DISASTER PREVENTION RESEARCH INSTITUTE

BULLETIN No. 19 MARCH, 1958

ON THE APPLICATION OF THE UNIT HYDROGRAPH METHOD TO RUNOFF ANALYSIS FOR RIVERS IN JAPAN

BY

TOJIRO ISHIHARA

AND

AKIHARU KANAMARU

KYOTO UNIVERSITY, KYOTO, JAPAN

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Contents

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Introduction

The flood protection projects in Japan have been advanced as a first considration of hydraulic engineering for many years, though many fundamental problems are still remained unsolved. The possible way to reach the clear formulation contributive to the further development of engineering works in such projects is to make basic and applied researches of the physics of stream behaviours, and especially, of greatest importance, but most difficult, is to establish the essential features of runoff distribution by rainfall in the light of past experience and advanced knowledge in modern hydraulics and hydrology, while the practical approaches of runoff distribution like the unit hydrograph method, distribution graph method, and so on, have been progressed.

In the Hydraulics Laboratory, Kyoto University, a variety of basic and applied research programs on runoff distribution by rainfall and stream behaviours by several approaches of scientific and engineering methods; unit hydrograph method, computation of runoff by the characteristic curves, and flood routing by electronic analog computer ; has been promoted for recent several years.

In this paper, some interesting results of hydrological behaviours of runoff distribution based on the concept of unit hydrograph method are preseneted.

Part 1

On the Characterstics of Unit Hydrographs for Rivers in Japan

The unit hydrograh method presented by L. K. Sherman in 1932 have been widely used as a highly effective and simple tool for the engineering practice of runoff estimation in Japan as well as in U.S.A. However, the applicability of the original procedure developed in U.S.A. to hydrological projects of Japanese valleys of which channel and basin characteristics are different from those in U.S.A. has to be considered, to obtain the real success of this procedure. As the characteristics of the river are of steep slope and those of valleys, subjected to the hydrological research program, are small in area, and therefore, areal and hourly distribution of rainfall is expected to be more complicated, so the non-linear effects of runoff behaviours will become appreciable, resulted in the possibility of limitation of application to hydrological projects in Japan.

In order to make the basic research of hydrological characteristics of runoff behaviours based on the unit hydrograph method in Japanese valleys, the detailed field observations have been continued for these seven years in the model catchment basin of the Yura River valley.

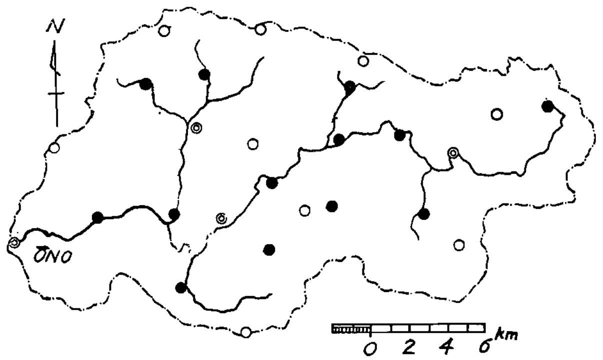
As the conclusive results, it is seen that individual unit hydrographs calculated by the ususal method[1] developed in U.S.A. for individual rainfalls vary together in their shape and ordinate. It means that, therefore, the character of unit hydrograph applicable in hydrological projects in Japanese valleys is different from that in U.S.A. Hence, the accent of research program of runoff behaviours must be directed to the clear formulation of non-linear effects in stream beheviours and boundaries characteristics and the establishment of devised method contributive to future advance of engineering practice in hydrological programs in Japan.

In this part, the characteristics of runoff distribution in the model basin of the Yura River valley calculated by the usual hydrological method and resulted unit hydrographs are indicated.

(1) General view of model basin

For practical convenience of couducting the hydrological observations for individual storms, the upstream part of valley above Ōno, Kyoto Prefecture, was considered as the model basin of research program. The length of main stream is 55 km, the basin has the catchment area of 350.3 sq.km, which involves hills and fields from 300 to 800 m in height above sea level, and most parts are covered with broad-leaf trees. The secton of upstream in the model basin is V-shaped, while the lower part, being of comparatively gentle midstream type, has some cultivated land along the stream. The geological feature of the valley is mainly composed of the Chichibu paleozoic strata.

In this model basin, hydrological observations of rainfall and resulted runoff have been carried out by use of various types of raingage located at 26 points,



◎ Self-recording raingage station ● Daily rainfall gaging station
○ Cumulative rainfall gaging station

Fig. 1 Location of raingage stations in the model basin of the Yura River valley

and a self-recoding water stage gage, located at the basin outlet, Ōno, since 1951. For estimation of discharge at Ōno, the water stage-discharge curve provided by the direct velocity measurement was used. Fig. 1 indicates the model basin of the Yura River valley and location of raingages.

(2) Separation of runoff into flow components

Stream flows by a storm are generally composed of four flow components; surface flow, secondary flow, ground water flow, and direct rainfall into channels. As each flow component indicates different characters in hydrological behaviours from others, so seperation of runoff into flow components is considered for practical convenience of runoff analysis. Although B. S. Barnes[2], R. K. Linsley and W. C. Ackermann[3] proposed some methods of separation into flow components, the correct separation of runoff seems impossible.

The hydrological characteristics of secondaly flow, readily influenced by rainfall intensity and discharged into channels at an earlier stage of flood, seems to be nearly equal to those of surface flow, and therefore, the concept of "direct flow" involved three flow components of surface and secondary flows and direct rainfall into cannels was assumed for engineering practice in hydrological analysis, as done by many investigators.

