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# STUDY ON ELASTIC STRAIN OF THE GROUND IN EARTH TIDES

BY

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## 1. Introduction

Observation of the strain of the earth's surface caused by the tide-generating force by means of extensometer was inititated by K. Sassa [1], and the value of the constant $l$, usually called the Shida's number [2] was, at the first time, discussed separately from other constants, $k$ and $h$ (Love's numbers) [4]. E. Nishimura [3] obtained the value of $D=1+k-h=0.66$ from tiltmetric observation at Barin and the value of $L=1+k-l=1.20$ by analysing tidal latitude variations at six stations of the International Latitude Service. The Loves's number $k$ can be obtained from observation of the Chandler's period [5] of latitude variations, and according to Pollack [6], [7] it is estimated to be $k=0.287$. If the earth is considered homogenous and incompressible, the relations of $l=3/10h$ [2] and $k=3/5h$ [4] exist theoretically. These relations do not concord with the practical observations of the earth tide obtained by means of tiltmeters, gravimeters and others.

The problem of determining the strain of gravitating elastic sphere, under the action of small disturbing force having a spherical harmonic potential, has been solved by Kelvin and Tait [34], A.E.H. Love [8] and others for the case in which the density and elastic moduli are uniform throughout the body. Thereafter L. M. Hoskins [9] has also solved the problem in a case where the density and the elastic moduli are functions of distance from the earth's center. Recently, H. Takeuchi [10] calculated tidal deformation of the earth with internal constitutions inferred from recent seismological works [11], [12] combined with K. E. Bullen's [13] model of density distribution within the earth.

The tidal strain of the earth crust due to the attraction of heavenly bodies includes the so-called oceanic effect or load-tide which is the strain due to tidal variation of the load of sea water besides the so-called direct effect or bodily tide which is the deformation under the direct influence of the tide-generating force.

The direct effect means the tidal deformation which is assumed to occur if there were no ocean on the earth. The investigation of this oceanic effect is, needless to say, necessary and important in order not only to study the direct effect of the earth tide, but also to investigate the nature and structure of the earth crust by applying the load effect of sea water to the earth surface. In the present paper, the results obtained from the observations of the earth's tidal strain by using various types of extensometers at three stations are presented. The three stations concerned are Osakayama in Shiga Prefecture, 65 km distant from the nearest sea, Kishu Mine in Mie Prefecture, 15 km distant from the nearest sea, and Suhara in Wakayama Prefecture, close by the sea. From the observational results, the strain of ground caused by the tidal load of sea-water is discussed in some detail. Further, the tidal extensions in individual six directions at Osakayama Observatory are analysed and the vertical strain, horizontal areal strain and cubical dilatation in direct effect of the earth tides are also calculated.

## 2. Fundamental theory

Assuming that the tide-generating potential $W_2$ is proportional to $S_2 r^2$, where $S_2$ is a spherical surface harmonic of order 2 and a simple harmonic function of the time, and $r$ is the radius vector from the earth's center. The components of radial, meridional and prime vertical displacements, $u_r$, $u_\theta$ and $u_\phi$ due to the tide-generating potential $W_2$ are written as follows:

$$\left.\begin{aligned} u_r &= \frac{2F(r)+G(r)r^2}{r} W_2 \\ u_\theta &= \frac{F(r)}{r}\frac{\partial W_2}{\partial \theta} \\ u_\phi &= \frac{F(r)}{r\sin\theta}\frac{\partial W_2}{\partial \phi} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(1),$$

where $F(r)$ and $G(r)$ are function of $r$ only, $\theta$ is colatitude, and $\phi$ is longitude.

In polar coordinate $r, \theta, \phi$, the following formulae will obtain: for the strains,

$$\left.\begin{aligned} &e_{rr}=\frac{\partial u_r}{\partial r}, \quad e_{\theta\theta}=\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r}{r}, \quad e_{\phi\phi}=\frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi}+\frac{u_\theta}{r}\cot\theta+\frac{u_r}{r} \\ &e_{\theta\phi}=\frac{1}{r}\left(\frac{\partial u_\phi}{\partial \theta}-u_\phi\cot\theta\right)+\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \phi}, \quad e_{\phi r}=\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \phi}+\frac{\partial u_\phi}{\partial r}-\frac{u_\phi}{r} \end{aligned}\right\} \quad (2),$$

$$e_{r\theta}=\frac{\partial u_\theta}{\partial r}-\frac{u_\theta}{r}+\frac{1}{r}\frac{\partial u_r}{\partial \theta}$$

for the cubical dilatation

$$\Delta=\frac{1}{r^2\sin\theta}\left\{\frac{\partial}{\partial r}(r^2u_r\sin\theta)+\frac{\partial}{\partial\theta}(ru_\theta\sin\theta)+\frac{\partial}{\partial\phi}(ru_\phi)\right\}.$$

Using (1), (2) and the partial differential equation

$$\frac{1}{\sin^2\theta}\frac{\partial^2 S_2}{\partial\phi^2}+\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial S_2}{\partial\theta}\right)+6S_2=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots(3),$$

the components of strain due to the disturbing potential $W_2$ may be written as follows:

for the strains,

$$\left.\begin{aligned}
e_{rr}&=\frac{1}{r^2}\left\{2r\frac{dF(r)}{dr}+2F(r)+r^3\frac{dG(r)}{dr}+3r^2G(r)\right\}W_2\\
e_{\phi\phi}&=\frac{1}{r^2}\left[\frac{F(r)}{\sin^2\theta}\frac{\partial^2 W_2}{\partial\phi^2}+F(r)\cot\theta\frac{\partial W_2}{\partial\theta}+\{2F(r)+r^2G(r)\}W_2\right]\\
e_{\theta\theta}&=\frac{1}{r^2}\left[F(r)\frac{\partial^2 W_2}{\partial\theta^2}+\{2F(r)+r^2G(r)\}W_2\right]\\
e_{r\phi}&=\frac{1}{r^2\sin\theta}\left\{r\frac{dF(r)}{dr}+r^2G(r)+2F(r)\right\}\frac{\partial W_2}{\partial\phi}\\
e_{r\theta}&=\frac{1}{r^2}\left\{r\frac{dF(r)}{dr}+r^2G(r)+2F(r)\right\}\frac{\partial W_2}{\partial\theta}\\
e_{\theta\phi}&=\frac{F(r)}{r^2\sin\theta}\left\{2\frac{\partial^2 W_2}{\partial\theta\partial\phi}-\cot\theta\frac{\partial W_2}{\partial\phi}\right\}
\end{aligned}\right\}\cdots(4\text{-}1),$$

for the horizontal areal strain,

$$\Sigma=e_{\theta\theta}+e_{\phi\phi}=\frac{2}{r^2}\{r^2G(r)-F(r)\}W_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots(4\text{-}2),$$

for the cubical dilatation

$$\Delta=\left\{\frac{2}{r}\frac{dF(r)}{dr}+r\frac{dG(r)}{dr}+5G(r)\right\}W_2 \quad \cdots\cdots\cdots\cdots\cdots\cdots(4\text{-}3).$$

And potential and strain components of component tide are as follows:

1. Semi-diurnal tide at $r=a$ ($a$ is the radius of the earth).

For the potential of tide-generating force,

$$W_2\left(\frac{1}{2}\right)=a^2A_2\sin^2\theta\cos 2(t_2+\phi) \quad \cdots\cdots\cdots\cdots\cdots\cdots(5\text{-}1),$$

where $t_2$ is the hour angle of the heavenly body at Greenwich and $A_2$ is a factor containing declination of the heavenly body.

For the strains,

$$\left.\begin{aligned}
&e_{rr}=A_2\left\{2F(a)+2a\frac{dF(a)}{dr}+3a^2G(a)+a^3\frac{dG(a)}{dr}\right\}\sin^2\theta\cos 2(t_2+\phi)\\
&e_{\theta\theta}=A_2\{2F(a)(\cos 2\theta+\sin^2\theta)+a^2G(a)\sin^2\theta\}\cos 2(t_2+\phi)\\
&e_{\phi\phi}=A_2\{-2F(a)+a^2G(a)\sin^2\theta\}\cos 2(t_2+\phi)\\
&e_{\theta\phi}=A_2(-6\cos\theta)F(a)\sin 2(t_2+\phi)\\
&e_{r\theta}=e_{r\phi}=0\\
&\sum=e_{\theta\theta}+e_{\phi\phi}=2A\{a^2G(a)-F(a)\}\sin^2\theta\cos 2(t_2+\phi)\\
&e_{\theta\theta}-e_{\phi\phi}=2A_2F(a)(\cos 2\theta+\sin^2\theta+1)\cos 2(t_2+\phi)\\
&\Delta=\left\{2a\frac{dF(a)}{dr}+a^3\frac{dG(a)}{dr}+5a^2G(a)\right\}A_2\sin^2\theta\cos 2(t_2+\phi)
\end{aligned}\right\}\quad(5\text{-}2).$$

2. Diurnal tide at $r=a$.

For the potential of tide-generating force,

$$W_2(1)=a^2A_1\sin 2\theta\cos(t_1+\phi)\qquad\cdots\cdots\cdots\cdots\cdots\cdots(6\text{-}1),$$

where $A_1$ is another factor containing declination of the heavenly body.

For the strains,

$$\left.\begin{aligned}
&e_{rr}=A_1\left\{2F(a)+2a\frac{dF(a)}{dr}+3a^2G(a)+a^3\frac{dG(a)}{dr}\right\}\sin 2\theta\cos(t_1+\phi)\\
&e_{\theta\theta}=A_1\{a^2G(a)-2F(a)\}\sin 2\theta\cos(t_1+\phi)\\
&e_{\phi\phi}=A_1\{a^2G(a)\}\sin 2\theta\cos(t_1+\phi)\\
&e_{\theta\phi}=A_1F(a)\{4-2\cot^2\theta\}\sin(t_1+\phi)\\
&e_{r\phi}=e_{r\theta}=0\\
&\sum=e_{\theta\theta}+e_{\phi\phi}=2A_1\{a^2G(a)-F(a)\}\sin 2\theta\cos(t_1+\phi)\\
&e_{\theta\theta}-e_{\phi\phi}=-2F(a)A_1\sin 2\theta\cos(t+\phi)\\
&\Delta=\left\{2a\frac{dF(a)}{dr}+a^3\frac{dG(a)}{dr}+5a^2G(a)\right\}A_1\sin 2\theta\cos(t_1+\phi)
\end{aligned}\right\}\quad(6\text{-}2).$$

According to formulae (5) and (6), the strains $e_{\theta\phi}$ and $e_{\theta\theta}-e_{\phi\phi}$ are function of $F(r)$ only, and according to formulae (6), $e_{\phi\phi}$ is a function of $G(r)$ only. Introducing the following notations

$$\left.\begin{aligned}
\frac{H(r)}{g(r)}&=\frac{2F(r)+r^2G(r)}{r}\\
\frac{L(r)}{g(r)}&=\frac{F(r)}{r}\\
h&=H(a)\\
l&=L(a)
\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(7),$$

where $g$ is the undisturbed gravity at $r$, we get for horizontal areal strain $\Sigma(a)$,

$$\Sigma(a)=e_{\theta\theta}(a)+e_{\phi\phi}(a)=2(h-3l)\frac{W_2}{ag} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(8).$$

Since normal and tangential stress must vanish on the earth surface, we get

$$\left.\begin{aligned} \widehat{rr}&=\lambda\Delta(a)+2\mu\, e_{rr}(a)=0 \\ \widehat{r\theta}&=\qquad\quad \mu\, e_{r\theta}(a)=0 \\ \widehat{r\phi}&=\qquad\quad \mu\, e_{r\phi}(a)=0 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(9),$$

where $\lambda$ and $\mu$ are Lamé's elastic constants.
The extension $E_i$ in any direction $i$ will be given by

$$E_i=e_{rr}\alpha^2+e_{\theta\theta}\beta^2+e_{\phi\phi}\gamma^2+e_{\theta\phi}\beta\gamma \qquad \cdots\cdots\cdots\cdots\cdots\cdots(10),$$

where $\alpha$, $\beta$ and $\gamma$ denote the direction cosines of the direction $i$ taken to radius, meridian and prime vertical circles respectively. And we are able to obtain the components of the strain from relation (10) by means of observation of $E_i$ in more than four different directions.

