frac{2}{g}\frac{1}{\lambda_s J_1(\lambda_s)} J_0\left(\frac{\lambda_s}{h}y\right) a_0(s)\,\pi, \\ K_2 &= \frac{8}{g\pi}\frac{1}{n\lambda_s J_1(\lambda_s)}\sin\frac{n\pi}{a}x\, J_0\left(\frac{\lambda_s}{h}y\right) a_0(n,s)\,\pi. \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(16)$$

When the ground accelerations $a_0(s)$ and $a_0(n, s)$ are constant, the coefficients in the above Eqs. show the ratios of the seismic coefficients in the resonance of s th and n,s th order for the constant acceleration, respectively. Similarly, for the case of constant velocities the coefficients are given by

$$\left.\begin{aligned} K_1 &= \frac{2c_0}{gh}\frac{1}{J_1(\lambda_s)}J_0\left(\frac{\lambda_s}{h}y\right)v_0(s)\,\pi, \\ K_2 &= \frac{8c_0}{g\pi h}\frac{1}{n\lambda_s J_1(\lambda_s)}\left\{\left(\frac{n\pi}{a}\right)^2+\lambda_s^2\right\}^{1/2}\sin\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)v_0(n,s)\,\pi, \end{aligned}\right\}\cdots(17)$$

where $v_0(s)$, $v_0(n,\ s)$ mean the constant velocities. Moreover, the shearing stresses can be computed from Eqs. (13)~(15) as follows:

For the case of constant acceleration

$$\left.\begin{aligned} S_{1y} &= -\,2h\rho\frac{1}{\lambda_s^2 J_1(\lambda_s)}J_1\left(\frac{\lambda_s}{h}y\right)a_0(s)\,\pi, \\ S_{2y} &= -\frac{8h\rho}{\pi}\frac{1}{nJ_1(\lambda_s)}\frac{1}{(n\pi/k)^2+\lambda_s^2}\sin\frac{n\pi}{a}x\,J_1\left(\frac{\lambda_s}{h}y\right)a_0(n,s)\,\pi, \\ S_{2x} &= 8\rho h\frac{1}{\lambda_s J_1(\lambda_s)}\frac{1}{k\{(n\pi/k)^2+\lambda_s^2\}}\cos\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)a_0(n,s)\,\pi. \end{aligned}\right\}\cdots(18)$$

For the case of constant velocity;

$$\left.\begin{aligned} S_{1y} &= -\,2c_0\rho\frac{1}{\lambda_s J_1(\lambda_s)}J_1\left(\frac{\lambda_s}{h}y\right)v_0(s)\,\pi, \\ S_{2y} &= -\frac{2c_0\rho}{\pi}\frac{1}{nJ_1(\lambda_s)}\frac{1}{\sqrt{(n\pi/k)^2+\lambda_s^2}}\sin\frac{n\pi}{a}x\,J_1\left(\frac{\lambda_s}{h}y\right)v_0(n,s)\,\pi, \\ S_{2x} &= 8\,c_0\rho\frac{1}{\lambda_s J_1(\lambda_s)}\frac{1}{k\sqrt{(n\pi/k)^2+\lambda_s^2}}\cos\frac{n\pi}{a}x\,J_0\left(\frac{\lambda_s}{h}y\right)v_0(n,s)\,\pi. \end{aligned}\right\}\cdots(19)$$

Fig. 6 and 7 illustrate the distributions of seismic coefficient and of shearing stress for the one-dimensional case. Since there hardly occur the resonances of higher order for the sake of the correlative relation with the period of the seismic motion, the resonances of the higher order less than the 2 nd are shown in the Figs..

As seen from these Figs., for the case of the constant acceleration without damping, the shearing stress shows the maximum value in the resonance of the fundamental mode; accordingly, the dam is safe for the resonance of higher order, only when the distribution of seismic coefficient of the fundamental mode is taken into account in the design. For the case of constant velocity, however, the stress in the resonance of higher order is larger in the upper part than that in the resonance of the fundamental mode. The dotted lines in Fig. 7 b) show the distribution of the shearing stress, when the internal viscous damping ($h_1 = 0.05$, $h_2 = 0.12$, $h_3 = 0.18$) acts. It also shows that there is no remarkable difference between the stress distributions in the resonance of the fundamental mode and in that of higher order. Therefore, it may be safe as well for the resonance of higher order to take into account the distribution of seismic coefficient only for the fundamental mode.