On the other hand, as the motion of ground water flow is controlled by the hydraulic gradient of ground water and inflow, influenced by infiltration rate of soils which is independent of the rainfall intensity, so the ground water flow is considered independent of rainfall intensity. The ground water flow, subtracted direct flow from total runoff, is called "base flow" The practical approximation of separation of runoff into two components of flows is as follows. It has been widely recognized by many investigators that the recession curve of base flow is represented in a form of exponental curve. In Fig. 2, A–A″ indicates the base flow re-resulted from the antecedent storms. In the hydrograph, the area above A–A″ is total runoff resulted from the present rainfall. This area must be divided into two components of direct and base flows for the hydrological analysis of ruuoff. As aforementioned, however, no definite method of separation exists, and thus, the elementary method by dividing the area into two parts through a horizontal straight line A–A′ from the point A located at the beginning of increment of runoff will be considered as a first approximation. The merits of this method are simple and useful for practical application to runoff analysis with few personal error, while the demerits are that it has no theoretical basis of hydrology and furthermore, the point A varies for individual storms. In a flood duration, as the variaton of base flow seems negligible compared with that of direct flow, and the ordinate at A is generally less than 100 cu.m/sec, so any method of separation of base flow from runoff has less practical influence for the estimation of peak discharge.

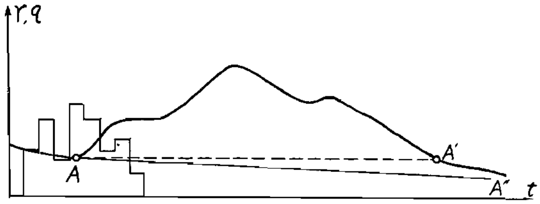


Fig. 2 Separation of runoff hydrograph into to components of flow

(3) Rainfall and its separation

The depth and distribution of precipitation are also most fundamental for the hydrological analysis of research, so that the favourable condition for hydrological observations is obtained by uniform areal distribution of rainfall gaging stations. Since most hydrological problems require the amount of rainfall over a valley, the following two procedures are commonly used to convert the actual mesurements to the estimation of areal averages. The isohyetal method is expected most accurate for computing average precipitation, but its procedure is complicated for obtaining hourly isohyetal maps and resulted maps of distribution will be influenced by persons. As the hydrological observations in the model basin indicated the areal distribution of precipitation was rather uniform, so the Thiessen method was used for estimation of average rainfall depth. Thus, the hourly depth of rainfall over the model basin for calculation of unit hydrographs is expressed in a form of

$$\bar{r}_t=\left[\left(\sum_{t=1}^{n} r_{it}/R_i\right)/n\right]\bar{R}, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \ (1)$$

where $\bar{r}_t$ is the hourly average rainfall, r_{it} the hourly rainfall at self-recording raingage station i (i is the number of such stations.), R_i the total rainfall at i station, and $\overline{R}$ the average rainfall in the model basin. Next procedure for calculating unit hydrographs is to separate the depth of rainfall into two parts of "effective rainfall" contributive to direct flow and "lost rainfall" to base flow.

The hourly variation of lost rainfall is mostly controlled by the rate of infiltration of rainwater through soils, R. E. Horton[4] presented an empirical formula of infiltration rate with the aid of his observations. It is expressed as follows.

$$f=f_c+(f_0-f_c)e^{k't}, \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \ (2)$$

where f : lost rainfall per hour at a time t,

f_0 : 〃 at the beginning time of rainfall,

f_c : 〃 at the final period,

k' : a constant.

The estimation of lost rainfall and resulted effective rainfall contributive to direct flows will be obtained with the use of this formula. For the estimation of values of f_0, f_c and k', the following procedure is available.

Total rainfall R and resulted total volume of direct flow Q are obtained from pluviograph for individual storms, so that the total loss of rainfall F is expressed as $R-Q=F$, and furthermore, F is indicated as follows by the use of empirical formula of Horton.

$$F=\int_0^t f\,dt=f_c t+\frac{f_0-f_c}{k'}(1-e^{-k't}). \cdots\cdots\cdots\cdots\cdots\cdots (3)$$

Therefore, values of f_0, f_c and k' are calculated with the aid of rainfall duration t and lost rainfall F measured by field observation for storms. As the value of f_0 is influenced by the reserved-water in soils as ground water, so Fig. 3 indicates the relation of lost rainfall to rainfall duration as a parametric expression of T, where T is the time from the end of antecedent rainfall to the beginning of the present rainfall. The tangent of a curve indicates the hourly rate of lost rainfall, f.

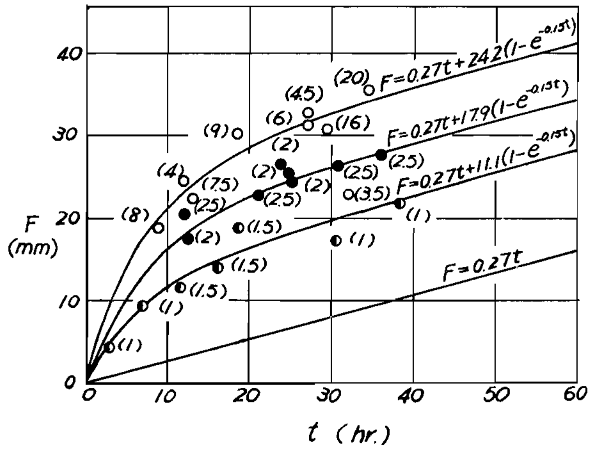


Fig. 3 Relation between F and t as a parametric expression of T(day) indicated in the bracket

If the values of k' and f_c are assumed constant, the empirical formula of Horton is linear, for given value of f_0, and therefore, it is seen a family of curves of rate of lost rainfall characterised by individual initial conditions is represented by a standard infiltration curve, which is, in the model basin of the Yura River valley, of a type

$$f = 0.27 + (4.30 - 0.27)e^{-0.15t}. \quad \cdots\cdots\cdots\cdots\cdots\cdots (4)$$