## 3. The influence of oceanic tide on tidal strain*

The results obtained from the earth tidal observation at a station near the sea contain, by any means, the large amount of ground deformation caused by the oceanic tide of neighboring sea, as well as the deformation caused by the direct effect of the sun and the moon. The oceanic effect of tidal tilt was studied by T. Shida [14], K. Sekiguchi [15]. D. Nukiyama, R. Takahashi [16], E. Nishimura [17], R. H. Corkan [18] and others [19], T. Hagiwara [20] and his group carried out the observation of tidal extensions at Aburatsubo close by the sea (20 m distant from the sea-side) and studied the influence of oceanic tide in a case where the sea bed is inclined at the coast. The present author [21] observed tidal strains at Osakayama, and compared the results of his observation with that at Makimine, discussing the oceanic effect at the stations, both considerably distant from the sea.

Theoretically the problem is concerned with the so-called "Boussinesq's

* cf. Appendix

Problem" which was first discussed by J. Boussinesq [22], with regard to the point load problem on semi-infinite elastic body, and since then it has been treated by many researchers such as K. Terazawa [23], H. Nagaoka [24], H. Lamb [25], A.E.H. Love [26], K. Sezawa and G. Nishimura [27]. M. Matsumura [28], [29] and G. Nishimura [30] has obtained the surface deformation of semi-infinite elastic solid with surface layer due to any loading.

Let us suppose that the earth consists of uniform semi-infinite elastic medium. Let $x$ and $y$ axes be horizontal along the earth's surface, and $z$ axis vertical inward. When a force $P$ due to a point load acts at the origin parallel to the $z$ axis, the displacement $u$, $v$, $w$ of $x$, $y$, $z$ components are, according to J. Boussinesq, given by

$$\left.\begin{aligned} u &= \frac{P}{4\pi\mu}\frac{xz}{r^3} - \frac{P}{4\pi(\lambda+\mu)}\frac{x}{r(z+r)} \\ v &= \frac{P}{4\pi\mu}\frac{yz}{r^3} - \frac{P}{4\pi(\lambda+\mu)}\frac{y}{r(z+r)} \\ w &= \frac{P}{4\pi\mu}\frac{z^2}{r^3} + \frac{P(\lambda+2\mu)}{4\pi\mu(\lambda+\mu)}\frac{1}{r} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(11),$$

where

$$r^2 = x^2 + y^2 + z^2$$

From the equation (11) the components of strain are given as follows.

$$\left.\begin{aligned} e'_{xx} &= \frac{\partial u'}{\partial x} = \frac{P}{4\pi\mu}\left(\frac{z}{r^3} - \frac{3x^2z}{r^5}\right) - \frac{P}{4\pi(\lambda+\mu)}\left\{\frac{(r^2-x^2)(z+r)-rx^2}{r^3(z+r)^2}\right\} \\ e'_{yy} &= \frac{\partial v'}{\partial y} = \frac{P}{4\pi\mu}\left(\frac{z}{r^3} - \frac{3y^2z}{r^5}\right) - \frac{P}{4\pi(\lambda+\mu)}\left\{\frac{(r^2-y^2)(z+r)-ry^2}{r^3(z+r)^2}\right\} \\ e'_{zz} &= \frac{\partial w'}{\partial z} = \frac{P}{4\pi\mu}\left(\frac{2z}{r^3} - \frac{3z^3}{r^5}\right) - \frac{P(\lambda+2\mu)}{4\pi\mu(\lambda+\mu)}\frac{z}{r^3} \\ e'_{xy} &= \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} = -\frac{P}{4\pi\mu}\frac{3xyz}{r^5} + \frac{P}{4\pi(\lambda+\mu)}\frac{2xy(z+2r)}{r^3(z+r)^2} \\ e'_{zx} &= \frac{\partial w'}{\partial x} + \frac{\partial u'}{\partial z} = -\frac{P}{4\pi\mu}\frac{6xz^2}{r^5} \\ e'_{yz} &= \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} = -\frac{P}{4\pi\mu}\frac{6yz^2}{r^5} \\ \Delta' &= e'_{xx} + e'_{yy} + e'_{zz} = -\frac{P}{4\pi(\lambda+\mu)}\frac{2z}{r^3} \end{aligned}\right\} \cdots(12),$$

where the sign "$'$" refers to the displacement and strain related with the oceanic effect. From (12), the components of strain at the surface except the

origin become as follows.

$$\left.\begin{aligned}
&(e'_{xx})_{z=0}=\frac{P}{4\pi(\lambda+\mu)}\frac{x^2-y^2}{r^4}\\
&(e'_{yy})_{z=0}=\frac{P}{4\pi(\lambda+\mu)}\frac{y^2-x^2}{r^4}=-(e'_{xx})_{z=0}=0\\
&(e'_{zz})_{z=0}=0\\
&(e'_{xy})_{z=0}=\frac{P}{4\pi(\lambda+\mu)}\frac{4xy}{r^4}\\
&\Sigma'=(e'_{xx}+e'_{yy})_{z=0}=0\\
&\Delta'_{z=0}=0
\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots\cdots(13).$$

Let a new coordinate $x'$, $y'$, $z'$ be parallel to the former coordinate $x$, $y$, $z$ and its orign be removed from the load point $P$ to an observing point $O$, and let $r$ and $\psi$ be the length of the vector $OP$ and the angle between $x'$-axis and $OP$ respectively. Then the equations (13) will become

$$\left.\begin{aligned}
&(e'_{x'x'})_{z'=0}=\frac{P}{4\pi(\lambda+\mu)}\frac{\cos 2\psi}{r^2}\\
&(e'_{y'y'})_{z'=0}=-\frac{P}{4\pi(\lambda+\mu)}\frac{\cos 2\psi}{r^2}\\
&(e'_{x'y'})_{z'=0}=\frac{P}{4\pi(\lambda+\mu)}\frac{2\sin 2\psi}{r^2}\\
&(e'_{z'z'})_{z'=0}=(e'_{y'z'})_{z'=0}=(e'_{x'z'})_{z'=0}=(e'_{x'x'}+e'_{y'y'})_{z'=0}=(\Delta')_{z'=0}=0
\end{aligned}\right\}\quad(14).$$

where

$$\begin{aligned}
x'&=-r\cos\psi\\
y'&=-r\sin\psi\\
z'&=z.
\end{aligned}$$

When each equation of (14) is integrated over the surface load, surrounded by concentric circles which have the radii $r_n$ and $r_{n+1}$ respectively, and whose center is the point of observation, the movable radii making polar angles $\psi_n$ and $\psi_{n+1}$ with $x'$-axis, the components of strain will be expressed by

$$\begin{aligned}
(e'_{x'x'})_{z'=0}&=\frac{P}{4\pi(\lambda+\mu)}\int_{r_n}^{r_{n+1}}\int_{\psi_n}^{\psi_{n+1}}\frac{\cos 2\psi}{r^2}r\,dr\,d\psi\\
&=\frac{P}{4\pi(\lambda+\mu)}\log\frac{r_{n+1}}{r_n}\left(\frac{1}{2}\right)(\sin 2\psi_{n+1}-\sin 2\psi_n)\\
(e'_{y'y'})_{z'=0}&=\frac{P}{4\pi(\lambda+\mu)}\int_{r_n}^{r_{n+1}}\int_{\psi_n}^{\psi_{n+1}}\frac{-\cos 2\psi}{r^2}r\,dr\,d\psi\\
&=\frac{P}{4\pi(\lambda+\mu)}\log\frac{r_{n+1}}{r_n}\left(\frac{-1}{2}\right)(\sin 2\psi_{n+1}-\sin 2\psi_n)=-(e'_{x'x'})_{z'=0}
\end{aligned}$$

$$\left.\begin{aligned}(e'_{x'y'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)}\int_{r_n}^{r_{n+1}}\int_{\psi_n}^{\psi_{n+1}}\frac{2\sin 2\psi}{r^2}r\,dr d\psi \\ &= \frac{P}{4\pi(\lambda+\mu)}\log\frac{r_{n+1}}{r_n}(-1)(\cos 2\psi_{n+1}-\cos 2\psi_n) \\ &= \pm\frac{P}{4\pi(\lambda+\mu)}\log\frac{r_{n+1}}{r_n}\left\{\sin 2\left(\psi_{n+1}\mp\frac{\pi}{4}\right)-\sin 2\left(\psi_n\mp\frac{\pi}{4}\right)\right\} \\ (e'_{z'z'})_{z'=0} &= (e'_{x'z'})_{z'=0}=(e'_{y'z'})_{z'=0}=0 \\ (\Delta')_{z'=0} &= (e'_{x'x'}+e'_{y'y'})_{z'=0}=0\end{aligned}\right\}(15),$$

where $P$ is a unit surface load and is equivalent to $\rho\Delta hg$, where $\rho$ and $\Delta h$ are density of sea water and tidal height respectively.

