33.6.

DISASTER PREVENTION RESEARCH INSTITUTE
BULLETIN No. 22 MARCH, 1958

ON THE SECOND VOLCANIC MICRO-TREMOR AT THE VOLCANO ASO

BY

MICHIYASU SHIMA

KYOTO UNIVERSITY, KYOTO, JAPAN

K

DISASTER PREVENTION RESEARCH INSTITUTE
KYOTO UNIVERSITY
BULLETINS

Bulletin No. 22 March, 1958

On the Second Volcanic Micro-Tremor at the Volcano Aso

By

Michiyasu SHIMA

Contents

On the Second Volcanic Micro-Tremor at the Volcano Aso

by

Michiyasu Shima

Geophysical Institute, Faculty of Science, Kyoto University

(Communicated by Prof. K. Sassa)

Abstract

At Volcano Aso, four kinds of volcanic micro-tremors have been observed by K. Sassa. And each of them has been named the micro-tremor of the first kind, of the second, of the third and of the fourth, of which the second one is of a long period, 3.5～7 sec. and its origin seems to be considerably different from the rest. So it is supposed that this micro-tremor is generated with the explosive motion of gas-riched magma in a reservoir and that its period, 3.5～7 sec. is the free one of magmatic reservoir, and then the dimension of the reservoir is calculated. In case that the reservoir begins to vibrate by the explosion of gas-riched magma, the mode which vibrates only to the propagating direction of wave appears chiefly, and therefore the equations of the wave motion of such a mode are solved and the radius of the reservoir, 696～1392 m., is gotten, being applied the velocity of seismic wave calculated from the records of volcanic earthquakes. This result suggests that the above mechanism may be expected for the micro-tremor of the second kind.

1.

At Volcano Aso four kinds of volcanic micro-tremors have been observed by K, Sassa.[1] And each of them has been named the micro-tremor of the first kind, of the second, of the third and of the fourth. Generally, the micro-tremor of the first kind, of the second and of the third have been observed independently each other, but the second one has been frequently observed to be accompanied by the first one, and the latter has been recorded from 0 to 20 sec. before the recorded initial phase of the principal part of the former. The micro-tremer of the second kind is of a long period, 3.5～7 sec., while the other periods are less than 1 sec.. These facts suggest that the origin of the second kind is considerably different from the rest. With respect to their vibrating modes, every direction (of the horizontal displacement) accords with the propagating direction of wave, in the six observing stations and these waves propagate as compression waves except a weak tension wave observed in one observing station. Thus, this long wave should consist of a kind of longitudinal waves generated with the explosive motion of gas-riched magma in a reservoir. So, it is supposed in the following part that the period of the microtremor of the second kind, $3.5 \sim 7$ sec. is the free one of the magmatic reservoir, and then the dimension of reservoir is tried to calculate.

2.

The equations of wave motion in a homogeneous and isotropic medium are expressed as follows using the polar coordinate,[2)]

$$\begin{aligned}
\rho\frac{\partial^2\Delta}{\partial t^2} &= (\lambda+2\mu)\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Delta}{\partial r}\right)+\frac{1}{r^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Delta}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\Delta}{\partial\phi^2}\right]\\
\rho\frac{\partial^2\omega_r}{\partial t^2} &= \mu\left(\frac{\partial^2\omega_r}{\partial r^2}+\frac{4}{r}\frac{\partial\omega_r}{\partial r}+\frac{2}{r^2}\omega_r+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\omega_r}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\omega_r}{\partial\phi^2}\right)\\
\rho\frac{\partial^2\omega_\theta}{\partial t^2} &= \mu\left[\frac{1}{r}\frac{\partial^2(\omega_\theta r)}{\partial r^2}+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\omega_\theta}{\partial\phi^2}-\frac{1}{r^2\sin^2\theta}\frac{\partial^2(\omega_\phi\sin\theta)}{\partial\phi\partial\theta}-\frac{1}{r}\frac{\partial^2\omega_r}{\partial r\partial\theta}\right] \qquad (1)\\
\rho\frac{\partial^2\omega_\phi}{\partial t^2} &= \mu\left[\frac{1}{r}\frac{\partial^2(\omega_\phi r)}{\partial r^2}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial(\omega_\phi\sin\theta)}{\partial\theta}\right)\right.\\
&\quad\left.-\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\omega_\theta}{\partial\phi}\right)-\frac{1}{r\sin\theta}\frac{\partial^2\omega_r}{\partial r\partial\phi}\right]
\end{aligned}$$