Fig. 4 indicates the relation of rate of lost rainfall to rainfall duration and Fig. 5 describes the recovery of rate of lost rainfall as a function of T. The

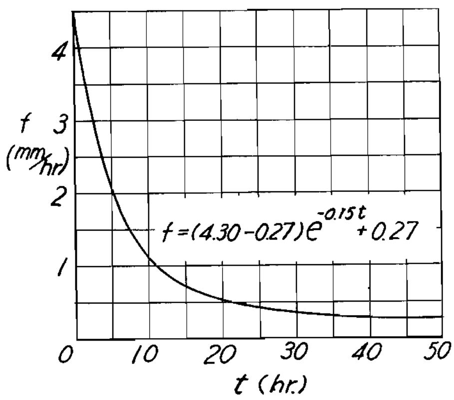


Fig. 4 Standard f-curve

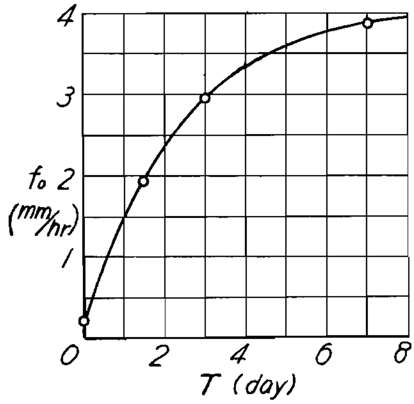


Fig. 5 Relation between f_0 and T

separation of any depth of rainfall, therefore, will be possible by the use of results in Figs. 4 and 5.

The present procedure must be applied to the case for which the hourly rainfall depth is more than that of rate of lost rainfall. For the case of non-excess rainfall, the devised method as H. L. Cock[5] proposed is favourable for the correct estimation of separation. Further discussion of the present procedure of separation of rainfall will be presented, after field and indoor observations are completed.

(4) Estimation of unit hydrograph

a) *Estimation of unit time and unit rainfall.* The pertinent selection of unit time and unit rainfall is required for the correct estimation of flood discharge in hydrological research. Although the unit of a day is commonly used in U.S.A., its application to Japanese rivers, which are of steep slope in smaller basins, will not become preferable. The uniform averaging of areal distribution of precipitation, based on the concept of unit hydrograph method, requires that the longer unit time is used, the better results is expected, while the difficulty of estimation for correct distribution will be given a rise. On the other hand, the selection of short period for the unit time yields complication of computation and appreciable non-linear influence of stream behaviours. Commonly, the lag time of runoff is used as a standard of unit time and the better selection is to use a half of lag time for the estimation of unit time. A quarter of lag time is required as an unit time for the correct hydrological analysis of increasing part of flood flow.

Careful consideration to the selection of unit time in the model basin indicated that the best unit time was assumed two hours and the most pertinent depth of unit rainfall 20 mm.

b) *Estimation of unit hydrograph.* Both algebraic and geometrical procedures of method are commonly used for the estimation of unit hydrograph with the aid of direct flow and effective rainfall. However, the cut and try method is only the possible way to obtain the fruitful results applicable to natural streams. As a first approximation, the geometrical procedure[6] gives the shape of unit hydrograph in a form of

$$\left.\begin{aligned} S_o &= S_n'/\bar{r}, \\ \text{and} \qquad q_0 &\fallingdotseq \left[Q_m + \frac{S_n' \cdot \Delta t}{\bar{r}} \left\{ (n-1) r_1 + \cdots\cdots\cdots + r_{n-1} \right\} \right] \bar{r}, \end{aligned}\right\} \quad \cdots\cdots\cdots (4)$$

where, S_n' is the slope of a line from the ordinate of peak to that after one unit time in hydrograph, $\bar{r}=\sum_{i=1}^{n} R_i/R_0$, R_i the rate of rainfall, R_0 the unit rainfall, S_0 the slope of a line from the peak ordinate to that after one unit time in unit hydrograph, q_0 the maximum ordinate of unit hydrograph, Q_m the maximum ordinate of hydrograph, Δt the unit time, and $r_i=R_i/R_0$.

The values in the right hand term of Eq. (5) are calculated by the measurement of field observation for individual storms, so that the values of S_0 and q_0 become known. Consideration that the grade of recession curve of a hydrograph is equal to that of unit hydrograph provides the determinaton of ordinates of recession curve of unit hydrograph. As a first approximation, the time from the peak of rainfall intensity to the peak of flood discharge is used as the time from the beginning of flood to the peak in unit hydrograph, and the time base is also assumed the time from the end of a series of successive rainfall to the end of direct flows. Thereafter, successive procedures of cut and try method are required for the correct estimation of unit hydrograph. Fig. 6 indicates some typical unit hydrographs in the model basin of the Yura River valley by the above described procedure.