The extensions of $E(0°)$ in the direction of $x'$-axis and $E(\pi/2)$ in the direction of $y'$-axis are equal to $(e'_{x'x'})_{z'=0}$, $(e'_{y'y'})_{z'=0}$ respectively. The extension of $E(\pi/4)$ in the direction of $\psi=\pi/4$ will be

$$E\left(\frac{\pi}{4}\right)=(e'_{x'y'})_{z'=0}\cos^2\frac{\pi}{4}=\frac{1}{2}(e'_{x'y'})_{z'=0} \quad \cdots\cdots\cdots\cdots\cdots(16).$$

Therefore

$$(e'_{x'y'})_{z'=0}=2E\left(\frac{\pi}{4}\right) \quad \cdots\cdots\cdots\cdots\cdots\cdots(17),$$

namely, the longitudinal component of strain $(e'_{x'y'})_{z'=0}$ is twice as large as the extension in the direction of $\psi=\pi/4$. Therefore we are able to calculate easily strain componts $(e'_{x'x'})_{z'=0}$, $(e_{y'y'})_{z'=0}$ and $(e'_{x'y'})_{z'=0}$ due to any local surface loads.

Let $r$, $\theta$, $\phi$ be radius, colatitude and east longitude of the earth and $u'_r$, $u'_\theta$, $u'_\phi$ be components of displacement caused by load respectively. Since it is probable to suppose that the strain caused by oceanic tides has very short wave length compared with the radius of the earth $a$, it may be assumed,

$$\left.\begin{aligned}&\left(\frac{u'_r}{a},\frac{u'_\theta}{a},\frac{u'_\phi}{a}\right)\ll\left(\frac{1}{a}\frac{\partial u'_\theta}{\partial\theta},\frac{1}{a\sin\theta}\frac{\partial u'_\phi}{\partial\phi}\right) \\ \text{and}\quad &(\theta,\phi)\ll 1\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots(18).$$

From (18), a relation between strain componts with respect to both systems of coordinate $(r, \theta, \phi)$ and $(x', y', z')$ will be obtained as follows.

$$\left.\begin{aligned}&e'_{\theta\theta}=e'_{x'x'}, \quad e'_{\phi\phi}=e'_{y'y'}, \quad e'_{rr}=e'_{z'z'} \\ &e'_{\theta\phi}=-e'_{x'y'}, \quad e'_{r\theta}=e'_{z'x'}, \quad e'_{r\phi}=-e'_{y'z'}\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots(19),$$

where $x'$, $y'$ and $z'$ axes are directed toward the north, the east and the

underground respectively.

In actual calculation of the oceanic effect $e'_{\theta\theta}$, $e'_{\phi\phi}$ and $e'_{\theta\phi}$, we prepare a transparent section paper sectioned with the graduation of concentric circles centering on the observational point and with the radial vector graduation in such a way as we shall have

$$\log\frac{r_2}{r_1}=C_1,\quad \sin 2\psi_2-\sin 2\psi_1=C_2$$

and put this section paper right on the tide chart, fitting its center on the observational spot at an appropriate rotation angle. By counting the number of sections involved within the sea area on the tide chart which we intend to calculate, the computation of the related equations (15) and (17) can easily be done.

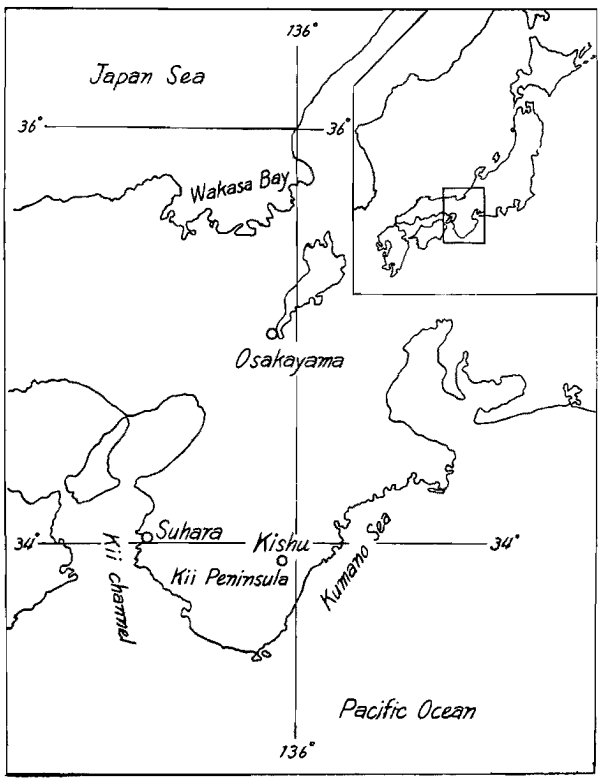



Fig. 1. Location of observatories

## 4. Observation

Observational stations concerned in tidal strain of the earth are Osakayama Observatory, Kishu Mine Observatory and Suhara Observatory, their locations being shown in Fig. 1.

1) Observation at Osakayama : Osakayama Observatory is situated at 34°59′6N and 135°51′5E. Long. The nearest sea is the Wakasa Bay which is at a distance of 65 km from the observatory. The observing room is located in the middle of an abandoned railroad tunnel of 700 m length, 150 m under the earth's surface. No artificial disturbance preventing us from the observation is in neighborhood. The geological condition is of the clayslate belonging to the Chichibu palaeozoic system. The change of temperature in the observing room is about 0.4°C in annual variation and smaller than 0.01°C in daily variation, and the amplitude of linear strain is smaller than $10^{-9}$ c.g.s. in daily variation. At this observatory, extensometers are placed in six directions three horizontal, two diagonal and one vertical directions. Besides, four additional extensometers are set up in order to make comparative observations of particular directions. The types and sensitivity of instruments are shown in Table 1.

The full informations on the instruments were given in previous reports [1], [31]. By the ordinary method of harmonic analysis the values of component tides of $M_2$, $S_2$, $K_1$, $O_1$ and others were calculated. Further adjustment necessitated by differences in time length of running mean and the period of com-

| Direction of observation | Type of instruments | Span of measured two points | Sensitivity in $10^{-8}$c.g.s./mm |
|---|---|---|---|
| Azimuth N38°E | Horizontal Sassa-type (wire) | 20 m | 0.41~0.63 |
| Azimuth N29°W | Horizontal Sassa-type (wire) | 4.2 m | 3.0 ~9.8 |
| Azimuth N61°E | Horizontal Spring-type (wire) | 9.6 m | 1.82 |
| Vertical | Spring-type (wrie) | 4.4 m | 1.85~2.24 |
| Vertical | Pivot-type (wire) | 4.0 m | 3.1 |
| Azimuth S52°E Elevation 45° | Rod-type | 5.1 m | 1.83 |
| Azimuth N38°E Elevation 48° | Pivot-type (wire) | 6.4 m | 2.46 |

Table I.

Table III. Analysed values of component tide.

| Division | Direction of observation | Epoch analysed | Sensitivity in $10^{-8}$ c.g.s./mm | $M_2$-component Ampli-tude in $10^{-8}$ | Phase lag | $S_2$-component Ampli-tude in $10^{-8}$ | Phase lag | $K_2$-component Ampli-tude in $10^{-8}$ | Phase lag | $K_1$-component Ampli-tude in $10^{-8}$ | Phage lag | $O_1$-component Ampli-tude in $10^{-8}$ | Phase lag |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | N38°E | 1947 Oct. 24.– 1949 Feb. 24.– | 0.41 –0.63 | 0.33 | 43° | | | | | | | | |
| 1′ | N38°E | 1948 Feb. 24.– 1949 Feb. 28.– | 0.41 –0.63 | | | 0.177 | 350.1° | 0.084 | 22.3° | 0.187 | 290.1° | 0.123 | 358.4° |
| 2 | N29°W | 1953 Feb. 19.– Mar. 19.– | 9.80 | 1.98 | 23° | 0.66 | 25° | 0.18 | 25° | | | | |
| 3 | N61°E | 1953 Feb. 12.– Mar. 12.– | 3.10 | 0.50 | 9° | 0.22 | 72° | 0.06 | 72° | | | | |
| 4 | Vertical | 1952 Dec. 25.– 1953 Jan. 23.– | 2.24 | 0.44 | 215° | | | | | | | | |
| 5 | Vertical | 1953 Feb. 12.– Mar. 12.– | 2.22 | 0.63 | 201° | | | | | | | | |
| 6 | Vertical | 1953 Mar. 12.– Apr. 10.– | 2.22 | 0.64 | 193° | | | | | | | | |
| 7 | N29°W | 1954 May 2.– May 31.– | 3.00 | 1.38 | 8° | 0.77 | 346° | 0.21 | 346° | 0.86 | 18° | 1.19 | 34° |
| 8 | N61°E | 1954 Aug. 31.– Sep. 29.– | 1.82 | 0.56 | 14° | 0.13 | 30° | 0.04 | 30° | 0.18 | 359° | 0.36 | 29° |
| 9 | Vertical | 1954 Oct. 7.– Nov. 5.– | 1.85 | 0.74 | 174° | 0.65 | 158° | 0.16 | 158° | 0.73 | 187° | 0.61 | 232° |
| 10 | Vertical | 1954 Nov. 5.– Dec. 4.– | 1.85 | 0.84 | 192° | 0.39 | 142° | 0.11 | 142° | 0.62 | 193° | | |
| 11 | Azimuth S52°E Elevation 45° | 1955 Jan. 7.– Jan. 21.– | 1.34 | 0.12 | 56° | | | | | | | | |
| 12 | Azimuth S52°E Elevation 45° | 1955 Sep. 20.– Oct. 19.– | 1.34 | 0.06 | 271° | | | | | | | | |
| 13 | Azimuth N38°E Elevation 48° | 1956 Jul. 13.– Aug. 11.– | 2.46 | 0.26 | 331° | | | | | | | | |

ponent tides is made at the final step as follows.

| Component tide | Period of component tide in hour | Time length of 24 hours | Running mean 25 hours |
|---|---|---|---|
| $M_2$ | 12.42 | 1.035 | 1.006 |
| $S_2$ | 12.00 | 1.000 | 1.040 |
| $K_1$ | 23.93 | 1.003 | 1.043 |
| $K_2$ | 11.97 | 1.003 | 1.043 |
| $O_1$ | 25.47 | 1.072 | 1.018 |

Table II. Coefficients of adjustment

The amplitude and phase of component tide thus obtained and the ratio of amplitude between component tides $S_2/M_2$, $K_1/M_2$ and $O_1/M_2$ are shown in Table III and Table IV.

| Division | Direction | $S_2/M_2$ | $K_1/M_2$ | $O_1/M_2$ |
|---|---|---|---|---|
| 2 | N29°W | 0.33 | | |
| 3 | N61°E | 0.44 | | |
| 7 | N29°W | 0.56 | 0.62 | 0.86 |
| 8 | N61°E | 0.23 | 0.32 | 0.64 |
| 9 | Vertical | 0.88 | 0.99 | 0.82 |
| 10 | Vertical | 0.92 | 0.74 | |
| 1′ | N38°E | 0.53 | 0.56 | 0.37 |

Table IV. Ratio of component tide.

