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Kyoto University
CONSIDERATION ON MECHANISM OF STRUCTURAL CRACKING OF REINFORCED CONCRETE BUILDINGS DUE TO CONCRETE SHRINKAGE

BY

Y. YOKCO AND S. TSUNODA

KYOTO UNIVERSITY, KYOTO, JAPAN
Consideration on Mechanism of Structural Cracking of Reinforced Concrete Buildings due to Concrete Shrinkage

By

Y. Yokoo and S. Tsunoda

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Consideration on Mechanism of Structural Cracking of Reinforced Concrete Buildings due to Concrete Shrinkage

By

Y. Yokoo* and S. Tsunoda**

Preface

As the causes of crack production in reinforced concrete buildings, following factors can be distinguished:
1. Shrinkage of concrete,
2. Thermal deformation,
3. Differential settlement.

Cracks due to the above seem to be scattered without any rule in various portions of building. Careful investigation, yet, shows that there are some rules for their distribution. Accordingly the existence of patterns of crack in reinforced concrete buildings will be explained in following paragraph "A" with intention to give brief explanations for mechanism of their development.

Among the three factors above, thermal deformation and differential settlement produce tension or bending moment in the body of building, resulting in cracks. The process of crack production is considered to be the same, as in case of the members extended or bent due to external loads. In relation with this case, much have already been discussed by R. Saliger and others. Consequently paragraph "B", the procedure of crack production due to concrete shrinkage will be treated. Being almost every part of reinforced concrete buildings more or less prevented from free contraction, a fundamental consideration on the cracking of the reinforced concrete member will be made on the assumption that the both ends are fixed.

* Professor, Disaster Prevention Research Institute, Kyoto University, Japan.
** Chief, Architectural Section, Kanto Bureau, Ministry of Construction, Japan.
(A) Crack Patterns in Reinforced Concrete Buildings

1. Crack patterns

As the result of series of crack investigations conducted by the authors jointly on more than 30 reinforced concrete barracks of Japanese Guard Forces and other buildings, it happened to be found that crack patterns exist as explained in the following sub-paragraphs.

a) Cracks under windows (Fig. 1)

Cracks can be observed in almost all the cases under windows. In this case the lower the windows are, the more conspicuous are the cracks. The location and tendency of the cracks produced in buildings in long size are as explained later—namely “V” Pattern (due to contraction) or “Reversed V” Pattern (due to thermal change). Among buildings constructed on poor earth, a couple of examples can be pointed out, showing a great number of cracks under windows.

![Fig. 1 Example of cracks under window.]

b) Transverse Cracks (Fig. 2)

Cracks of this type can be observed on the floor transversally to the building as shown in Fig. 2. Most of them run through the center of

![Fig. 2 Example of the transverse cracks.](image1)

![Fig. 3 Example of the corner cracks.](image2)
the slabs, and extend to the adjacent beams, though not so much conspicuous as in the slab. It is noteworthy that the cracks can be observed from the compression side of slab in many cases.

c) Corner Cracks

Cracks can be observed on four corners of every floor of building with very few exceptions, in the diagonal directions as shown in Fig. 3.

d) "V"-type Cracks (Fig. 4)

On the exterior walls in the direction of longitudinal axis of long buildings, the cracks of this type can be noticed. The lines of cracks of this pattern at the ends or its neighbouring tend to show "V" as shown in Fig. 4 and in this case the lower the location is, the more this tendency becomes distinguished. In case of windows installed in the wall, each crack line starts from the internal corner of the window sill to the below and from the external corner of the window lintel to the above respectively as shown in Fig. 4. The cracks on the center or its vicinity show almost vertical lines, starting from the window sill.

![Fig. 4 Example of "V" pattern of cracks.](image)

e) "Reversed V"-type cracks (Fig. 5)

This pattern of cracks can also be observed on the wall of long buildings and the direction of cracks is in quite reverse as compared with that of in the preceding sub-paragraph (d). Although contraction of concrete is considered as the cause of "V" pattern of cracks, those of this pattern are deemed as follows.

1. Differential settlement of building due to consolidation of the ground.
2. Increase of temperature due to the solar heat over the roof. (This pattern of cracks is conspicuous on the top)

There are interesting examples that in some long and low buildings X-type cracks can be observed in walls near the end span, showing "V" type in winter and "Reversed V" type in summer.
2. Brief consideration on the crack patterns

a) Cracks under windows

Inasmuch as the foundations of columns at both sides of window are fixed firmly by the ground, horizontal tensile stress is produced, resulting in this type of cracks. The portion of wall under window sill, however, mostly is constructed in the ground and considered usually contracted little because of moist condition of the ground. For this reason, therefore, cracks under window sills become more conspicuous.