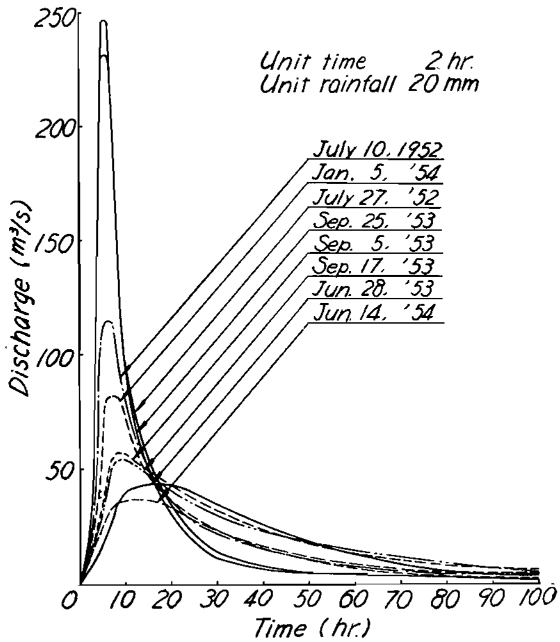


Fig. 6 Some unit hydrographs in the model basin

(5) Characteristics of unit hydrograph

The hydrological significance of Fig. 6 indicates the range of applicability of practical procedures of unit hydrograph method, based on the basic assumption of an unit hydrograph for a river basin, for the estimation of runoff distribution, as many investigators already recognized. The basic assumption of unit hydrograph method is emphasized on the engineering practice for hydrological research, and therefore, it will be of common observation that the application of unit hydrograph method to the estimation of runoff distribution becomes effective for gentle rivers in large basins of U.S.A., while in small basins of Japan, the non-linear behaviours of rainfall and stream have a primary influence on the unit hydrograph method. Hence, after the clear formulation of complicated behaviours of stream in non-linearity is advanced, the real evaluation of unit hydrograph method has to be expected as engineering practice for hydrological research.

As the conclusive summary, some future problems to be discussed will be presented.

a) *Influence of rainfall and channel characteristics.* Non-uniform distribution in a basin for individual storms requires the change of the shape of resulted hydrograph and therefore, the unit hydrograph will be influenced to some extent. The variation in rainfall intensity influences also the runoff distribution and thus boundary characteristics. As the motion of stream in channel is non-linear, so the resulting effects of rainfall intensity will be of primary importance for the basic assumption of unit hydrograph. An approach of estimation of the unit hydrograph resulted from the rainfall characteristics in intensity will be described in the next part.

b) *Influence of separation methods of runoff and rainfall.* The present separation method between direct and base flows, as described herein, is not subjected to theoretically correct procedure in the light of knowledge in hydrology and hydraulics but to engineering practice for a first approximation, and furthermore, it is seen that the base flow is not influenced by the rainfall intensity. The separation of successive rainfalls is of primary importance for the separation of resulted runoff. However, the successful method has not been established. In 1954 the experimental study of filtration of water into sands in a cylinder of 15 cm in diameter was made in the Hydraulics Laboratory, Kyoto Universiy[7]. The conclusive summary of the experientation is indicated that the infiltration rate is expressed in a form of the Horton formula. On the other hand, the separation of rainfall divided by means of the formula seems unsatisfactory for the hydrological analysis bacause of the difference in boundary characteristics between the model and the actual field. However, the error of unit hydrograph in its shape will be cancelled by mutual influences of other hydrological factors, and thus the final shape is rather satisfactory for the engineering purpose.

The estimaton of lost rainfall is of primary importance for the analysis of runoff behaviours. The present methods of estimation for lost rainfall are the following.

(1) The lost rainfall is estimated from the total rainfall in a subjected basin and the resulted total runoff flowed through a stream.

(2) The lost rainfall is explicitely assumed by the cut and try procedure for the convenience of runoff estimation as an engineering approximation.

The model study of the transient behaviours of rainwater supplied with artifical rainfall along the model hill side is now under investigation at the Hydraulics Laboratory, and the establishment of dynamics of transient behaviours will be expected to contribute to the further progress in hydrological analysis.

Part 2

Variation of Unit Hydrograph Resulted from the Variation of Rainfall Characteristics

In this part, the hydrological influences of rainfall characteristics on the unit hydrograph method are concerned. The main purpose of the present part is to evaluate the hydrological significance of unit hydrograph as a useful tool for engineering practice, with the aid of the theory of flood propagation of S. Hayami[8], though the hydraulic behaviours of stream are not concerned.

The effect of retardation at the hill side will play a primary part on the resulted retardation of runoff compared with other factors of channel storage and the like. In this part, the main purpose is to evaluate the variation of unit hydrograph resulted from the variation of rainfall characteristics, and therefore, the retardation effect at the hill side is divided into two parts of the adjustment of propagation velocity of flood waves and the separation method of rainfall for the convenience of practical application. The dynamics of retardation of rainwater at the hill side will be discussed in another opportunity.

At first, the hydrological consideration is accented on the clear formulation of relations between the unit hydrograph and an assumed solitary flood wave, or unit flood, resulted from Hyami's theory, and next, the behaviour of rainfall characteristics on the propagation of unit flood is treated. Finally, the variation of unit hydrographs resulted from the change of rainfall characteristics is concerned.

For the convenience of later discussion, Hayami's theory on flood propagation, based on the diffusive process of channel characteristics, will be presented.

Denoting h_0 the water stage at the upstream end, H the water stage at a point x from the upstream end, H_0 that in steady regime at the same point, t the time, $\mu=\left\{HU\Big/2\left(i-\frac{\partial H}{\partial x}\right)\right\}+\eta$, η the diffusive coefficient, U the mean velocity, i the slope of channel bed, and $\omega=3U/2$, the first approximation of timely distribution of original water stage h_0 held constant for the time of Δt at a downstream point of distance x is expressed as

$$(H-H_0)/h_0=1-\frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{\mu t}}}\exp\left\{\frac{\omega x}{2\mu}-\xi^2-\frac{(\omega/2\mu)^2x^2}{4\xi^2}\right\}d\xi, \quad \cdots\cdots(6)$$

under the assumption of constant ω and x, and the unit flood wave is composed of two curves calculated by Eq. (6) under the following conditons of Eqs. $(7)_a$ and $(7)_b$.

$$\left.\begin{array}{ll} t\leqq 0 \cdots\cdots\cdots\cdots (H-H_0)/h_0=0, \\ t>0 \cdots\cdots\cdots\cdots \quad '' \quad >0. \end{array}\right\} \quad \cdots\cdots\cdots\cdots(7)_a$$

$$\left.\begin{array}{ll} t\leqq t \cdots\cdots\cdots\cdots \quad '' \quad =0, \\ t>t \cdots\cdots\cdots\cdots \quad '' \quad <0. \end{array}\right\} \quad \cdots\cdots\cdots\cdots(7)_b$$