2) Observation at Kishu Mine : Kishu Mine Observatory is situated at the point of 135°53.4′E Long. and 33°51.7′N. The observation point is about 15 km distant from the sea of Kumano which is on the east coast of the Kii peninsula. The observing room is located within the gallery at a curved distance of about 300 m from the mine gate on the ground, and about 100 m below the earth surface, and in the sand-stone of the tertiary system. The instruments used for the observation are shown in Table V.

| Direction of observation | Type of instrument | Span of measured two points | Sensitivity in $10^{-8}$c.g.s./mm | Epoch of observation |
|---|---|---|---|---|
| N79.5°W | Horizontal spring-type (wire) | 9.00m | 1.62 | Mar. 1955 |
| N60°E | Horizontal spring-type (wire) | 7.53 | 1.27 | Sep. 1955 |
| N30°W | Horizontal spring-type (wire) | 4.54 | 1.22 | Sep. 1955 |

Table V Instruments used at Kishu Mine.

The component tides were obtained by harmonic analysis of the records of one month for each of three components in a same manner as was done in the case of Osakayama. The amplitude and phase of $M_2$-term are shown in Table VI.

| Azimuth of observation | Epoch analysed | Sensitivity in $10^{-8}$c.g.s./mm | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) |
|---|---|---|---|---|
| N79.5°W | Apr. 9~May 8, 1955 | 1.62 | 0.063 | 32.9° |
| N30°W | Feb. 2~Mar.2, 1956 | 1.22 | 1.271 | 331.1 |
| N60°E | Feb. 2~Mar.2, 1956 | 1.27 | 0.710 | 44.2 |

Table VI. Observed values of $M_2$-term at Kishu.

Using observed values of extensions in three directions, the three components of horizontal strain of $M_2$-term were calculated (Table VII.)

| Component | Amplitude in $10^{-8}$ | Phase-lag ($\kappa$) |
|---|---|---|
| $e_{\theta\theta}$ | 1.691 | 5.2° |
| $e_{\phi\phi}$ | 0.244 | 256.2 |
| $e_{\theta\phi}$ | 1.477 | 258.4 |

Table VII. $M_2$-term of horizontal strain components at Kishu

3) Observation at Suhara : Suhara Observatory is situated on the sea shore and in an old copper mine at 135°11.7′E Long. and 34°02.6′N. The extensometers are installed at a place 50-90 m distant from the sea shore, 30-70 m from the mine gate and 30-60 m below the ground-surface. Neighboring geology is of chlorite schist and breccia belonging to Chichibu palaeozoic system and mesozoic system, and the station is just at the south side of Central Median Dislocation Line traversing through the Kii peninsula and Sikoku. The extensometers concerned are shown in Table VIII.

| Azimuth of observation | Type of extensometer | Span of measured two points | Sensitivity in $10^{-8}$c.g.s./mm | Epoch of observation |
|---|---|---|---|---|
| N | Horizontal spring-type (wire) | 19.55m | 3.2～ 4.6 | Dec. 1955 |
| E | Horizontal spring-type (wire) | 8.82 | 2.1～ 4.0 | Dec. 1955 |
| S70°E | Horizontal spring-type (wire) | 11.00 | 10.1～12.6 | Dec. 1955 |

Table VIII. Instrument used at Suhara.

The $M_2$-term of the extension was analysed in a same process as at Kishu and Osakayama. And amplitude and phase angle of $M_2$-term are shown in Table IX.

| Azimuth of observation | Epoch Analysed | Sensitivity in $10^{-8}$c.g.s./mm | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) |
|---|---|---|---|---|
| NS | Mar. 9～Apr. 8, 1956 | 3.2 | 1.940 | 49.9° |
| EW | Mar. 9～Apr. 8, 1956 | 2.1 | 1.876 | 36.4 |
| S70°E | Mar. 19～Apr. 18, 1956 | 10.1 | 2.307 | 52.3 |

Table IX.

Using the above mentioned observed values, $M_2$ components of horizontal strains were obtained which are shown in Table X.

| Component | Amplitude in $10^{-8}$ | Phase-lag ($\kappa$) |
|---|---|---|
| $e_{\theta\theta}$ | 1.940 | 49.1° |
| $e_{\phi\phi}$ | 1.876 | 36.4 |
| $e_{\theta\phi}$ | 2.092 | 96.0 |

Table. X. $M_2$-terms of horizontal strain-component of obtained from observation at Suhara

In Table XI, $M_2$-terms of horizontal strain components $e_{\theta\theta}$, $e_{\phi\phi}$, $e_{\theta\phi}$ and horizontal areal strain at Osakayama, Kishu and Suhara are shown together.

From the reaults of Table XI, it is clearly seen that the amplitudes of horizontal areal strain are not so different at three observatories, and their phase angles are nearly 0°. The relation between amplitudes and phase angles of $M_2$-term and their azimuth obtained from the observations are shown in Table XII and Figs. 2(a), (b), (c), (d). As is seen from Fig. 2, it appears that the amplitude of extension reaches its maximum in the azimuth of southeast and northwest and the amplitude reaches

| Component | Osakayama | | Kishu | | Suhara | |
|---|---|---|---|---|---|---|
| | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) |
| $e_{\theta\theta}$ | 0.957 | 29.6° | 1.691 | 5.2° | 1.940 | 49.1° |
| $e_{\phi\phi}$ | 1.315 | 5.5 | 0.244 | 256.2 | 1.876 | 36.4 |
| $e_{\theta\phi}$ | 1.650 | 74.8 | 1.477 | 258.4 | 2.092 | 96.0 |
| Horizontal areal strin | 2.223 | 12.3 | 1.628 | 356.9 | 2.780 | 42.8 |

Table XI $M_2$-term of horizontal strain component (observation)

| Azimuth | Osakayama | | Kishu | | Suhara | |
|---|---|---|---|---|---|---|
| | Amplitude in $10^{-8}$ | phase lag ($\kappa$) | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) | Amplitude in $10^{-8}$ | Phase lag ($\kappa$) |
| 0° | 0.724 | 22.6° | 1.691 | 5.2° | 1.940 | 49.1° |
| 15 | 1.060 | 12.9 | 1.514 | 351.1 | 2.384 | 58.3 |
| 30 | 1.400 | 7.1 | 1.260 | 335.7 | 2.595 | 62.0 |
| 45 | 1.650 | 3.8 | 1.021 | 311.4 | 2.658 | 61.8 |
| 60 | 1.810 | 1.2 | 0.811 | 288.4 | 2.493 | 57.7 |
| 75 | 1.670 | 0.1 | 0.576 | 269.0 | 2.240 | 49.0 |
| 90 | 1.260 | 0.3 | 0.244 | 256.2 | 1.875 | 36.4 |
| 105 | 0.986 | 0.3 | 0.198 | 49.0 | 1.704 | 22.5 |
| 120 | 0.662 | 9.4 | 0.699 | 44.3 | 1.550 | 11.7 |
| 135 | 0.344 | 31.7 | 1.172 | 35.5 | 1.500 | 9.9 |
| 150 | 0.338 | 49.9 | 1.534 | 26.6 | 1.496 | 20.3 |
| 165 | 0.463 | 36.9 | 1.584 | 19.2 | 1.681 | 36.0 |
| 180 | 0.724 | 22.6 | 1.691 | 5.2 | 1.940 | 49.1 |

Table XII. Observed values : Relation between amplitudes and phase lag of $M_2$-term and their azimuth.

its minimum in the direction of the right angle to it at Osakayama and Suhara. At Kishu mine, the amplitude reaches its maximum in the direction of north and south, and reaches its minimum in the direction of the right angle to it. In the three observatories, the phase angles ($\kappa$) of extension are positive, that is, cosine term is positive in all azimuth. The observ-

ed values of amplitude at Suhara are, as is expected, larger than those values at the two other stations.

## 5. Comparison of observational resultsat two stations

Now, let $\theta$ and $\phi$ be the co-latitude and longitude of observatory respectively. The potential of tide-generating force is given by

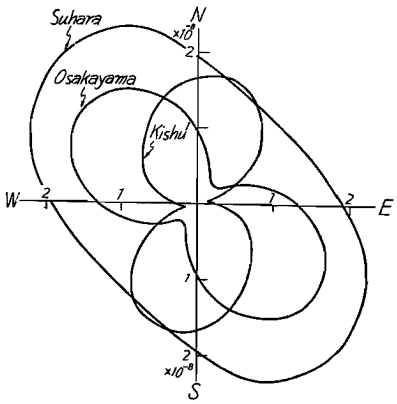



Fig. 2.(a) Relation between the amplitudes of $M_2$-term and their azimuth obtained from the observations at Osakayama, Kishu and Suhara.

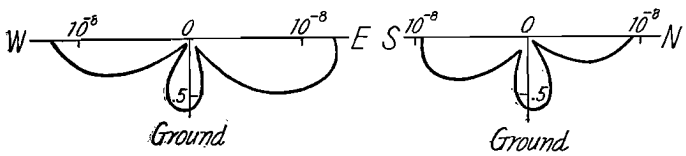



Fig. 2.(b) Relation between amplitudes of $M_2$-term and their elevation obtained from the observation at Osakayama.

$a^2A_2 \sin^2\theta \cos 2(t_2+\phi)$ for the semi-diurnal tide

$a^2A_1 \sin 2\theta \cos(t_1+\phi)$ for the diurnal tide.

Since maximum difference of longitude between these three observatories is only 0°42′ in angle and 2.8 minutes in the mean solar time, it may be assumed that their longitude are equal to each other. The difference between the tide-generating potential at any two observatories $(\theta_i, \phi_i)$ and $(\theta_j, \phi_j)$ will be

$a^2A_2\{\sin(\theta_i+\theta_j)\sin(\theta_i-\theta_j)\}\cos 2(t_2+\phi)$ for the semi-durnal tide

$a^2A_1\{2\cos(\theta_i+\theta_j)\sin(\theta_i-\theta_j)\}\cos(t_1+\phi)$ for the durnal tide

and these numerical values are shown in Table XIII.

Since the greatest differences are only 1.8% in the semi-diurnal tide and

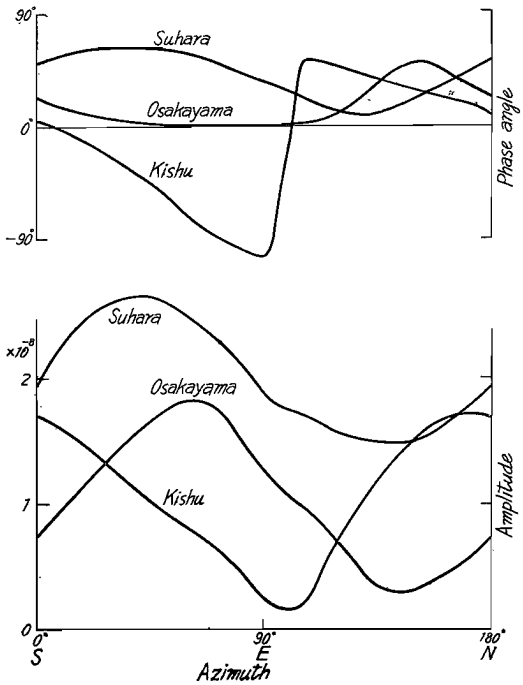



Fig. 2.(c) Relation between the amplititudes and phase angle of $M_2$-term and their azimuth obtained from the obsertations.

