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## Abstract

In order to research the effect of the moisture content in connection with strength characteristics of undisturbed saturated clay, a large number of tests such as U-test, vane test and triaxial compression test, have been performed. As the result, it is shown that the compressive strength of saturated clay has a linear relationship to the moisture content on the semi-logarithmic paper, and that the above linear plotting is parallel to the virgin compression line of the consolidation test. Applying the concept of soil water bond introduced in the stretched membrane theory, to the unconfined compressive strength of clay under the air-drying process, it is found that the strength of clay can be determined uniquely from the moisture content, independent of the types or kinds of testing method. And some investigations have been also tried with respect to the effect of the lateral drainage in the triaxial consolidation test.


## 1. Introduction

Undertaking the large task of obtaining our comprehensive knowledge of

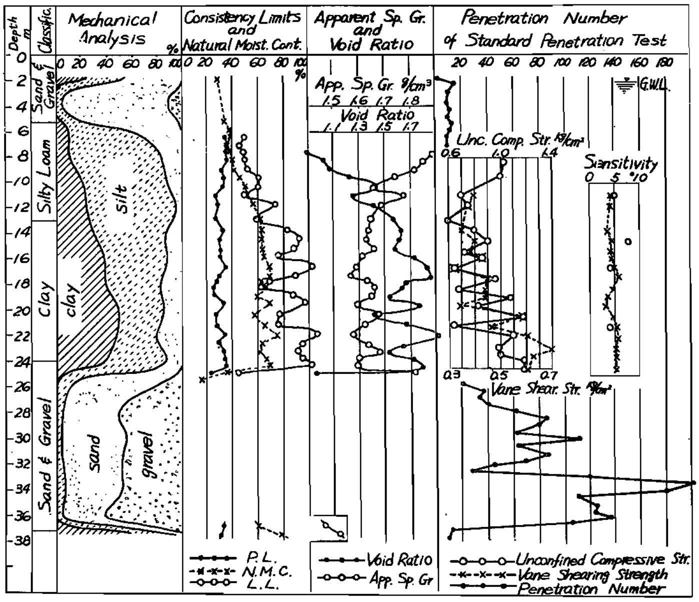


Fig. 1 Soil profile and its character at the proposed situation of constructing the Umeda Southern Station of Hankyū Electric Railway Co..

the geological formation of Ōsaka subsoil by means of a large number of soil explorations during several years, it has been anticipated that there may be a clear correlation between the unconfined compressive strength and the natural water content in undisturbed clay. Fig. 1 shows a soil profile at the proposed situation of constructing the Umeda Southern Station of Hankyū Electric Railway Co., Ltd.. The upper clay stratum which appears at a depth of 13 - 24 m below ground level is called Umeda Alluvial Stratum. Comparing its natural water content or void ratio with the results of U-tests or vane tests performed in the laboratory using undisturbed samples, it is known that, in spite of great variation of water content or void ratio with the depth of the stratum, the strength of undisturbed clay varies according to the moisture change, and that the larger the moisture content or the void ratio in the stratum, the smaller the strength of clay becomes, and vice versa. This fact, showing the effect of the moisture content on the compressive strength of undisturbed clay, is quite remarkable.

In some papers already published,[1),2),3)] it has been noted that the linear relationship exists approximately between the moisture content of clay and the unconfined compressive strength, provided the latter is plotted in the logarithmic scale. But this relationship has been ambiguous[4)]. This fact may be caused from the difficulty of getting a pretty large amount of undisturbed clay samples of the same origin.

In this paper, the effect of the moisture content on the strength characteristics of an undisturbed clay is explained by means of the soil water bond made by the pore water of soils. Some types of tests concerning thes above-mentioned problem have been performed, and some hypotheses are attempted in accordance with these experimental results.