It is seen the value of μ is assumed a constant of magnitude of $10^6\sim10^7$ c.g.s. Therefore, the values of $(H-H_0)/h_0$ are calculated for given ω and x and considered to be proportional to the two third power of discharge. Then the following relation is obtained,

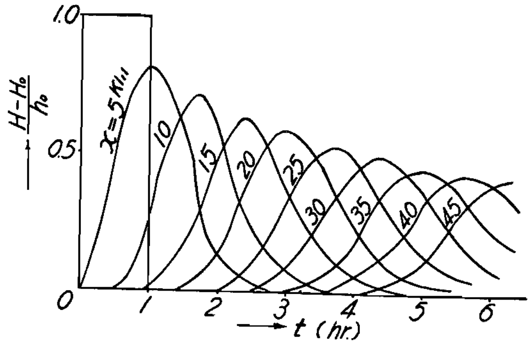


Fig. 7 Timely distribution of $(H-H_0)/h_0$ as a parametric expression of distance x

$$\left.\begin{aligned} C\{h_t(\omega, x)\}^{3/2} &= q_t(\omega, x), \\ C &= 1 \Big/ \int_t h_t^{3/2} dt, \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(8)$$

where h_t is the value $(H - H_0)\ h_0$ and q_t the discharge corresponding to h_t. When the sum of ordinates which are calculated for each unit time in the q_t curve becomes unity, the q_t curve is considered the hydrograph resulted from rainfall for an unit time in an unit area. Fig. 7 shows the behaviours of h_t curves under the condtion of $\omega = 2$ m/sec and $\mu = 10^7$ c.g.s.

(1) Consideration to the variation of areal rainfall distribution

The basin under consideration is divided into some zones to make the distance from any midpoint of a zone to the lower outlet of the basin along the stream, x_0, equivalent. Assuming $x = x_0$ and $\omega = \omega_0$, q_t curve represents a hydrograph for an unit rainfall in an unit area of the zone $x = x_0$, and the propagation velocity is ω_0. Consequently, the hydrograph for uniform unit rainfall in the whole area of the zone of $x = x_0$ is obtained by the product of q_t curve and area.

The basin is also divided into some regions in which the uniform areal distribution of rainfall may be expected and one or more self recording raingage stations are located. The discharge hydrograph in a region is a compound of hydrographs in all zones of a region. Thus, the present procedure leads to make the influence of areal characteristics of rainfall distribution in any region on hydrographs clear.

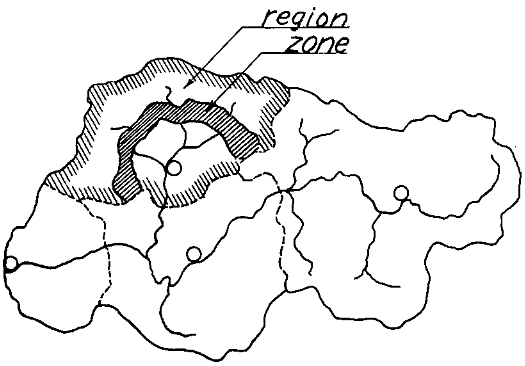


Fig. 8 Schematic diagram of dividing a basin into regions and zones

Fig. 8 indicates the schematic diagram of dividing a basin into regions and zones.

(2) Consideration to the variation of rainfall intensity

Considering the non-linearity of propagation velocity ω, and assuming effective rainfall intensity r is linearly proportional to the discharge q resulted from the rainfall for an unit time in an unit area, the following approximate expression will be obtained with the aid of simple knowledge in hydraulics.

$$\left.\begin{aligned} r/r_0 &= q/q_0 = (h/h_0)^{3/2} \\ &= (\omega/\omega_0)^3, \\ \text{and}\quad \omega &= \omega_0(r/r_0)^{1/3} = \omega_0 a^{1/3}, \\ a &= r/r_0, \end{aligned}\right\} \qquad \cdots\cdots\cdots(9)$$

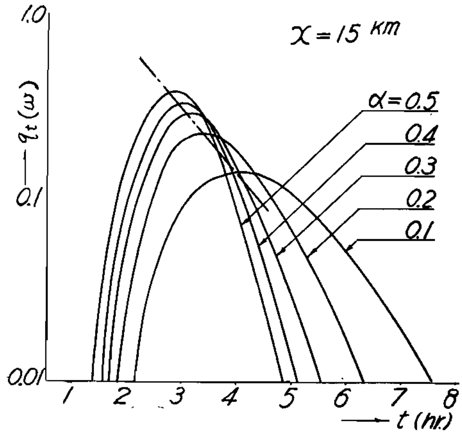


Fig. 9 Variation of unit hydrograph resulted from the variation of α

where r_0 is a standard rainfall. Inserting Eq. (9) into Eq. (6), the approximate hydrological behaviours of rainfall intensity to the shape of unit hydrograph will be seen. Fig. 9 indicates q_t curves at $x = 15$ km as a parametric expression of a, under the assumption of $\omega = 2a^{1/3}$

For the practical convenience of calculation, the following parameters are used in a form of

$$a = k\beta, \quad \omega = \omega_0(k\beta)^{1/3}, \quad \cdots\cdots\cdots(10)$$

where $k = r_0'/r_0$, $\beta = r/r_0'$, and r_0' is the optimum rainfall for hydrological research (10 mm or 1 in.). The approximate determination of k is selected by the following procedure for a given ω_0.