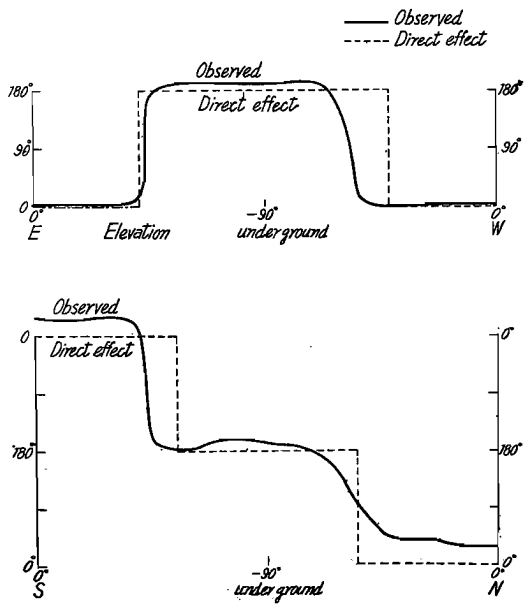



Fig. 2.(d) Relation between elevation and phase angle of linear strain of $M_2$-term at Osakayama.

$\times a^2 A_n \cos n\ (t_n+\phi)$

| | Osakayama-Kishu | Kishu-Suhara | Suhara-Osakayama |
|---|---|---|---|
| Semi-diurnal tide $n=2$ | 0.01840 | -0.00286 | -0.01548 |
| Diurnal tide $n=1$ | -0.01410 | 0.00232 | 0.01108 |

Table XIII. Differences between tide-generating potential at pairs of observatories.

| pair of observatory / component of strain | Osakayama-Kishu | | Kishu-Suhara | | Suhara-Osakayama | |
|---|---|---|---|---|---|---|
| | Amplitude in $10^{-8}$ | Phase angle ($\kappa$) | Amplitude in $10^{-8}$ | Phase angle ($\kappa$) | Amplitude in $10^{-8}$ | Phase angle ($\kappa$) |
| $e'_{\theta\theta}$ | 0.913 | 159.7° | 1.402 | 288.0° | 1.095 | 67.6° |
| $e'_{\phi\phi}$ | 1.414 | 14.9 | 2.051 | 221.2 | 1.003 | 79.8 |
| $e'_{\theta\phi}$ | 2.653 | 45.2 | 3.532 | 181.0 | 2.389 | 139.4 |

Table XIV. The Differences between observed strain components at pairs of observatories.

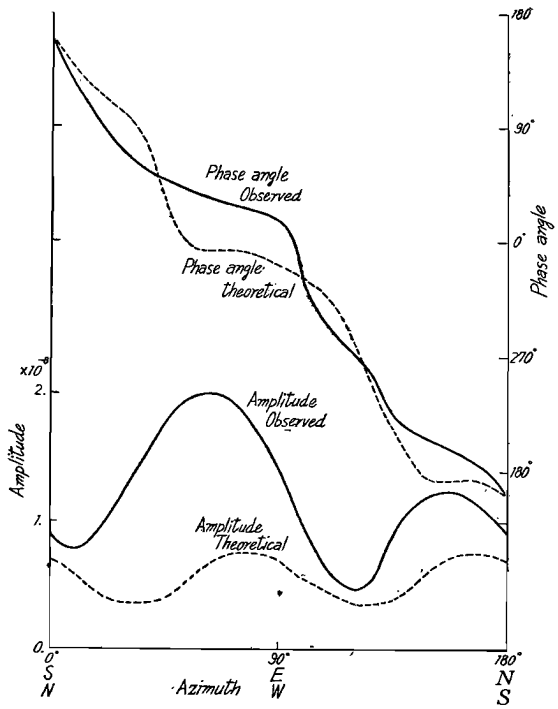



Fig. 3. The difference between the linear strains at pair of observatories.
(a) Osayama-Kishu.

1.4 % in the diurnal tide, it is reasonablly assumed in a first approximation that the direct effects of earth tide is nearly equal for three observational stations concerned. Accordingly, the direct effect of the earth tide is considered to be cancelled in the present treatment and the oceanic effect alone will be contained in the differences of these observed values. In Table XIV are shown the differences between the observed strain components at these three pairs of observatories ; Osakayama-

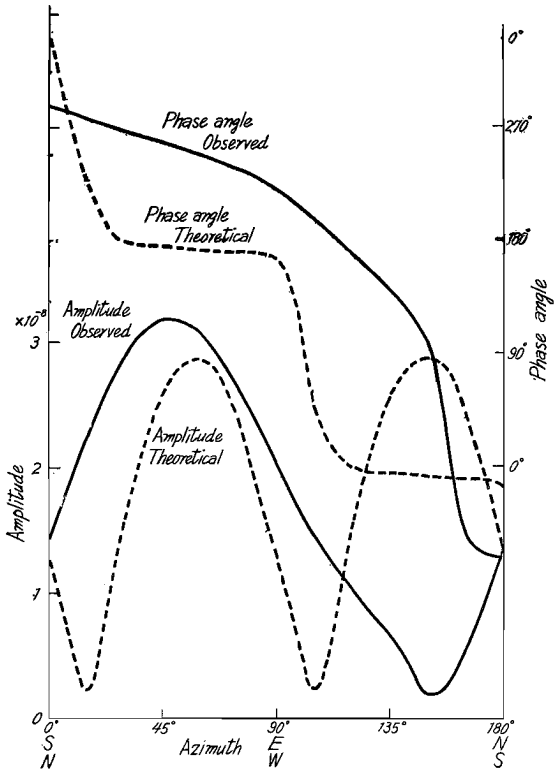



Fig. 3. The difference between the linear strain at pair of observatories.

(b) Kishu-Suhara

Kishu, Kishu-Suhara and Suhara-Osakayama.

Using these differences between components of strain at pairs of observatory, the relations of azimuth amplitude, and azimuth phase angle were calculated and shown in Table XV and are illustrated in Figs. 3. (a), (b), (c), (d), (e).

It is to be remarked that the azimuth in the table is reckoned from the south in counter-clockwise direction.

On the other hand, applying the $M_2$ co-tidal chart [32] and ($M_2$+$S_2$) co-range chart of the adjacent sea of Japan to

| Pair of observatory / Azimuth | Osakayama-Kishu | | Kishu-Suhara | | Suhara-Osakayama | |
|---|---|---|---|---|---|---|
| | Amplitude in $10^{-8}$ | Phase angle ($\kappa$) | Amplitude in $10^{-8}$ | Phase angle ($\kappa$) | Amplitude in $10^{-8}$ | Phase angle ($\kappa$) |
| 0° | 0.913 | 159.7° | 1.402 | 288.0° | 1.095 | 67.6° |
| 15 | 0.837 | 105.5 | 2.234 | 278.0 | 1.392 | 91.8 |
| 30 | 1.266 | 66.1 | 2.868 | 268.0 | 1.731 | 104.2 |
| 45 | 1.756 | 47.3 | 3.164 | 259.3 | 1.848 | 108.8 |
| 60 | 1.994 | 35.5 | 3.072 | 249.5 | 1.753 | 108.0 |
| 75 | 1.855 | 26 4 | 2.649 | 237.6 | 1.401 | 100.6 |
| 90 | 1.414 | 14.9 | 2.051 | 221.2 | 1.003 | 79.8 |
| 105 | 0.755 | 304.9 | 1.468 | 198.5 | 0.883 | 43.1 |
| 120 | 0.476 | 270.0 | 1.037 | 170.0 | 1.070 | 17.6 |
| 135 | 0.900 | 221.1 | 0.674 | 141.0 | 1.222 | 10.5 |
| 150 | 1.204 | 186.8 | 0.194 | 99.3 | 1.188 | 16.3 |
| 165 | 1.177 | 186.8 | 0.550 | 305.2 | 1.060 | 35.6 |
| 180 | 0.913 | 159.7 | 1.402 | 288.0 | 1.095 | 67.6 |

Table XV. Relation between the difference of linear strain and its azimuth obtained at pair of observatories.

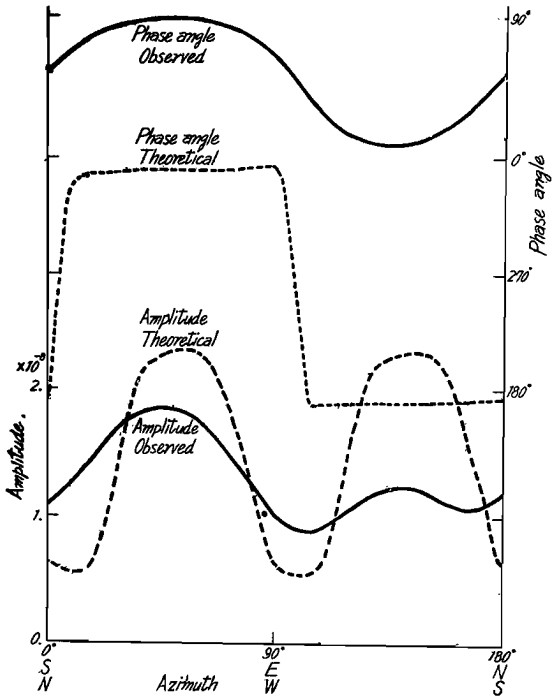



Fig. 3. The difference between the linear strains at pair of observatories.
(c) Suhara-Osakayama.

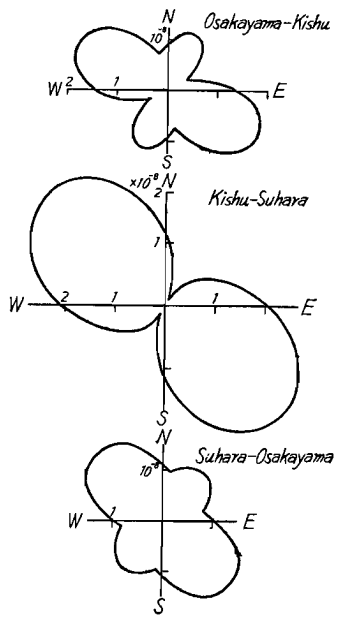



Fig. 3(d) Relation between differences of observed strain amplitude and their azimuth at pair of observatories.

each observatory, strain due to the tidal load of sea water on the surface of semi-infinite elastic body was calculated by means of above mentioned formula (15) for each observatory. The calculations were made for oceanic tide in concentric rings bounded by concentric circles, 0–100 km, 100–200 km,.....700 km–800 km around each observation point as their center and the calculated components of strain are shown in Table XVI where strains caused by load represent $M_2$-terms. Then, the differences of the components of Table XVI among three observatories : Osakayama-Kishu, Kishu-Suhara and Suhara-Osakayama, were taken. They are listed in Table XVII. The relations between the azimuth-amplitude or the azimuth-phase angle of relative extension of oceanic effect for the couples of Osakayama-Kishu, Kishu-Suhara and Suhara-Osakayama, are shown in Table XVIII and Fig. 3, in which the furthermost distance of sea area used for calculation is 800 km.