Cracks of the same pattern under window sills in buildings constructed on poor ground may be more widened due to upward bending moment produced with excessive settlement of columns.

b) Transverse Cracks

A long building is subjected to a fairly large tensile stress in the direction of long axis due to contraction and in this case the nearer the center, the stress is larger. In case of windows installed at center between columns, in other words, in case of the smaller area of cross-section of building the tensile stress becomes more striking. The center of slab, besides, is subjected to larger bending moments which presumably help cracking through the center portion of slabs.

c) Corner Cracks

The rigidity of exterior wall in the direction of circumference is great. On the contrary that of in the direction vertical to it is very less, thus resulting in the deformation as shown in Fig. 6, and producing tensile stresses in corner in the diagonal direction.

d) “V quo”-type Cracks
The contraction of lower portion in exterior wall parallel to the long axis of building is little due to the restraint by ground and moist condition in ground as explained in the preceding paragraphs. Upper portions, however, being contracted comparatively freely. Consequently in wall at the end span, the striking larger tensile stresses are produced in “V” destination as shown in Fig. 7. The tendency of “V” type of cracks becomes less in the wall aparting from the end. e) “Reversed V”-type Cracks

In the case that the roof of a building is expanded due to high temperature in summer, the diagonal tension near the ends of wall acts as shown in Fig. 8 in the reverse direction as compared with that of in the preceding paragraph. Therefore, cracks are also reversed in the direction and the most outstanding at the top story. Due to differential settlement of building on soft clay ground, the center of building sinks more than the ends as a simply supported beam as shown in Fig. 9. This diagonal tension causes the reversed tendency of cracks as compared with that of in the preceding sub-paragraph.

(B) A Fundamental Consideration on the Cracking of the Reinforced Concrete Member

a) The free contraction of a reinforced concrete member

With the reinforcement, the contraction of concrete is reduced to some
extent. When the free shrinkage of concrete takes value \( \gamma \), the stresses and strains of the reinforced concrete member are given by the following equations.

\[
\sigma_e = \frac{n\mu}{1+n\mu}E_c\gamma \\
\sigma_s = -\frac{n}{1+n\mu}E_c\gamma \\
\varepsilon_s = \frac{\sigma_s}{E_s}
\]

Assuming \( \gamma = 5 \times 10^{-4} \), \( E_c = 2 \times 10^6 \) kg/cm\(^2\) and \( n = 10 \) in Eqs. (1), (2) and (3), the stresses and the contraction per 10 m are shown in Table 1. From Table 1 it will be seen that the reinforcement is effective against the shrinkage, but to the extent that the contraction of the member with lower steel ratio, i.e. slab or wall is only reduced by several percents from the free shrinkage of concrete and that with higher steel ratio \( 1\% \sim 3\% \), i.e. column or beam, by \( 10\% \sim 20\% \). Consequently the free contraction of usual reinforced concrete members is more than 80\% of that of concrete. As in a building this contraction is more or less restricted, internal forces are provoked and aggravate the cracking.

<table>
<thead>
<tr>
<th>( \mu % )</th>
<th>( \sigma_e ) kg/cm(^2)</th>
<th>( \sigma_s ) kg/cm(^2)</th>
<th>contraction per 10m mm</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>—</td>
<td>5.0</td>
</tr>
<tr>
<td>1</td>
<td>9.1</td>
<td>-910</td>
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</tr>
<tr>
<td>2</td>
<td>16.7</td>
<td>-834</td>
<td>4.2</td>
</tr>
<tr>
<td>3</td>
<td>23.0</td>
<td>-770</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>28.8</td>
<td>-715</td>
<td>3.6</td>
</tr>
</tbody>
</table>

b) The cracking of a reinforced concrete member with fixed ends due to the concrete shrinkage

As a most fundamental case, a member with uniform section, uniform reinforcement, both ends fixed and without load, will be studied here.

In the early state before cracking, the member is not subject to any deformation, though the free shrinkage of concrete \( \gamma \) has been increased, because the both ends are fixed, and the stress of steel stay zero. However, the concrete is stressed to \( \sigma_e = E_c\gamma \) (tension) and the axial force has the value \( N = A \sigma_e \) (tension). When the concrete stress and the concrete strain due to the stress
reach the maximum values, the first crack comes out at a certain section. At the instant of the cracking the axial tension $N$ at the crack section, which has been undertaken by the concrete, is shifted to the steel, and the steel is tensioned remarkably, though $N$ is relaxed to some extent, as the extensional rigidity around the crack section is decreased. If the relaxation is neglected, the steel stress at the crack section just after the first cracking is given by

$$\sigma_{s \text{ max}} = \frac{\sigma_{s}}{\mu}$$  \hspace{1cm} (4)

This formula is valid only for an ideal member with infinite length. The steel stress is less than the value given by Eq.(4) in members with practical length.