The shape of a solitary flood wave is expressed in a form of

$$\int_{g(x,t')}^{g(x,t)} f(\xi)d\xi, \quad \cdots\cdots\cdots(11)$$

where

$$f(\xi) = \exp\left\{\frac{\omega x}{2\mu} - \xi^2 - \frac{(\omega/2\mu)^2 x^2}{4\xi^2}\right\}, \quad g(x,t) = x/2\sqrt{\mu t}, \quad t' = t - \Delta t.$$

The location of the peak of flood wave at any time t is given as a solution of

$$\frac{\partial}{\partial x}\left\{\int_{g(x,t')}^{g(x,t)} f(\xi)d\xi\right\} = -\frac{1}{\sqrt{t^3}}f\{g(x,t),x\} - \frac{1}{\sqrt{t^3}}f\{g(x,t'),x\} = 0$$

and it becomes

$$x^2 = [\omega^2 \Delta t - 6\mu \log \{(t-\Delta t)/t\}]\, t\,(t-\Delta t)/\Delta t. \quad \cdots\cdots\cdots\cdots(12)$$

Fig. 10 indicates the above relation of the arrival time, t, to the location from the downstream outlet, x. The maximum rainfall intensity r_P is assumed to influence mainly the peak flow, and consequently, for a storm in which the peak flow is recorded at the time t_1 and the maximum rainfall intensity also at the time t_2 in the zone of $x = x_P$, the first approximation of k is obtained by

$$k = \alpha_P / \beta_P \; .$$

where $\beta_P = r_P / r_0'$ and α_P is calculted from Fig. 10, by the use of $x = x_P$ and $t = t_1 - t_2$. When the areal distribution of rainfall is uniform, x_P is simply the distance from the center of area to the downstream end. Hence, the estimation of variation of unit hydrograph resulted from the change of rainfall characteristics will be possible.

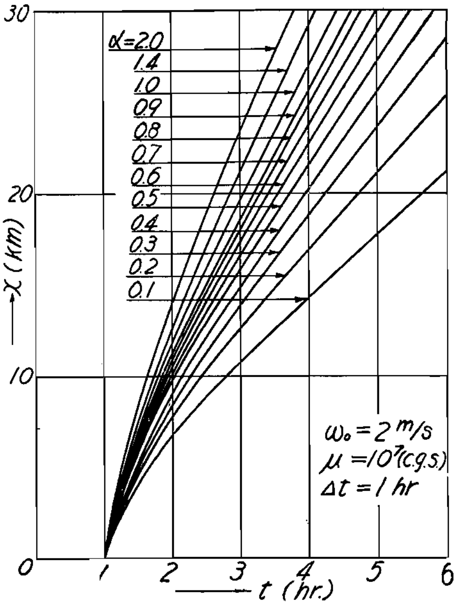


Fig. 10 Relation between the arrival time of of peak, t and the location from the downstream outlet, x

While, selecting ω_0 and r_0 constant standards for any river basin and considering the velocity of stream is given by the Manning formula as a first approximation of engineering practice for flood waves, the following hydrologically important expression is obtained.

$$\omega_0 k^{1/3} \propto U \propto \sqrt{i}/n \, , \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(13)$$

where i is the slope of channel bed and n the Manning roughness. It means the value of k is determined by channel and boundary characteristics of a basin, and therefore the establishment of relation between k, i and n for any river basin is of significance for the hydrological application to river basins without observations and the further advance of synthetic hydrograph method.

(3) Application to the model basin of the Yura River valley

In the model basin, as already described, the change of areal distribution of rainfall gives few influences on the characteristics of unit hydrograph, and then, only the influence of variation of rainfall intensity is considered. Fig. 11 indicates a family of unit hydrographs for various r under the assumption

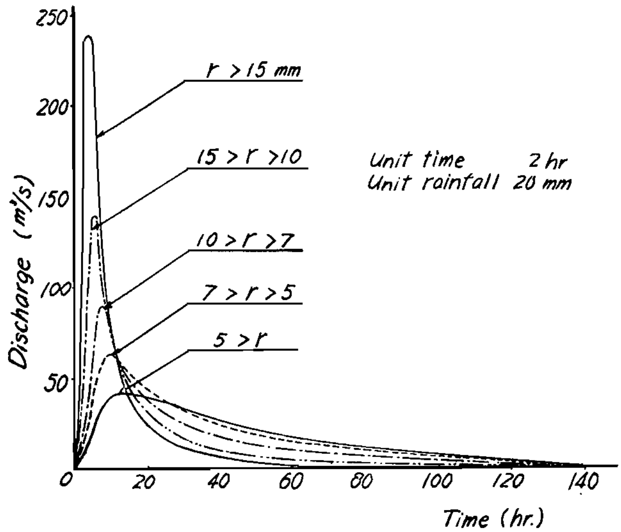


Fig. 11 Variation of unit hydrographs resulted from the variation of rainfall intensity (The Yura River)

that $\omega=2$ m/sec, $r_0'=20$ mm and the unit time is 2 hours. For the present calculation, the value of k is used 0.1 by the cut and try procedure on field observatons. It is, however, readily recognized by Eq. (13) that k is a function of i and n, and therefore, for the correct estimation of unit hydrograph, the behaviours of i and n must be clearified.

Part 3

Synthetic Unit Hydrograph Applicable to Rivers in Japan

The usual procedure of method for obtaining the unit hydrograph, as shown in Part 1, requires long period measurements of hydrological observations.

On the contrary, in the procedure presented in Part 2, the direct runoff for any river basin without hydrological observations will be estimated, if the value of k is given. The method which is used to obtain the unit hydrograph in any river basin is called the synthetic unit hydrograph mothod[8),9)] The purpose of hydrological research, therefore, applicable to the development of synthetic hydrograph method is to establish the relation between channel and boundary characteristics in a valley and the coefficients involved in the basic theory. For the present case, it is to make the formulation of behaviours of i and n against k clear.