Let us compare the observational and theoretical relation between exten-

Table XVI. Oceanic-effect obtained by theoretical calculation.

$$c=\frac{2(M_2+S_2)}{M_2}\Big/\frac{\rho g}{4\pi(\lambda+\mu)},$$ where $M_2$ and $S_2$ represent the $M_2$ and $S_2$ ranges respectively

| Radius of sea region | Osakayama | | | | Kishu | | | | Suhara | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $ce'_{\theta\theta}=(-ce'_{\phi\phi})$ | | $ce'_{\theta\phi}$ | | $ce'_{\theta\theta}(=-ce'_{\phi\phi})$ | | $ce'_{\theta\phi}$ | | $ce'_{\theta\theta}(=-ce'_{\phi\phi})$ | | $ce'_{\theta\phi}$ | |
| | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ |
| 0-100 km | 7.80 | − 2.57 | −12.74 | 20.00 | 32.15 | −11.09 | −97.02 | 18.98 | − 2.46 | − 6.39 | 374.90 | −20.80 |
| 100-200 km | − 6.15 | 7.53 | − 9.18 | 30.28 | 15.53 | − 2.72 | −10.31 | 0.02 | −43.32 | 5.91 | 7.78 | 7.40 |
| 200-300 km | −13.56 | 5.16 | −17.67 | 28.42 | − 3.78 | 6.12 | −26.11 | 27.08 | −10.21 | − 1.81 | 6.45 | 13.38 |
| 300-400 km | − 7.55 | 4.32 | − 5.71 | 30.50 | 0.74 | 4.24 | 14.84 | 12.34 | − 5.35 | −10.19 | 9.97 | 13.54 |
| 400-500 km | − 4.29 | 1.59 | 2.60 | 14.21 | − 3.72 | − 2.06 | 6.34 | 10.12 | −13.30 | − 3.58 | 2.78 | 7.89 |
| 500-600 km | − 3.03 | 1.03 | 1.55 | 9.82 | − 2.64 | − 0.60 | 1.77 | 9.11 | −12.57 | 15.78 | 3.58 | 12.83 |
| 600-700 km | − 3.48 | 1.78 | 5.15 | 13.59 | − 2.22 | − 4.10 | 7.31 | 14.66 | − 4.24 | 8.97 | 8.26 | 16.21 |
| 700-800 km | − 3.40 | 1.22 | 10.24 | 5.86 | − 3.29 | − 0.07 | 10.29 | 6.97 | − 5.94 | 12.54 | 4.10 | 12.35 |

Table XVII. Difference of oceanic-effect at pair observatories obtained by theoretical calculation.

$$c=\frac{2(M_2+S_2)}{M_2}\Big/\frac{\rho g}{4\pi(\lambda+\mu)}$$

| Radius of sea region | Osakayama-Kishu | | | | Kishu-Suhara | | | | Suhara-Osakayama | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $ce'_{\theta\theta}(=-ce'_{\phi\phi})$ | | $ce'_{\theta\phi}$ | | $ce'_{\theta\theta}(=-ce'_{\phi\phi})$ | | $ce'_{\theta\phi}$ | | $ce'_{\theta\theta}(=-ce'_{\phi\phi})$ | | $ce'_{\theta\phi}$ | |
| | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ | $\times\cos 2t$ | $\times\sin 2t$ |
| 0-100 km | 24.35 | 8.52 | 84.28 | 1.02 | 34.61 | − 4.70 | −471.92 | 39.78 | −10.26 | − 3.82 | 387.64 | −40.80 |
| 100-200 km | −21.60 | 10.25 | 1.13 | 30.24 | 58.85 | − 8.63 | − 18.09 | − 7.38 | −37.17 | − 1.62 | 16.96 | −22.88 |
| 200-300 km | − 9.78 | − 0.96 | 8.44 | 1.34 | 6.34 | 7.93 | − 32.56 | 13.70 | 3.44 | − 6.97 | 24.12 | −15.04 |
| 300-400 km | − 8.29 | 0.08 | −20.55 | 8.16 | 6.09 | 14.43 | 4.87 | − 1.20 | 2.20 | −14.51 | 15.68 | − 6.96 |
| 400-500 km | − 0.57 | 0.47 | − 3.74 | 4.09 | 9.58 | 1.47 | 3.56 | 2.23 | − 9.01 | − 1.94 | 0.18 | − 6.32 |
| 500-600 km | − 0.39 | − 0.43 | − 0.22 | 0.71 | 9.93 | −16.38 | − 1.81 | − 3.72 | − 9.54 | 16.81 | 2.03 | 3.01 |
| 600-700 km | − 1.26 | 5.88 | − 2.16 | − 1.07 | 2.02 | −13.07 | − 0.95 | − 1.55 | − 0.76 | 7.19 | 3.11 | 2.62 |
| 700-800 km | − 0.11 | − 1.15 | − 0.05 | − 1.08 | 2.65 | −12.61 | 6.19 | − 5.38 | − 2.54 | 13.76 | − 6.14 | 6.46 |
| 0-800 km | −66.43 | 22.66 | 67.09 | 43.43 | 130.07 | −31.56 | −507.04 | 36.48 | −63.64 | 8.40 | 443.58 | −79.91 |

sion and its azimuth with each other. In case of Osakayama-Kishu, the amplitude of extension reaches its maximum in the azimuth of N12°E-S12°W, and N78°W-S78°E in the calculated values by means of elastic theory, and both maximum amplitudes are equal. In the observed relation a grater maximum of amplitude of the linear strain is found in the direction of N65°W-S65°E, and a smaller maximum of amplitude in the direction of N 25°E-S25°W. The ratio of the amplitude of greater maximum to that of smaller one is 1.6 and the phase angles are 32° and 198° respectively, and the phase difference is nearly 180° in the directions at right angles to each other. The relations obtained by observation are in a good agreement with theoretical ones obtained by elastic theory in the pair of Osakayama and Kishu. In case of Kishu-Suhara, the calculated

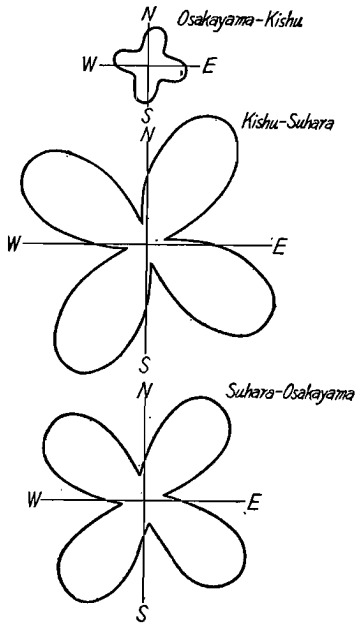



Fig. 3(e) Relation between theoretical relative strain amplitude and its azimuth at pair of observatories.

| Azimuth | Osakayama-Kishu | | Kishu-Suhara | | Suhara-Osakayama | |
|---|---|---|---|---|---|---|
| | Amplitude $\times c$ | Phase angle ($\kappa$) | Amplitude $\times c$ | Phase angle ($\kappa$) | Amplitude $\times c$ | Phase angle ($\kappa$) |
| 0° | 70 | 161.1° | 134 | 346° | 64 | 173° |
| 15 | 50 | 143.1 | 23 | 232 | 57 | 374 |
| 30 | 36 | 97.9 | 155 | 180 | 163 | 349 |
| 45 | 39 | 32.9 | 254 | 176 | 225 | 350 |
| 60 | 63 | 353.2 | 286 | 173 | 227 | 350 |
| 75 | 75 | 353.3 | 242 | 171 | 165 | 353 |
| 90 | 70 | 341.1 | 134 | 166 | 64 | 353 |
| 105 | 50 | 323.1 | 23 | 52 | 57 | 194 |
| 120 | 36 | 277.9 | 155 | 0 | 163 | 169 |
| 135 | 39 | 212.9 | 254 | 356 | 225 | 170 |
| 150 | 63 | 173.2 | 286 | 353 | 227 | 170 |
| 165 | 75 | 173.3 | 242 | 351 | 165 | 173 |
| 180 | 70 | 161.1 | 134 | 346 | 64 | 173 |

Table XVIII Theoretical difference of oceanic effect at pair observatories. Relation between linear strain and its azimuth (calculation).

$$c=\frac{2(M_2+S_2)}{M_2}\Big/\frac{\rho g}{4\pi(\lambda+\mu)}$$

amplitudes reach their maximum in the azimuth of N32°E-S32°W and N58°W-S58°E, and the amplitudes are equal at their maximum, and the difference of phase angle is 180°. In the observation, there is a greater maximum in the azimuth of N45°W-S45°E, and smaller one is not remarkable. But the derivative of phase angle of the linear strain with respect to azimuth obtained by observation is parallel to the derivative derived by means of theoretical calculation. In the case of Suhara-Osakyama, the calculated values of amplitude reaches maximum in the direction of N36°E-S36°W and in the direction of its right angle, and the maximum amplitudes in both directions are equal. In the observed values the amplitude reaches maximum in the azimuth of N 47°W-S47°E and a smaller maximum in the azimuth of its right angle and the ratio of amplitudes in the two maximum azimuths is 1.7 and the phase are 109° and 10° respectively: the amplitudes are not exactly equal in the orthogonal directions, but their phase-difference is 99°. The phase angle obtained by observations is not equal to theoretical one but the azimuth of maximum amplitude, and the derivative of phase angle with respect to azimuth are equal to each other. From these results, it is naturally assumed that the horizontal areal strain is nearly equal to 0 by tidal load of distant sea water. It may be considered that the elastic moduli of the earth is not homogeneous in the depth, but in the difference of strain at the pair of Osakayama-Kishu, the crust showed deformation of such a kind as if it is a homogeneous elastic body which had been theoretically treated by J. Boussinesq and others. The ratios of the amplitude in azimuths of maximum extensions (linear strain) obtained from observations to the corresponding amplitude obtained from theoretical calculations were computated and shown in Table XIX. The numerical values in Table XIX are in an arbitrary unit.

| Pair of observatory | Osakayama-Kishu | Kishu-Suhara | Suhara-Osakayama |
|---|---|---|---|
| Ratio | 2.08 | 2.22 | 1.60 |

Table XIX. Ratio of amplitude observed to that calculated in the azimuth of maximum linear strain.