At the section near the crack, the bond between concrete and steel is overcome and the steel slips from the concrete. The author calls this part of the member “Slip Zone”, and the other part, where the bond is still kept, “Bond Zone” The slip zone has lower extentional rigidity than the bond zone, so that it works in a sense like an expansion joint, by retarding the in-
creasing rate of $N$. However, after a certain time the concrete stress at the bond zone may reach again its maximum and the second crack may appear at another section. Until the value $\tau$ reaches the final, cycles as such will be repeated, bringing in cracks one by one. In some cases with small steel ratio, the steel stress at cracks may reach the yielding point. Then the cracks will work as expansion joints for further shrinkage and will be opened widely.

In order to treat the member in such a state, the authors introduce here the following assumptions.

1) The stress strain curve of concrete is given by the ideal plastic curve shown in Fig. 10 (a). For unloading from the plastic range and reloading after that, the curve is parallel to the elastic line for the first loading. See Fig. 10 (b) and (c).

2) The bond stress distribution along the bar in the slip zone is approximated by the sinusoidal curve given by R. Saliger, for the reinforced tensile member with many cracks and without bond zone. (see Fig. 11)

3) The stresses of a slip zone are continuous to those of the adjacent bond zones.

From the assumptions 2) and 3) the stresses and strains distribution along the length can be drawn as in Fig. 11.

In the slip zone, from the assumption 2) the bond stress

$$\tau = \tau_1 \sin \frac{2\pi x}{\lambda}$$

Considering the assumption 3) and the balance of stresses, the concrete stress and the steel stress

$$\sigma_c = \frac{\sigma_e^*}{2} \left(1 - \cos \frac{2\pi x}{\lambda}\right),$$

$$\sigma_s = (\sigma_0 - \sigma_c)/\mu,$$

and the length of the slip zone

$$\lambda = \frac{\pi A_0 \sigma_e^*}{4\mu \tau_1}$$

Substituting $u = \Sigma d$ and $A_e = \frac{\Sigma d^2}{4\mu}$ in Eq. 8,

$$\lambda = \frac{\pi d \sigma_e^*}{4\mu \tau_1},$$

and with $\tau_1 = 2\sigma_e$ assumed,
\[ \lambda = 0.4 \frac{d}{\mu} \frac{\sigma_v^*}{\sigma_s} \]  \tag{10} 

From Eqs. (6) and (7) the steel strain

\[ \varepsilon_s = \frac{1}{\mu E_s} \left( \sigma_0 - \frac{\sigma_v^*}{2} \left(1 - \cos \frac{2\pi x}{\lambda}\right) \right) \]  \tag{11} 

Therefore the total elongation of slip zones of the member

\[ \Delta = \frac{m\lambda}{\mu E_s} \left( \sigma_0 - \frac{\sigma_v^*}{2} \right) \]  \tag{12} 

where \( m \) = number of cracks.

In the bond zone, considering the assumption 1) the concrete strain is expressed by

\[ \varepsilon_c = -\delta + \frac{\sigma_v^*}{E_c} \]  \tag{13} 

where

\[ \delta = -\gamma + f \]  \tag{14} 

the steel strain

\[ \varepsilon_s = \frac{\sigma_v^*}{E_s} \]  \tag{15} 

Equating the both strains,

\[ \frac{\sigma_v^*}{E_s} = -\delta + \frac{\sigma_v^*}{E_c} \]  \tag{16} 

From the balance of the stresses and the axial force,

\[ \sigma_0 = \sigma_v^* + \mu \sigma_s^* \]  \tag{17} 

From Eqs. (16) and (17)

\[ \sigma_v^* = \frac{1}{1 + n\mu} (\sigma_0 + n\mu E_s \delta) \]  \tag{18} 

\[ \sigma_s^* = \frac{n}{1 + n\mu} (\sigma_0 - E_c \delta) \]  \tag{19} 

Then the total elongation of the bond zones of the member

\[ \Delta^* = \frac{1}{(1 + n\mu)E_c} (L - m\lambda)(\sigma_0 - E_c \delta) \]  \tag{20} 