## 2. Strength Tests of Undisturbed Clay

To determine the strength of clay, U-tests, triaxial compression tests and vane tests have been performed. The vane test has a considerable reliability for clayey soils, especially for soft clays whose consistency are too high to extrude specimens from sampler tubes. As seen from the right column in Fig. 1, there is a definite relationship $q_u=2C$ between the unconfined compressive strength $q_u$ and the cohesive strengh $C$ obtained from the vane test with the same specimen.

Specimens used in a series of tests are fully saturated undisturbed clays obtained from the ground at the North Harbor of Ōsaka, by means of the thin-walled sampler of stationary piston type. This sampler has an inner diameter of 73 mm, a thickness of tube of about 1 mm, and a length of specimen of 760 mm. As the clay in this situation has the sensitivity ratio of about 3 in Terzaghi's representation, it belongs to the category of an ordinary clay. The depth of the clay layer is 14 - 18 m below the ground level, and the results of the physical test of the clay are as follows: specific gravity = 2.64, L.L. = 80 %, P.L. = 30%, natural water content = 76%, void ratio = 2.01, degree of saturation = 100%, and the grain size distribution is shown in Fig. 2.

Specimens of U-test are taken with a sectional area size of 2.5 cm×2.5 cm and height of 6 cm, and tested under the strain-control condition whose loading speed is 1% of the height per minute. 32 tests are performed with specimens of various water contents which are obtained in such a manner so that their water content is slowly decreased by air-drying process to have the various magnitudes beginning with the natural state of 76 % to about 35 %.* Vane tests are performed by a laboratory vane tester with four vanes whose height is 3.6 cm and diameter is 2.0 cm, and the rotating speed of the vane is 60° per minute. Triaxial compression tests are performed by the consolidated-undrained test ($Q_c$-test), in the same loading manner as U-tests after 100 % consolidation under ambient pressures. The size of specimens used in this tests is divided into two kinds; one is $\phi$ 3.5 cm×8.0 cm for the tests of lateral pressure less than 3kg/cm², and the other is $\phi$ 5.0cm×12.0cm for more than 3kg/cm². In this tests, paper drains are used to accelerate the lateral drainage, and their effect will be described later.

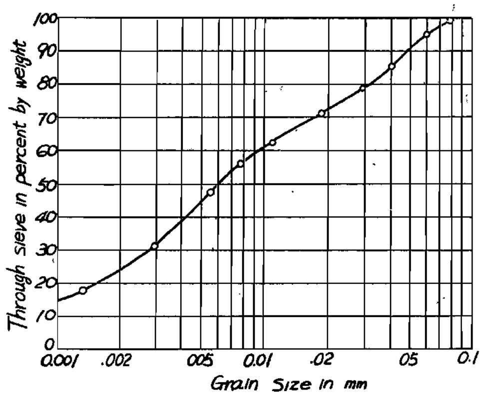

Fig. 2 Grain size distribution curve.

## 3. Relationship between the Water Content and the Strength of Clay

* These air-dried specimens can be still considered to have 100% degree of saturation.

(1) Independent of various kinds of strength test described in the preceding article, it is clear that a linear relationship holds, within a range of permissible error as in Fig. 3, when the strengths of the alluvial clay are plotted against their water contents on a semi-logarithmic paper. In this figure, the deviator stress is adopted as the triaxial compressive strength, and the compressive strength in vane tests is regarded as twice the cohesive strength $C$. Scattering points in U-tests and also in vane tests may be caused by the growth of cracks in specimens during the air-drying process, and by the existence of pieces of shell material. This effect is remarkable in vane tests, where there is a tendency to give lower strength than the actual one, especially when the moisture content is low. The relationship between the water content and the strength of clay from the results of these experiments can be represented as follows:

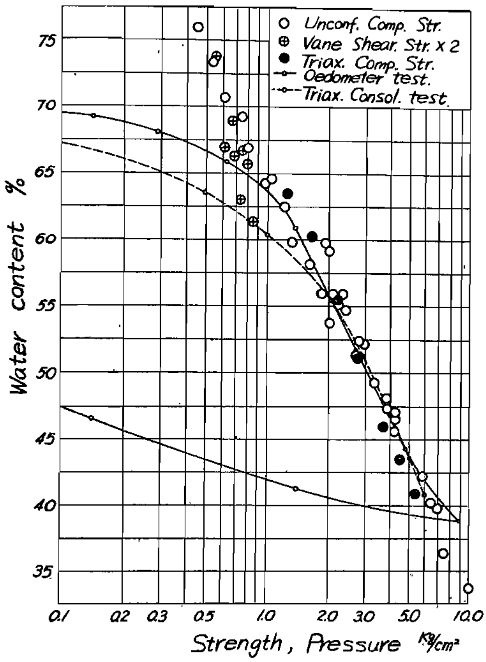

Fig. 3 Consolidation curves related to pressure and water content and relationship between water content and strength.

$$w = B(log_{10}A - log_{10}p), \qquad \cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where $w$: water content, $p$: compressive strength, and $A, B$: constants. Eq. (1) is transformed:

$$p = Ae^{-2.3\frac{w}{B}}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where $e$: the base of natural logarithm. From Eq. (2), therefore, the strength of clay is proportional to $e^{-Kw}$ ($K$: a constant).

To try to explain this relationship, the following assumption may be applicable. This assumption has been introduced by Salas-Serratosa[5] who have tried to explain the compressibility of clay, and is described as follows; "the compressive force $p$ of clay in equilibrium under the application of this force is equal to the force of repulsion by the electric charge between two particles laid at the mutual distance $d$, according to Debye's theory, and $p$ is proportional to $e^{-K'd}$. Besides, if it be assumed that the void ratio or water content in fully saturated clay is proportional to the distance between soil particles, the relationship between the moisture content $w$ at the instance of shear failure and the compressive force $p$ is:

$$p = A' e^{-K''w} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(3)$$

As this equation is equal to Eq. (2), it can be proved that the compressive strength of saturated clay has a linear relationship to the moisture content on the semi-logarithmic paper.

(2) Besides the above relationship between the compressive strength and the moisture content, the results of standard consolidation tests and of triaxial consolidation tests are also plotted in Fig. 3. The consolidation curves of these two kinds of tests coincide with each other, within the part of virgin compression lines. As it has already been described that the straight line representing the relationship between the compressive strength and the water content is parallel to the virgin compression line of standard consolidation test[6], the same statement holds for triaxial consolidation tests too. From this fact, it can be shown that the constant $B$ in Eqs. (1) and (2) is equal to the compression index $C_c$.

On the other hand, with a large number of consolidation tests, the relationship between the compression index and the liquid limit (*L.L.*) of undisturbed samples of Ōsaka alluvial clay is shown as Fig. 4, and is described as follows:

$$C_c \doteqdot 0.01\ (L.L. - 12\%). \quad \cdots(4)^*$$

As the compressive strength of clay is proportional to $e^{-Kw}$ and moreover the coefficient $K$ is determined from the liquid limit,

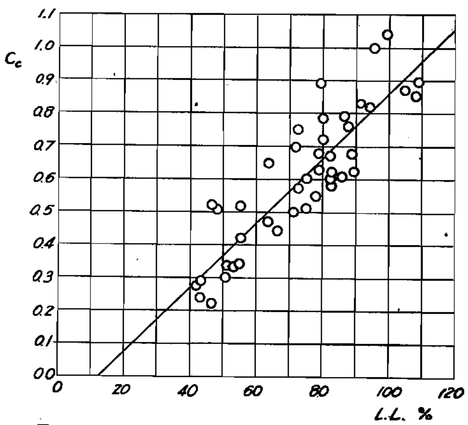


Fig. 4 Relationship between compression index and liquid limit.

* Skempton has given the following correlation: [7]

$$C_c \doteqdot 1.30 \cdot C_c' = 0.009(L.L. - 10\%),$$

where $C_c'$ is the compression index of remolded clay.

it is reasonable to consider the liquid limit as a principal factor which influences the strength of clay in accordance with the moisture content.