The shortest distances from each observational station to the nearest sea are 65 km from Osakayama to the Wakasa Bay (Japan Sea), 15 km from Kishu to the Kumano Sea (the Pacific Ocean) and 60 m from Suhara to the Kii Channel. The distances between observational stations are 122 km

| | | | Osakayama-Kishu | Kishu-Suhara | Suhara-Osakayama |
|---|---|---|---|---|---|
| Azimuth of maximum extension | greater | Calculation | N78°W | N58°W | N54°W |
| | | Observation | N65°W | N49°W | N47°W |
| | smaller | Calculation | N12°W | N32°E | N36°E |
| | | Observation | N25°E | (N31°E minimum) | N39°E |
| Ratio of amplitude between greater and smaller | | Calculation | 1.0 | 1.0 | 1.0 |
| | | Observation | 1.6 | (31) | 1.7 |
| Phase angle | greater | Calculation | 351° | 173° | 350° |
| | | Observation | 32° | 256° | 190° |
| | smaller | Calculation | 171° | 353° | 170° |
| | | Observation | 198° | 94° | 10° |
| Difference of phase angle between greater and smaller | | Calculation | 180° | 180° | 180° |
| | | Observation | 166° | (162°) | 99° |
| Derivative of phase angle with respect to azimuth | | Observed and Calculated | equal | parallel | parallel |

Table XX Summary of relation between observational and theoretical results of pair-observatories.

between Osakayama and Kishu, 70 km between Kishu and Suhara, and 126 km between Suhara and Osakayama. According to Table XIX, there are not large differences among the ratios, which indicates that the differences of observed values of tidal strains are in fairly good conformity with those of theoretical values of strains. By the way, it seems that Suhara has a considerable abnormality from the structural view-point of the earth crust. The author likes to study, in future on this point in minute details.

## 6. Primary earth tidal strain

As aforesaid, horizontal areal strain due to oceanic tidal load nearly vanishes at these observational stations. Consequently, as the cubical dilatation due to distant tidal load is equal to 0 at earth's surface because there acts no body force in the case of oceanic effect, it is naturally deduced that the vertical strain is also equal to 0. In other expressions,

$$\left.\begin{aligned} &e'_{\theta\theta}+e'_{\phi\phi}=0 \\ &\Delta'=e'_{rr}+e'_{\theta\theta}+e'_{\phi\phi}=0 \\ &e'_{rr}=0 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots\cdots\cdots(20).$$

Further, by the condition of free surface of the earth, we have at $r=a$

$$e'_{r\phi}=e'_{r\theta}=0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(21).$$

Therefore, unknown components of strains of oceanic effect are reduced to only two, and they are

$$e'_{\theta\theta}(=-e'_{\phi\phi}) \quad \text{and} \quad e'_{\theta\phi}.$$

Since strain of oceanic effect satisfies the formula (20), the vertical strain, horizontal areal strain and cubical dilatation of direct effect are obtained directly by observated extensions of the ground. The vertical strain is obtained by observation of vertical extension, the horizontal areal strain is obtained as the sum of linear strains in the directions orthogonal to each other along the surface of the earth, and the cubical dilatation is obtained as the sum of vertical and horizontal areal strains.

Since at the earth's surface, by the condition of free surface

$$e_{r\phi}=e_{r\theta}=0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(22),$$

and the unknowns of components of tidal strain of direct effect are reduced to four, and they are $e_{\theta\theta}$, $e_{\phi\phi}$, $e_{\theta\phi}$ and $e_{rr}$. As shown in (5) and (6), vertical strain $e_{rr}$, horizontal areal strain $\Sigma(a)$ and cubical dilatation $\Delta(a)$ at $r=a$ are given by

$$\left.\begin{aligned} &e_{rr}(a)=\frac{1}{ag}V_1W_2 \\ &\Sigma(a)=\frac{1}{ag}V_2W_2 \\ &\Delta(a)=\frac{1}{ag}V_3W_2 \\ \text{where}\quad &V_1=\frac{g}{a}\left\{2a\frac{dF(a)}{dr}+2F(a)+a^3\frac{dG(a)}{dr}+3a^2G(a)\right\} \\ &V_2=\frac{2g}{a}\{a^2G(a)-F(a)\} \\ &V_3=ag\left\{\frac{2}{a}\frac{dF(a)}{dr}+a\frac{dG(a)}{dr}+5G(a)\right\} \end{aligned}\right\} \cdots\cdots(23).$$

Using observed values at Osakayama, Kishu and Suhara, horizontal areal strain and cubical strain were obtained, and the numerical values of $V_1$, $V_2$ and $V_3$

calculated therefrom are shown in Table XXI.

In order to obtain the same accuracy with $M_2$-component derived from analysis of one month, it necessitates the period of three months for $O_1$, six months for $S_2$ and one year for $K_1$. From this reason, the weight of component tide is assumed to be inversely proportional to their necessary periods of observation.

| Observatory | Component Tide | Mean horizontal areal strain | | Mean vertical strain | | Mean cubical dilatation | |
|---|---|---|---|---|---|---|---|
| | | Amplitude in $10^{-8}$ | Phase ($\kappa$) | Amplitude in $10^{-8}$ | Phase ($\kappa$) | Amplitude in $10^{-8}$ | Phase ($\kappa$) |
| Osakayama | $M_2$ | 2.22 | 12.3° | 0.64 | 193.6° | 1.58 | 16.5° |
| 〃 | $S_2$ | 0.79 | 13.5 | 0.42 | 163.4 | 0.42 | 16.6 |
| 〃 | $K_1$ | 1.04 | 20.1 | 0.67 | 189.8 | 0.39 | 38.0 |
| 〃 | $O_1$ | 1.55 | 32.8 | 0.61 | 231.5 | 0.98 | 21.7 |
| Kishu | $M_2$ | 1.63 | 356.9 | | | | |
| Suhara | $M_2$ | 2.78 | 42.8 | | | | |

| Observatory | Component tide | $V_2$ | Weight | $V_1$ | Weight | $V_3$ | Weight |
|---|---|---|---|---|---|---|---|
| Osakayama | $M_2$ | 0.427 | 24 | -0.246 | 60 | 0.604 | 24 |
| 〃 | $S_2$ | 0.572 | 4 | -0.615 | 4 | 0.616 | 4 |
| 〃 | $K_1$ | 0.294 | 1 | -0.371 | 2 | 0.221 | 1 |
| 〃 | $O_1$ | 0.511 | 4 | -0.402 | 4 | 0.644 | 4 |
| Kishu | $M_2$ | 0.304 | 12 | | | | |
| Suhara | $M_2$ | 0.521 | 12 | | | | |

Table XXI.

From eqs. (8), (23) and Table XXI, we have

$$h-3l=0.434\pm0.026 \quad \text{(at Osakakayama, Kishu and Suhara)} \cdots\cdots\cdots(24),$$

and from (23) and Table XXI we get

$$\left.\begin{aligned} &2a\frac{dF(a)}{dr}+2F(a)+a^3\frac{dG(a)}{dr}+3G(a)a^2=(-1.82\pm0.23)\times10^5 \\ &a^2G(a)-F(a)=(2.82\pm0.17)\times10^5 \\ &a^2\left\{\frac{2}{a}\frac{dF(a)}{dr}+a\frac{dG(a)}{dr}+5G(a)\right\}=(3.90\pm0.18)\times10^5 \end{aligned}\right\} \cdots(25).$$

From the values of $h-3l$, and $h=0.600\pm0.006$ which is obtained from the tidal tiltmetric observation combined with the observation of Chandler's period of latitude variation, we will obtain

$$l=0.055\pm0.009 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(26).$$

Using the value of $h$, we get from (7) and the second eq. of (9) the follow-

ing relations :

$$\left.\begin{aligned} &2F(a)+a^2G(a)=(3.903\pm0.039)\times10^6 \\ &\frac{dF(a)}{dr}+\frac{2}{a}F(a)+aG(a)=0 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(27).$$

From (25) and (27) we get

$$\left.\begin{aligned} &F(a)=(0.36\pm0.06)\times10^6 \\ &a^2G(a)=(3.18\pm0.18)\times10^6 \\ &a\frac{dF(a)}{dr}=-(3.90\pm0.22)\times10^6 \\ &a^3\frac{dG(a)}{dr}=-(4.29\pm0.97)\times10^6 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(28).$$

(c.g.s.)

Using the values of (28), we can easily obtain the value of $dH(a)/dr$ and $dL(a)/dr$ from (7), provided the value of $\partial g/\partial r$ is known. For example, if we assume $g=g_0+\frac{2g_0(a-r)}{a}\left(1-\frac{3}{2}\frac{\rho}{\rho_m}\right)$ where $\rho$ (2.67) and $\rho_m$ (5.53) are the density of the surface layer and the mean density of the earth respectively, then we get

$$\left.\begin{aligned} &a\frac{dH(a)}{dr}=-2.58\pm0.58 \\ &a\frac{dL(a)}{dr}=-0.69\pm0.04 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(29).$$

Let us now change independent variable from $r$ to $\xi$ by a relation

$$\xi=\frac{r}{a}$$

then, we obtain

$$\left.\begin{aligned} &F(\xi)=(0.030\pm0.005)\times\frac{1}{4\pi fa^2} \\ &G(\xi)=(0.267\pm0.015)\times\frac{1}{4\pi fa^2} \\ &\frac{dF(\xi)}{d\xi}=(-0.328\pm0.018)\times\frac{1}{4\pi fa^2} \\ &\frac{dG(\xi)}{d\xi}=(-0.360\pm0.081)\times\frac{1}{4\pi fa^2} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(30),$$

where $f$ is the gravitation constant.

These observational values almost agree with the results which was calculated theoretically by H. Takeuchi [34] from the earth model derived from the results of velocity distribution of seismic wave and density distribution in the earth's interior. His values are as follows:

$$\left.\begin{aligned} F(\xi) &= 0.044557 \times \frac{1}{4\pi f a^2} \\ G(\xi) &= 0.24159 \times \frac{1}{4\pi f a^2} \\ \frac{dF(\xi)}{d\xi} &= -0.33069 \times \frac{1}{4\pi f a^2} \\ \frac{dG(\xi)}{d\xi} &= -0.29500 \times \frac{1}{4\pi f a^2} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(31).$$

Further, using observed values of six components of extensions at Osaka-yama, the six strain components at $r=a$ of $M_2$-tide will be calculated as follows.

$$\left.\begin{aligned} e_{rr} &= 0.642 \times 10^{-8} \cos(2t - 193.6^\circ) \\ e_{\theta\theta} &= 0.957 \times 10^{-8} \cos(2t - 29.6^\circ) \\ e_{\phi\phi} &= 1.315 \times 10^{-8} \cos(2t - 5.5^\circ) \\ e_{\theta\phi} &= 1.650 \times 10^{-8} \cos(2t - 74.8^\circ) \\ e_{r\theta} &= 0.463 \times 10^{-8} \cos(2t - 226.4^\circ) \\ e_{r\phi} &= 0.539 \times 10^{-8} \cos(2t - 183.3^\circ) \\ \Sigma &= e_{\theta\theta} + e_{\phi\phi} = 2.223 \times 10^{-8} \cos(2t - 12.3^\circ) \\ \Delta &= 1.582 \times 10^{-8} \cos(2t - 16.5) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(32).$$

In Figs. 2 (a) and (c) is shown the relation between the amplitude of $M_2$-term and their azimuth and in Figs. 2 (b) and (d) the relations between the amplitude and elevation (dip) in the sections of both directions north-south and east-west. The six components of $M_2$-term at $r=a$ calculated by the values of (31) which were obtained by H. Takeuchi are reduced as follows.

$$\left.\begin{aligned} e_{rr} &= 0.539 \times 10^{-8} \cos(2t - 180^\circ) \\ e_{\theta\theta} &= 1.085 \times 10^{-8} \cos 2t \\ e_{\phi\phi} &= 0.416 \times 10^{-8} \cos 2t \\ e_{\theta\phi} &= 0.872 \times 10^{-8} \cos 2t \\ e_{r\phi} &= e_{r\theta} = 0 \\ \Sigma &= e_{\theta\theta} + e_{\phi\phi} = 1.504 \times 10^{-8} \cos 2t \\ \Delta &= 0.965 \times 10^{-8} \cos 2t \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(33).$$

(c.g.s.)