Now, the condition of restriction at the fixed ends can be represented by

\[ \Delta + \Delta^* = 0 \]  \tag{21} 

Substituting Eqs. (12) and (20) in Eq. (21), and eliminating \( \lambda \) by Eq. (9),
the following equation for $\sigma_0$ is obtained.

$$
\sigma_0 + n\mu (2E_0\delta + \beta n\mu (1+n\mu)\sigma_s)\sigma_s + n^2\mu^2 E_0\delta (E_0\delta - \beta (1+n\mu)\sigma_s) = 0
$$

---------- (22)

From Eq. (22),

$$
\sigma_0 = \left[ -n\mu \left\{ \bar{S} + \frac{n\mu (1+n\mu)}{2}\beta \right\} + 2n\mu (1+n\mu) \sqrt{\beta \left( \bar{S} + \frac{n^2\mu^2\delta}{4} \right)} \right] \sigma_s,
$$

---------- (23)

where

$$
\bar{S} = E_0\delta/\sigma_s,
$$

---------- (24)

and

$$
\beta = \frac{8\tau_1 L}{mn\pi d\sigma_s}
$$

---------- (25)

Assuming $\tau_1 = 2\sigma_s$ and $n = 10$,

$$
\beta = 0.5 \frac{L}{md}
$$

---------- (26)

When a value of $\delta$ is given, from Eq. (23) many values of $\sigma_0$ are calculated for various $m$ values. Among them $\sigma_0$ values over $\sigma_s$ are meaningless. If cracks can not occur in more than one at a time, the number of cracks must be then the one which gives $\sigma_0$ the nearest to $\sigma_s$. The $m$ value as such will be decided by trial from Eq. (23). When $\sigma_0$ is obtained, the stresses in the slip and bond zones can be calculated by Eqs. (5), (6), (7), (18) and (19).

In accordance with increasing $\gamma$ consequently $\delta$, the number of cracks $m$, the length of a slip zone $\lambda$ increases and the total length of bond zones $\lambda^*$ decreases. Therefore the bond zones may vanish before the final state, when a great number of cracks is expected with higher steel ratio.

In this case the stresses are given by

$$
\sigma_0 = \frac{\sigma_s}{2} \left( 1 - \cos \frac{2\pi x}{\lambda} \right),
$$

---------- (27)

$$
\sigma_s = \frac{\sigma_0}{2} \cos \frac{2\pi x}{\lambda},
$$

---------- (28)

$$
\sigma_0 = \sigma_s/2,
$$

---------- (29)

and the length of a slip zone

$$
\lambda_1 = \frac{\pi d\sigma_s}{4\mu\tau_1} = 0.4 \frac{d}{\mu}
$$

---------- (30)
Therefore the maximum number of cracks

\[ \text{max. } m = \frac{L}{\lambda_1} \]  \hspace{1cm} \text{(31)}

Also, with lower steel ratio the steel stress at cracks may be expected to reach the yielding point \( \sigma_y \) before the final state.

In this case, \( \sigma_0 = \sigma_y \mu \),  \hspace{1cm} \text{(32)}

therefore the stresses in the bond zones from Eqs. (18) and (19) are

\[ \sigma_e = \frac{1}{1+n\mu} (\sigma_y \mu + n\mu E_0 \delta) \] \hspace{1cm} \text{(33)}

\[ \sigma_\delta = \frac{n}{1+n\mu} (\sigma_y \mu - E_0 \delta) \] \hspace{1cm} \text{(34)}

The bond zones in this case are in a state of the free contraction under the constant tension

\[ N = \sigma_y \mu A_e. \]

Numerical Examples: the following values are assumed here for concrete.

\[ E_c = 2 \times 10^6 \text{ kg/cm}^2, \sigma_e = 20 \text{ kg/cm}^2, n = 10 \]

and \( f = \varepsilon_1 = \) the maximum elastic strain.

Then \( f = 1 \times 10^{-4} \) and \( S = E_c \delta / \sigma_e = 10^{-4} \gamma - 1 \) \hspace{1cm} \text{(a)}