(3) Although it is somewhat strange at a glance to state the experimental result that the strength of clay can be determined uniquely from the moisture content, independent of the difference of stress conditions applied to specimens which have a certain minor principal stress in triaxial compression test but not in U-test, the concept of soil water bond is introduced to make way for solving the above paradoxical problem. According to this introduction, the water in soil specimens is kept by the surface tension and the absorptive force, which attract soil particles in the form of suction or reduced pressure. In the case of a fully saturated clay, the meniscus appears at the surface of a specimen with air contact, and the reduced pressure grows with the increase of surface tension on the meniscus. When the water content in the specimen is being decreased, the meniscus draws back into the inner part of the clay structure, causing the radius of curvature of the meniscus to grow smaller and smaller, and thus the soil water bond increases. Therefore, it may be permitted to consider that the surface tension appeared on the unconfined specimen as the suction force corresponds exactly to the lateral pressure in the triaxial compression test. And as it is assumed that the air-drying process to change the water content is analogous to the triaxial consolidation, the magnitude of the surface tension can be estimated from the triaxial consolidation curve. This assumption calls for the following procedure, in which the virtual pressure* caused by the surface tension on a specimen of an unconfined compression test is determined as the pressure at a point of the same water content on the triaxial consolidation curve shown in Fig. 3. Let this virtual pressure be adopted as the minor principal stress and the unconfined compressive strengh as the deviator stress, it is possible to draw Mohr's circle as shown in Fig. 5 (a) which is drawn by using the pressure of triaxial consolidation as the virtual lateral pressure. On the other hand, Fig. 5 (b) is drawn directly from the triaxial consolidated-undrained test. As the virgin compression line of the consolidation test is parallel to the line which represents the compressive strength, the angles of internal friction measured by these figures should coincide with each other. For example, the angle of internal friction $\phi$ obtained from Fig. 5 (a) is equal to abont 20°, whereas from Fig. 5 (b) it is about 19°.

* cf. the following Appendix.

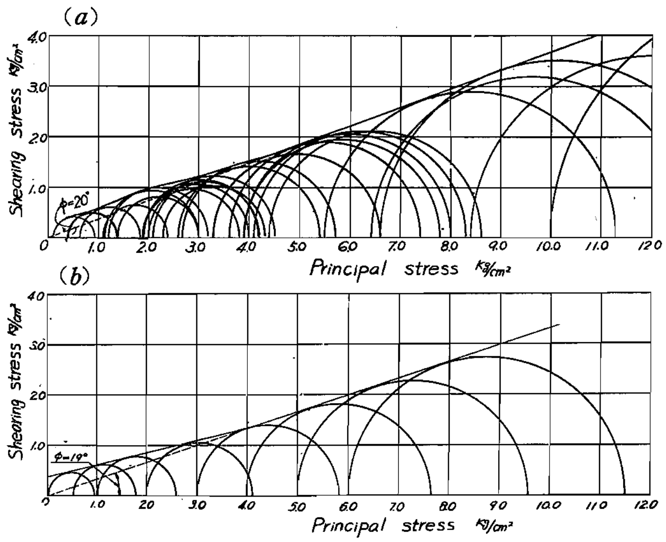


Fig. 5 (a) ; Mohr's circles and their envelope drawn by the new method using U-tests.
(b) ; Mohr's circles and their envelope drawn by the ordinary $Q_c$-tests.

As the virgin compression lines of standard consolidation curve and triaxial consolidation curve are represented as a same line, as described in (2), the Mohr's failure envelope corresponding to triaxial $Q_c$-test can be drawn by using only the data of the U-test and the standard consolidation test. As the drained test (S-test) permits the water decrease by the drainage during the shearing process, it may be recognized that the drained test gives larger strength than that of the consolidated-undrained test, although in this S-test, it must be considered that one part of the shearing resistance is spent in the volume change during the shearing process.[8]

## 4. Triaxial Consolidation Test

In this article, the triaxial consolidation before shearing procedure in the $Q_c$-test is treated. In the following description, let the term of uni-axial, bi-axial or tri-axial consolidation be used for the direction of drainage, but the load for consolidation is applied upon the whole surface of the test specimen by means of ambient pressure in each case.