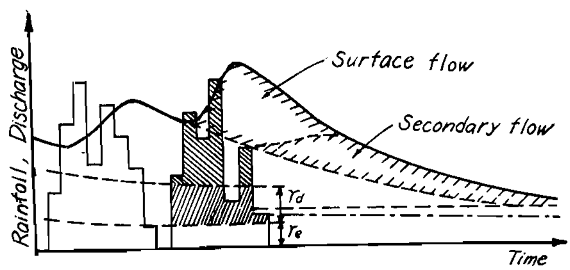
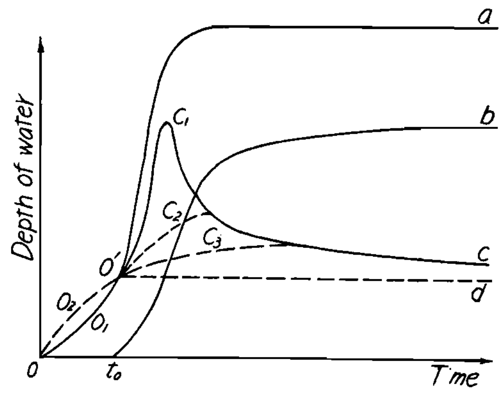


Fig. 12 Separation of runoff and rainfall

From Eq. (3), the value of k is proportional to the two third power of the slope i as an engineering approximation. However, as shown in the previous part, k is also influenced by the non-linear behaviours of stream flow. In order to consider these effects, the direct flow already defined is separated, again, into two components of surface and secondary flows. The approximate method of separation for the engineering practice is as follows.



$OO_1O'a$ Mass rainfall curve
t_0b : Mass runoff curve
$OO_1O'C_1c$ Water retention curve

Fig. 13 Analysis of retention of water in river basin

In Fig. 13, the rainfall begines at the time 0, and the resulted runoff at the time t_0. The curve $OO_1O'a$ indicates the mass curve of rainfall and the curve t_0b that of runoff. The difference between ordinates of two curves at the time of infinity represents the amount of retained water in the river basin, expressed in terms of depth. The curve $OO_1O'C_1c$ indicates the water retention curve, and the ordinate t_0O' the loss depth never appears again in channels. For the hydrological research of unit hydrograph, therefore, the part above the line $O'd$ is only considered. As the flow behaviours of secondary and ground water flows are assumed similar in a river basin for any storm, so in the water retention curve $O'C_1c$, the remain, after subtracting a part assumed equivalent to surface flow, is constructed by two components of secondary and ground water flows. Consequently, the shape of mass curves of infiltrated rainfall is represented by the curve $OO_2O'C_2$ for secondary flow and the curve $OO_2O'C_3$ for ground water flow may be considered similar for any storm. The difference of ordinates between two curves of $O'C_1$ and the curve $O'C_2$ indicates the component contributive to surface flow. In the same consideration, the difference of ordinates between the curve $O'C_2$ and the curve $O'C_3$ represents the component of secondary flow and that between two curves of $O'C_3$ and Od corresponds to the component of ground water flow.

For the practical procedure of separation, the semi-logarithmic sheet of paper is available. Plotting the curve C_1c on the semi-logarithmic sheet is divided into three straight lines. With the use of these lines divided by breaking points, two curves of $O'C_2$ and $O'C_3$ are estimated. Hence, as the possible shapes of curvse $O'C_2$ and $O'C_3$ are obtained by many measurements of observation, so coefficients in the formula of Horton are also decided.

Denoting subscripts 1 and 2 values for surface and secondary flows, the shape of unit hydrograph in a river basin is calculated by the procedure described in Part 2, with the aid of k_1 and k_2. The hydrological observations in some river basins of Japan give the following relations for k_1 and k_2.

$$k_1 = 435\,(\sqrt{\overline{i}})^3,$$

and

$$k_2 = 22.4\,(\sqrt{\overline{i}})^3,$$

where $\overline{\sqrt{i}}$ is the mean value of $\sqrt{i}$ in the basin.

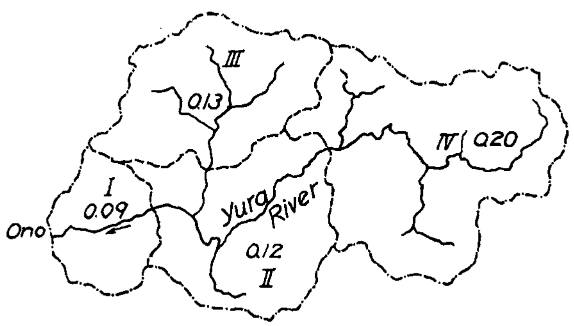


Fig. 14 Value of k_1 in divided regions for the model basin of the Yura River valley

The hydrological significance of above relation is the establishment of formulation between channel and boundary characteristics assumed constant for a river basin and numerical coefficients necessary to calculation of synthetic unit hydrograph for any river basin with or without hydrological observa-