Fig. 8 gives the distribution of the shearing stress S_x in the direction of the

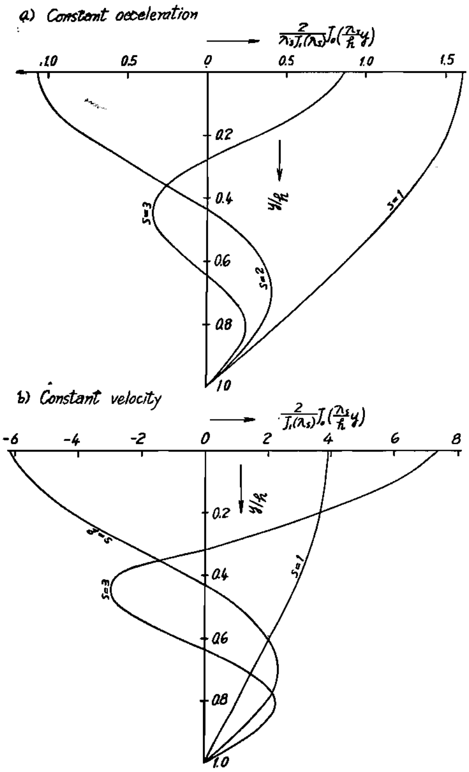

Fig. 6 Distributions of the seismic coefficient.

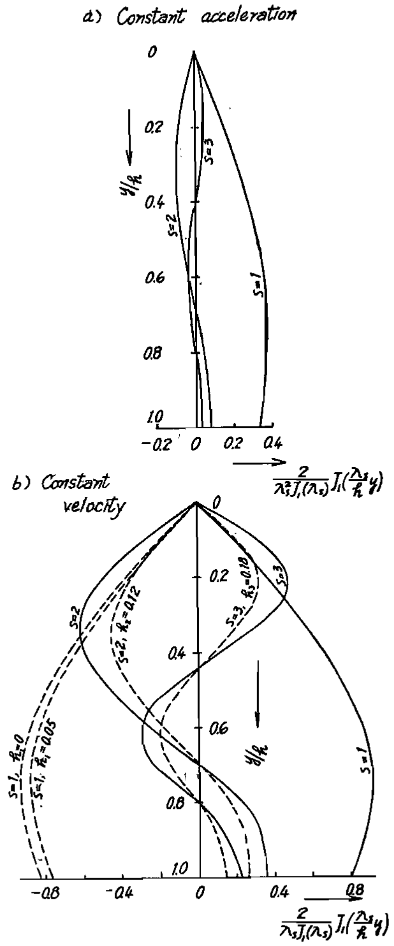

Fig. 7 Distributions of shearing stress.

crest length of the two-dimensional dam subjected to ground motion having constant velocity (damping coefficient $h=0$), showing that the vibration characters of the fundamental mode in the direction of the height and of the mode of higher order in the direction of the length ($n \geqq 2$, $s=1$) give the larger stress in the central part of dam than that caused by the resonance of the fundamental mode ($n=1$, $s=1$).

4. One Proposition for the Seismic Coefficient of Design

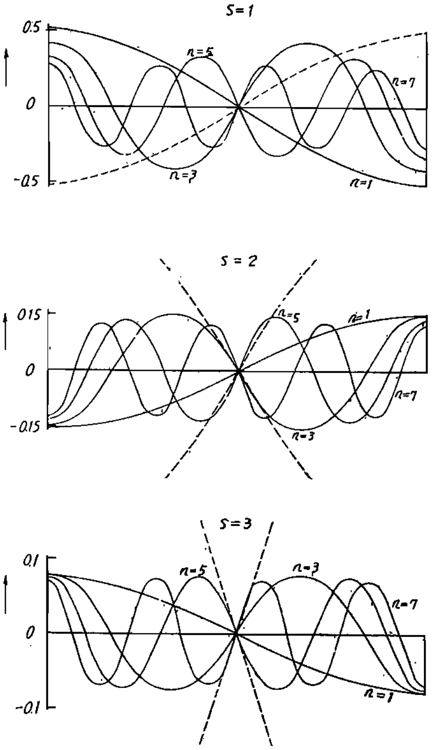

Fig. 8 Distributions of the shearing stress S_x in the direction of dam length.

The seismic coefficient as described above is obtained by presuming the constant acceleration or constant velocity. According to the latest studies, it is clarified that the constant velocity occurs in general. For example, the seismic spectrum proposed as the seismic coefficient of design of the upper part of building by R. Tanabashi and others[5], on the basis of the data recorded in Kanto Earthquake (1923), is of the constant velocity type much the same as the standard acceleration spectrum[6] proposed by the Joint Committee of the San Francisco, U.S.A., and it is practised in general that small seismic coefficient will be adopted for the structures having long period of vibration. T. Hatano[7] also points out that the stability during the earthquake is more widely affected by the velocity than by the acceleration.

As to the absolute value of the seismic coefficient, the discussion made on building is not always the same as that on dam. Nowadays, many points still remains unexplained on the character of ground motion. Hence, with reference to the value of seismic coefficient conventionally used in designing dam, the author dare propose a seismic coefficient of design described in the following space.