Using the cylindrical co-ordinate $(r, z)$, the differential equation of consolidation is written in the following forms for three drainage conditions, respectively[9].

Uni-axial consolidation :

$$\frac{\partial u}{\partial t} = c\frac{\partial^2 u}{\partial z^2}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(5)$$

Bi-axial consolidation :

$$\frac{\partial u}{\partial t} = c\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \qquad \cdots\cdots\cdots\cdots\cdots\cdots(6)$$

Tri-axial consolidation :

$$\frac{\partial u}{\partial t} = c\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2}\right). \qquad \cdots\cdots\cdots\cdots\cdots\cdots(7)$$

After solving these equations, the relationship between the degree of consolidation $U$ and the time facter $T$ is obtained as follows :

Uni-axial consolidation :

$$U_v = 1 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(1+2n)^2}e^{-(1+2n)^2\pi^2 T_v/4}, \qquad \cdots\cdots\cdots\cdots\cdots(8)$$

Bi-axial consolidation :

$$U_{r\omega^2_n} = 1 - 4\sum_{n=1}^{\infty}\frac{1}{\omega_n^2}e^{-T_r\omega_n^2}, \qquad \cdots\cdots\cdots\cdots\cdots\cdots(9)$$

where $T_v = ct/H^2$, $T_r = ct/R^2$, and $2H$, $2R$ are the height and the diameter of the cylindrical specimen, respectively, and $\omega_n$ is the $n$-th root of Bessel's function $J_0(\omega) = 0$ [10),11)] The degree of consolidation in the triaxial consolidation is given by the equation[12)] :

$$U_{v+r} = 1 - (1 - U_v)(1 - U_r). \qquad \cdots\cdots\cdots\cdots\cdots\cdots(10)$$

From Eqs. (8) and (9), the relationship between $U$ and $T$ has been solved by the numerical calculation applying the "Relaxation Method" for the case of $H/R = 2$ [13)].

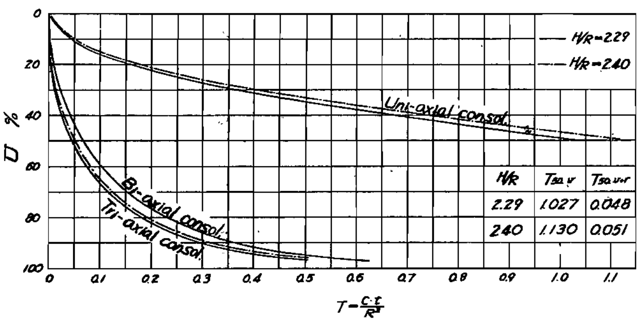


Fig. 6 Relationship between degree of consolidation and time factor.

Fig. 6 shows this relationship between $U$ and $T$, calculated for the dimension of specimens used in our test ($2H/2R=8.0/3.5=2.29$, and $2H/2R=12.0/5.0=2.40$), and the time factor $T_{50}$, corresponding to $U=50\%$, is written in this figure.

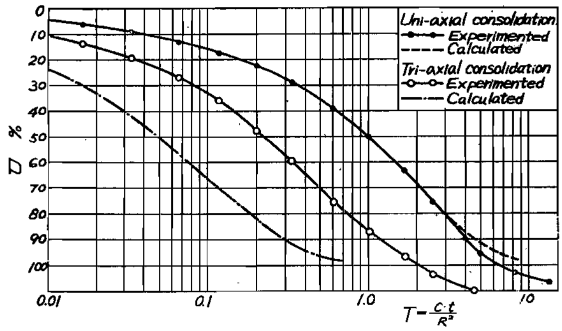


Fig. 7 Relationship between degree of consolidation and time factor (log. scale).