In Figs. 4 (a), (b) (c), (d) are shown the relation between amplitudes of $M_2$-term and $O_1$-term of extension and their azimuth or elevation (dip), calculated by the use of theoretical values (33). According to Fig. 2 (a), the maximum amplitude is seen in the direction of south-east to north-west. Its reason is

obvious, because there are not only the direct effect of the earth tide but also the influence of oceanic tide.

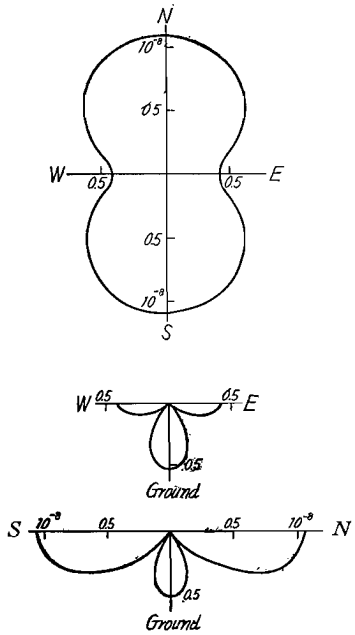



Fig. 4 (a) Relation between the amplitude of linear strain of $M_2$-tide and its azimuth or dip obtained by theoretical calculation at Osakayama.

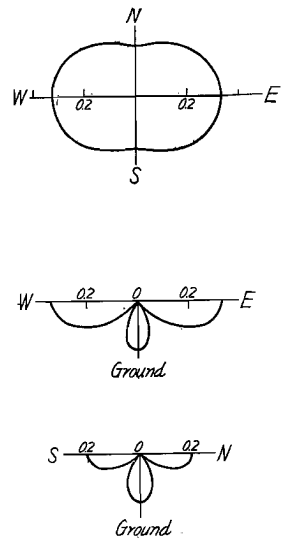



Fig. 4 (b) Relations between the amplitude of linear strain of $O_1$-tide and its azimuth or dip obtained by theoretical calculation at Osakayama.

## 7. Summary

We have observed the tidal strain in six individual directions at Osakayama Observatory, 65 km distant from the nearest sea, and in three horizontal-components at Kishu mine, 15 km distant from the Kumano Sea, and in three horizontal-components at Suhara Observatory on the sea-shore. From these observations, $M_2$-terms of the components of tidal strain at these three observation stations were analysed. And a comparison between the relative tidal strains obtained by the differences of observed values at pairs of these stations, and those obtained by means of elastic theory was made. The results obtained by the observations were in a fairly good agreement with the results of theoretical calculations at the pair of Osakayama-Kishu.

The values of vertical strain, horizontal areal strain and cubical dilatation in the primary earth tidal effect were obtained directly from observations.

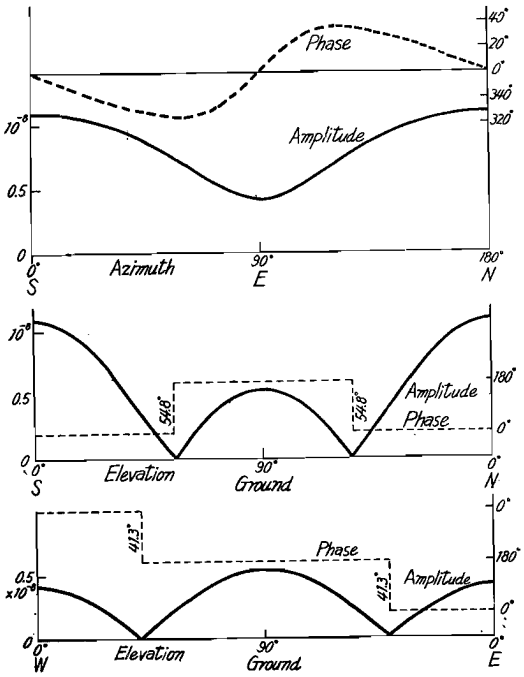



Fig. 4 (c) Theoretical relations between the azimuth or elevation (dip) and the amplitude and phase angle of linear strain of $M_2$-term at Osakayama.

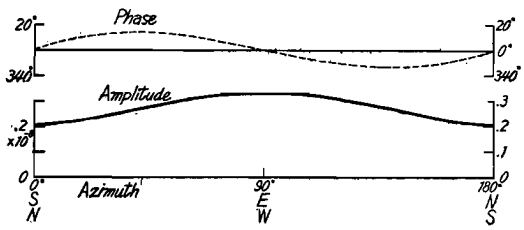
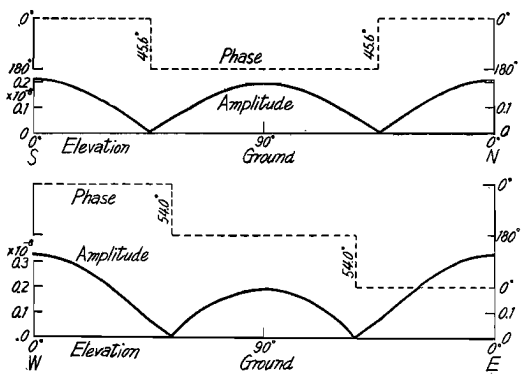



Fig. 4 (d) Theoretital relations between the azimuth or elevation (dip) and the amplitude and phase angle of linear strain of $O_1$-term at Osakayama.



From these materials the values of $l$, $F(a)$, $G(a)$, $\frac{dF(a)}{dr}$ and $\frac{dG(a)}{dr}$ at the earth's surface were ealculated. These values are in a good agreement with the theoretical values obtained by H. Takeuchi.

As we have seen, both observational and theoretical values showed good agremeent in the first approximation, but if we look into minute points, the observational results obtained at Suhara, for instance, showed a considerably large anomalous character compared with those obtained at other two stations, and this local character is supposed to be explained by the complex geological structure near Suhara. The detailed study on the peculiar nature of tidal strain at Suhara will be postponed to near future.

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## Appendix

*Another method of calculation of the influence of oceanic tide on tidal strain.*

Let us suppose that the ground consists of uniform semi-infinite elastic medium. Take a cylindrical coordinate $(r, \psi, z)$ such that $r$-axis is horizontal on the earth's surface, $z$-axis vertically inward, and the origin of coordinate at observing point. When a point load $P$, is applied at the surface $(r, \psi, 0)$, the stress components at observing point are given by

$$\left.\begin{aligned}
\widehat{rr} &= \frac{P}{2\pi}\left[(1-2\sigma)\left\{\frac{1}{r^2}-\frac{z}{r^2}(r^2+z^2)^{-\frac{1}{2}}\right\}-3r^2z(r^2+z^2)^{-\frac{5}{2}}\right] \\
\widehat{zz} &= -\frac{3P}{2\pi}z^3(r^2+z^2)^{-\frac{5}{2}} \\
\widehat{\psi\psi} &= \frac{P}{2\pi}(1-2\sigma)\left\{-\frac{1}{r^2}+\frac{z}{r^2}(r^2+z^2)^{-\frac{1}{2}}+z(r^2+z^2)^{-\frac{3}{2}}\right\} \\
\widehat{rz} &= -\frac{3P}{2\pi}rz^2(r^2+z^2)^{-\frac{5}{2}} \\
\widehat{r\psi} &= \widehat{\psi z}=0
\end{aligned}\right\}\cdots\cdots\cdots(1).$$

The strains and the stresses are connected by the equations

$$\left.\begin{aligned}
e'_{rr} &= \frac{1}{E}\left\{\widehat{rr}-\sigma(\widehat{\psi\psi}+\widehat{zz})\right\} \\
e'_{\psi\psi} &= \frac{1}{E}\left\{\widehat{\psi\psi}-\sigma(\widehat{rr}+\widehat{zz})\right\} \\
e'_{zz} &= \frac{1}{E}\left\{\widehat{zz}-\sigma(\widehat{\psi\psi}+\widehat{rr})\right\} \\
e'_{rz} &= \frac{1}{\mu}\widehat{rz} \\
e'_{z\psi} &= e'_{r\psi}=0
\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots\cdots(2),$$

where $E$, $\mu$ and $\sigma$ are Young's modulus, rigidity and Poisson's ratio respectively. From (1) and (2) the components of strain at the ground surface become as follows :

$$\left.\begin{aligned}
(e'_{rr})_{z=0} &= \frac{P}{2\pi E}\frac{1}{r^2}(1-\sigma-2\sigma^2) \\
(e'_{\psi\psi})_{z=0} &= -\frac{P}{2\pi E}\frac{1}{r^2}(1-\sigma-2\sigma^2)=-(e'_{rr})_{z=0} \\
(e'_{zz})_{z=0} &= (e'_{r\psi})_{z=0}=(e'_{z\psi})_{z=0}=(e'_{zr})_{z=0}=0
\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots\cdots(3).$$

Let cylindrical coordinate $(r, \psi, z)$ transform to rectangular coordinate $(x',$

$y'$, $z'$), and let $x'$-axis be in the direction of $\psi=0$, $y'$-axis be in the direction of $\psi=+\frac{\pi}{2}$. Then the formula (3) will be written as follows:

$$\left.\begin{aligned}(e'_{x'x'})_{z'=0}&=\frac{P}{4\pi(\lambda+\mu)}\frac{\cos 2\psi}{r^2}\\(e'_{y'y'})_{z'=0}&=-\frac{P}{4\pi(\lambda+\mu)}\frac{\cos 2\psi}{r^2}\\(e'_{x'y'})_{z'=0}&=\frac{P}{4\pi(\lambda+\mu)}\frac{2\sin 2\psi}{r^2}\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots\cdots\cdots(4),$$

which is the same with the formula (14) in page 8 of this paper.

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