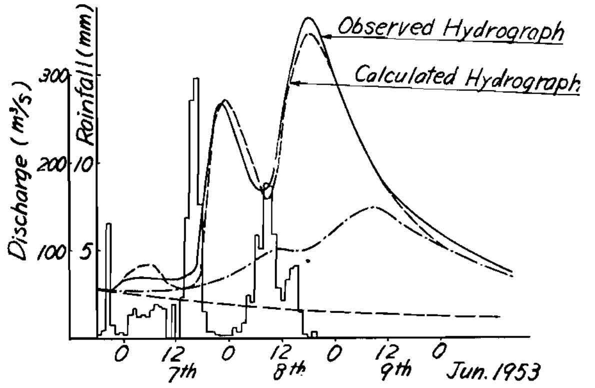


Fig. 15 Hydrological analysis of flood of the Yura River

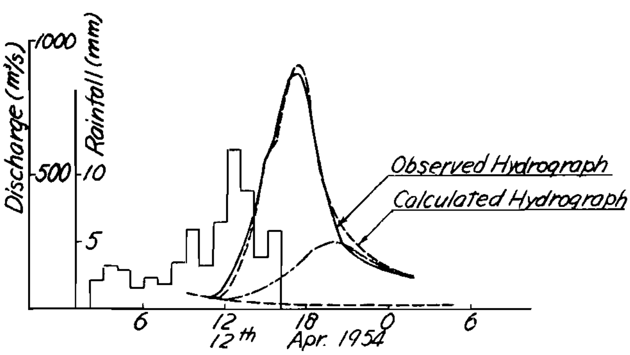


Fig. 16 Hydrological analysis of flood of the Kiso River

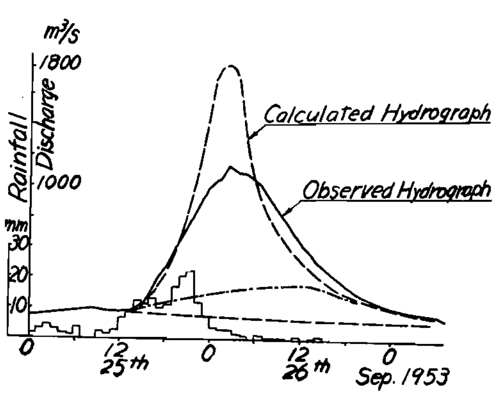


Fig. 17 Hydrological analysis of flood of the Oyabe River

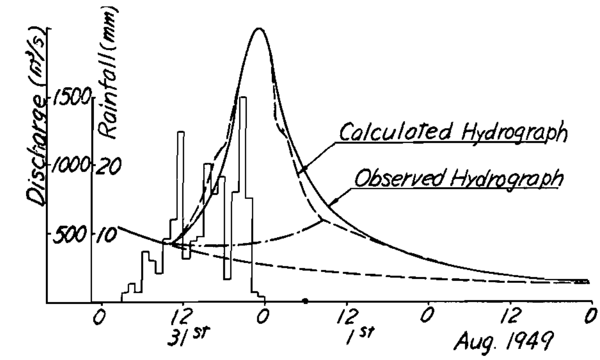


Fig. 18 Hydrological analysis of flood of the Kanna River

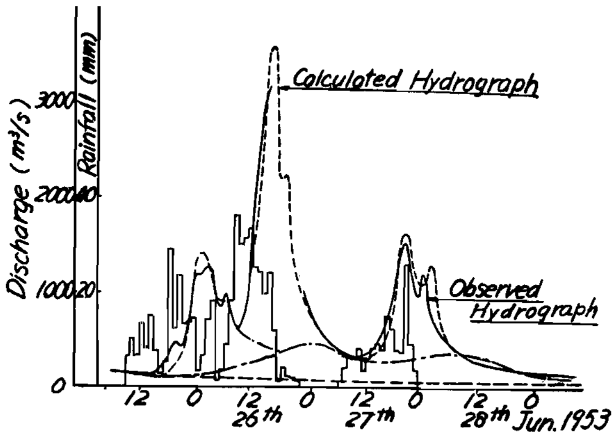


Fig. 19 Hydrological analysis of flood of the Ōita River

tions. Fig. 14 indicates the values of k_1 for divided four regions in the model basin of the Yura River valley, and Figs. 15 to 19 represent examples of calculation by the procedure proposed in the present paper.

Part 4

Some Problems to Be Furthermore Discussed

Although the method of separation of rainfall and resulted runoff into two or more components is commonly used by investigators and engineers for hydrological research, it is only for the practical convenience of engineering research. In part 1, a straight horizontal line from the beginning of runoff was used for the separation of direct and base flows, and the secondary flow was assumed to begin with the surface flow. Such assumption as well as the hydrological behaviours of infiltration like the equation of Horton must be solved by both experimental and theoretical researches for the further advance of unit hydrograph method.

Some investigators[11),12)] assumed the discharge was linearly proportional to the storage in a basin at the recession part of hydrograph. However, the preliminary exprimental results of storage conducted at our Hydraulics Laboratory indicate that the discharge is not proportional to the first power of storage and varies with the path of runoff as secondary or gound-water flows. The experimental result and physical analysis of such behaviours will be presented in the near future.

For the physical analysis of runoff behaviours, the following two methods of approach are considered: the first is the macroscopic method like unit hydrograph in which all factors are involved as the synthesized factor, and the other is the microscopic method in which the runoff behaviours are separated into individual ones like flows along the hill side and channels, and the loss of rainfall. The investigation of better applicability of macroscopic or microscopic methods to the clear formulation of runoff behaviours is also of fundamental importance. For example, in Part 3, the behaviours of k were estimated in term of the slope i, but as described in that part, it will be nonlinearly influenced by channel and boundary charcteristics, and especially k_2 will be influenced by surface coverings and soils in a basin. The microscopic mëthod is requested for the establishment of above behaviours, so that the detailed studies based on macro- and microscopic approaches must be continued.

The further hydrological observations in the model basin as well as other basins are now in progress and the clear formulation of functional diversity in channel and boundary characteristics and the establishment of synthetic unit hydrograph contributive to the future advance in hydrological research in Japan will be obtained by the detailed field observations and basic researches in laboratories.

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Bulletin No. 19 Published March, 1958

昭和 33 年 3 月 10 日 印刷
昭和 33 年 3 月 15 日 発行
編輯兼発行者 京都大学防災研究所
印刷者 山代多三郎
京都市上京区寺之内通小川西入
印刷所 山代印刷株式会社