In the tri-axial consolidation test, the paper drain, which consists of a leaching paper with vertical strips wrapped around the side of the specimen, is used for accelerating the lateral drainage. The amount of drained water squeezed from the specimen is recorded by observating the burette, in relation to the time elapsed. Comparing the times required for 50% consolidation in the case of using the paper drain and without it (uni-axial consolidation), the result is that the use of a paper drain shortens the time by one-fifth of that without paper drain. For example, Fig. 7 shows the result in the case of lateral pressure of $3.0 kg/cm^2$. Solid curves in the figure represent the theoretical ones traced from Fig. 6. It can be seen that uni-axial consolidation-time curve is quitely coincident with the theoretical value, and that the effect of the paper drain accelerates the consolidation of clay for a considerable degree, but still less than the theoretical value.*

## 5. Conclusion

This paper is a brief description about the results of a series of experimental studies to investigate the effect of the moisture content on the strength characteristics of a fully saturated undisturbed clay. From the experimental

* In this test, to get 100% consolidation, it takes 100 minutes and 330 minutes for the case of using the paper drain and without it, respectively.

result, the conclusions are as follows.

(1) The compressive strength of fully saturated clay has a linear relationship to the moisture content on the semi-logarithmic paper, and the above linear plotting is parallel to the virgin compression line of the consolidation test.

(2) As the above statement holds independent of the types or kinds of tests, the strength of clay can be determined uniquely from the moisture content and is proportional to $e^{-Kw}$, excepting the time effect appeared in the creep of clay.

(3) The virgin compression lines of standard consolidation test and triaxial consolidation test coincide with each other.

(4) The liquid limit can be taken as a principal factor which influences the strength of clay with the moisture content.

(5) In U-tests performed by the moisture control from the natural water content to lower values, it is possible to draw Mohr's circles as triaxial consolidated-undrained tests applying the concept of soil water bond introduced in the stretched membrane theory.

(6) The use of the paper drain in triaxial test accelerates the consolidation of clay to a considerable degree.

The above conclusions are, however, obtained only from the experimental results of a drying process from the natural water content to its lower values. It is hoped that more investigations will be forthcoming concerning the reversive procedure, namely wetting process.

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# Appendix

Introducing an idea of using pF which is the logarithm of the capillary potential of water in soil, Schofield* has simplified the concept of the energy with which water is held by the soil. To measure the capillary potential over the whole range of moisture content of soil, several testing methods should be used, because each of these methods has an applicable range. Fig. 8 shows a relationship between pF and water content of an undisturbed clay sample measured by some testing methods, i. e. for the lower part of water content, vacuum desiccator method and for the medium and high parts, suction plate method, centrifugal method and consolidation method. Though the drying and wetting curves differ considerably and have a remarkable hysteresis as shown in Fig. 8, it is noted that the soil suctions expressed by pF values in drying process have an approximate linear relationship to the moisture contents within the range in which the consolidation test is applicable. This relation seems to be analogous to the relationship between water content and log $p$ obtained by usual consolidation test.

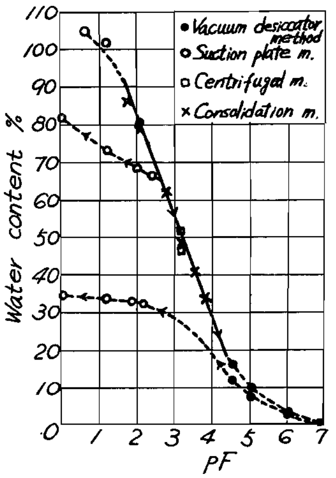

Fig. 8 Relationship between water content and pF value.

It is interesting and suggestive, therefore, to note that the relationship between soil suction caused by air drying of specimen and its water content is given by using the results of consolidation test. This fact makes possible to consider that the virtual lateral pressure caused by the surface tension on a specimen of an unconfined compression test is determined as the pressure at a point of the same water content on the triaxial consolidation curve.

* R.K. Schofield : The pF of the Water in Soil, Trans. 3rd Intern. Congr. Soil Sci., Vol. 2, 1935, pp. 37–48.

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