Consequently at the first cracking, for \( \gamma = \varepsilon_1 + \gamma = 2 \times 10^{-4} \)

\[ S = 1. \] \hspace{1cm} \text{(b)}

At the final state, where \( \gamma = 5 \times 10^{-4} \) is assumed,

\[ S = 4. \] \hspace{1cm} \text{(c)}

Numerical calculation is performed for the following two cases:

\[ S = 1, \beta (m = 1) = 500 \text{ and } 200 ; \]

\[ S = 4, \beta (m = 1) = 500. \]

where \( \beta (m = 1) = 500 \) and \( 200 \), correspond respectively to

\[ L = 10 \text{m}, \ d = 1 \text{cm} ; \]

and \( L = 4 \text{m}, \ d = 1 \text{cm} \).
The results are shown in Table 2. The stresses are expressed by the ratios to $\sigma_t$, namely

$$S_0 = \sigma_0/\sigma_t, \quad S_c = \sigma_c/\sigma_t \text{ and } S_s = \sigma_s/\sigma_t,$$

and the yielding of steel is neglected. If $\sigma_Y = 2,400$ kg/cm$^2$ consequently $\sigma_Y/\sigma_t = 120$ assumed, according to the Table 2, it can be said that the steel will yield at the first cracking with $\mu$ less than 0.6 for $\beta = 500$ and with $\mu$ less than 0.1 for $\beta = 200$.

Table 2

i) $\beta(m=1)=500$, $S=1$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$S_0$</th>
<th>$S_c$</th>
<th>$S_s$</th>
<th>$\lambda/L$</th>
<th>$m$</th>
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<tr>
<td>0.1</td>
<td>0.191</td>
<td>0.199</td>
<td>191</td>
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<td>0.977</td>
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ii) $\beta(m=1)=500$, $S=4$

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$S_0$</th>
<th>$S_c$</th>
<th>$S_s$</th>
<th>$\lambda/L$</th>
<th>$m$</th>
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<tbody>
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<tr>
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<td>0.500</td>
<td>1.000</td>
<td>17</td>
<td>0.013</td>
<td>75*</td>
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</table>

The line with * are calculated by Eqs. (27)~(30).
c) As the result of the foregoing consideration the following conclusions can be obtained.

1) Due to contraction of concrete, the stress of steel bar may exceed the yielding point in some cases, especially in case of less reinforcement ratio even if not loaded. The stress of steel bar advances with the additional tension or bending moment due to the external loads, or thermal change or differential settlement.

2) The factors which promote the stress of steel bar at the location cracked are as follows:
   (i) tensile intensity of concrete,
   (ii) length of member.

3) As the effective factor preventing from the increase of stress of steel bar at the location cracked, the reinforcement ratio is considered.

In the foregoing consideration, creep of concrete has been neglected. It has also been assumed that rigidity of "slip zone" is remained the same at the instant of another crack-production. This assumption may, however, not satisfy strictly the state of the rigidity of "slip-zone", which actually may show more rigidity when unloaded. Consequently above consideration may be available for qualitative understanding of crack problems and further betterment in relation with the assumptions is expected in future study.
(C) Notations

\(N\) = axial force
\(\sigma_c\) = concrete stress in the slip zone
\(\sigma_s\) = steel stress in the slip zone
\(\sigma_c^*\) = concrete stress in the bond zone
\(\sigma_s^*\) = steel stress in the bond zone
\(\sigma_0 = \frac{N}{A_c}\)
\(\bar{\sigma} = E_c \bar{\delta}\)
\(\tau_b\) = bond stress in the slip zone
\(\tau_s\) = bond strength of concrete
\(\sigma_c\) = tensile strength of concrete
\(\sigma_y\) = yield point of steel
\(\varepsilon_c\) = concrete strain
\(\varepsilon_s\) = steel strain
\(\Delta\) = total elongation of slip zones
\(\Delta^*\) = total elongation of bond zones
\(E_c\) = Young's modulus of concrete
\(E_s\) = Young's modulus of steel
\(n = E_s/E_c\)
\(f\) = max. flow of concrete
\(\dot{\varepsilon}_c\) = max. elastic strain of concrete
\(\gamma\) = free shrinkage of concrete
\(\delta = \gamma - f\)
\(A_c\) = cross-sectional area of concrete
\(A_s\) = cross-sectional area of steel bars
\(\mu = A_s/A_c\) steel ratio
\(S_0 = \sigma_0/\sigma_c\)
\(S_c = \sigma_c^*/\sigma_c\)
\(S_s = \sigma_s\max/\sigma_s\)
\(S = \bar{\sigma}/\sigma_s\)
\(\beta = 8r_1 L/\pi d s_l\)
\(L\) = the length of member between restricted ends
\(d\) = diameter of bar
\(u\) = perimeter of bar
Reference

Y. Yokoo and S. Tsunoda: Considerations on the Mechanism of Reinforced Concrete Buildings due to the Concrete Shrinkage; Bulletin of the Disaster Prevention Research Institute, Kyoto University, Memorial Issue of the Fifth Anniversary, Nov. 1956.

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