For the top of dam:

usually $0.3/T$ instead of $0.2/T$ proposed by R. Tanabashi and others, the upper limit 0.5 and the lower limit 0.15 (corresponding to the upper limit 0.3 and the lower limit 0.09 for the uniform distribution of seismic coefficient).

Along the height:

the Bessel's distribution

$$K=\frac{0.3}{T}J_0\left(2.4048\frac{y}{h}\right). \qquad \cdots\cdots\cdots\cdots\cdots (20)$$

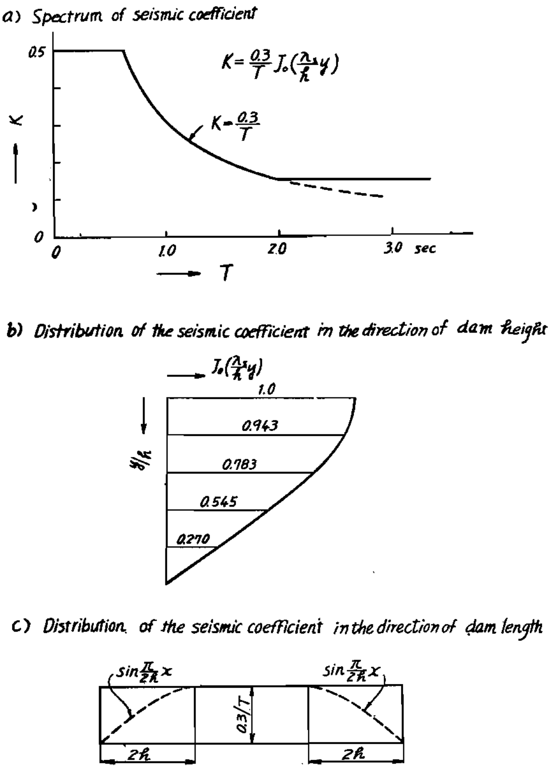

Fig. 9 Seismc coefficient of design.

The seismic spectrum and the distribution of seismic coefficient proposed above are illustrated in Fig. 9 a) and b). Since the earth dam, in general, has a constant grade of faces of slope in the length direction, it may be built uniformly of the same material and by the same construction method. Therefore, if the height is also constant, the dam may have the one-dimensionally uniform safety over its whole length, but, judging two-dimensionally, seismic coefficient is too large in both sides. Accordingly, the smaller seismic coefficient should be adopted in both sides, and the distribution of the coefficient may be determined, in entirely the same way as that was done in order to determnie the distribution in the direction of height, examining the stresses in two-dimensional dam by Eqs. (18) and (19). As stated before, however, the restraint of both sides has little effect upon the central part of dam when the ratio of length to height is larger than 3~5; thus, the distribution of seismic coefficient may be determined in such a way that it decreases in a sinusoidal form from the one-dimensional value at the central part to the zero value at both ends, at each of the parts having the length two times as large as the dam height from each end, as shown Fig. 9, c). Fig. 9, c) corresponds to the case of constant height, but the dams usually built decrease generally in heights from the central parts to both sides. It is necessary, considering one-dimensionally, to assume the larger seismic coefficient at the part of smaller height. In such dams, the distribution of seismic coefficient in the direction of the dam length may be assumed to be nearly constant.

5. Conclusion

This paper discusses mainly the distribution of seismic coefficient of design, and presents its absolute value, only as a provisional standard. Of course, the absolute value of seismic coefficient should be determined by taking into account the regional distribution of earthquake frequency[8] and the character of subbase soil.[9]

The studies reported in this paper are based on the theory that the dam is visco-elastic, but such a theory may have something to be questioned, as to considering the soil visco-elastic. However, consideration that the dam deforms due to seismic forces leads to the conclusion that the seismic coefficient to be used in dam design cannot be uniform in every part of the dam. Further investigations are needed in solving such problems.

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Acknowledgment

This research is one part of the investigation "Studies on the Earthquake Resistant Properties of Huge Structures" which has been carried out by the Grant in Aid for Fundamental Scientific Research of the Ministry of Education. Appreciation is expressed to Dr. Y. Yokoo, Dr. T. Ishihara and Dr. S. Murayama, professors in The Disaster Prevention Research Institute of The Kyoto University, for their useful suggestions and instructions, and also to Mr. K. Tanimoto, lecturer in The Kyoto University, Mr. H. Yuasa, research assistant in The Tokushima University and Mr. M. Kawamoto, engineer of the Minstry of Construction, for their capable works and to Mr. S. Hayashi, member of The Transportation Technical Research Institute, for his cooperation in using the acceleration recorder.

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Bulletin No. 11 Published December, 1955

昭和 30 年 12 月 5 日 印 刷
昭和 30 年 12 月 10 日 発 行
編輯兼発行者 京都大学防災研究所
印刷者 山代多三郎
京都市上京区寺之内通小川西入
印刷所 山代印刷株式会社