The solutions of these equations are given as follows by Sezawa.

$$\Delta=\frac{1}{\sqrt{r}}\begin{Bmatrix}A_{mn}J_{n+1/2}(hr)\\A'_{mn}N_{n+1/2}(hr)\end{Bmatrix}P_n^m(\cos\theta)\begin{Bmatrix}\cos\\\sin\end{Bmatrix}m\phi e^{ipt}$$

$$2\omega_r=\frac{1}{r^{3/2}}\begin{Bmatrix}B_{mn}J_{n+1/2}(kr)\\B'_{mn}N_{n+1/2}(kr)\end{Bmatrix}P_n^m(\cos\theta)\begin{Bmatrix}\sin\\-\cos\end{Bmatrix}m\phi e^{ipt}$$

$$\begin{aligned}
2\omega_\theta &= \left(\frac{1}{\sqrt{r}}\begin{Bmatrix}C_{mn}J_{n+1/2}(kr)\\C'_{mn}N_{n+1/2}(kr)\end{Bmatrix}\frac{P_n^m(\cos\theta)}{\sin\theta}+\frac{1}{n(n+1)}\frac{1}{r}\right.\\
&\quad\left.\times\frac{d}{dr}\begin{Bmatrix}\sqrt{r}\,B_{mn}J_{n+1/2}(kr)\\\sqrt{r}\,B'_{mn}N_{n+1/2}(kr)\end{Bmatrix}\frac{dP_n^m(\cos\theta)}{d\theta}\right)\begin{Bmatrix}\sin\\-\cos\end{Bmatrix}m\phi e^{ipt}
\end{aligned}$$

$$\begin{aligned}
2\omega_\phi &= \left(\frac{1}{\sqrt{r}}\begin{Bmatrix}C_{mn}J_{n+1/2}(kr)\\C'_{mn}N_{n+1/2}(kr)\end{Bmatrix}\frac{dP_n^m(\cos\theta)}{d\theta}+\frac{m}{n(n+1)}\frac{1}{r}\right. \qquad (2)\\
&\quad\left.\times\frac{d}{dr}\begin{Bmatrix}\sqrt{r}\,B_{mn}J_{n+1/2}(kr)\\\sqrt{r}\,B'_{mn}N_{n+1/2}(kr)\end{Bmatrix}\frac{P_n^m(\cos\theta)}{\sin\theta}\right)\begin{Bmatrix}\cos\\\sin\end{Bmatrix}m\phi e^{ipt}
\end{aligned}$$

$$h^2 = \rho p^2/(\lambda+2\mu) \qquad\qquad k^2 = \rho p^2/\mu$$
$$= \frac{p^2}{v_p^2} \qquad\qquad\qquad = \frac{p^2}{v_s^2} \quad ,$$

where $J_{n+1/2}$, $N_{n+1/2}$ are respectively the Bessel functions of the first kind and the second, and $P_n{}^m(\cos\theta)$ is the associated Legendre function. So we can obtain the displacement components, which we denote as u (r-component), v (θ-component) and w (ϕ-component) respectively, from (2). And these displacement components are resolved into three parts $u = u_1 + u_2 + u_3$, $v = v_1 + v_2 + v_3$, $w = w_1 + w_2 + w_3$. The second part of them, u_2, v_2, w_2 are not necessary since the rotation component ω_r vanishes in this case.

3.

For simplicity we treat the vibration of the reservoir fillled with the gas-riched magma in the infinite medium, discussing that of the reservoir under the surface of the Earth. For, although the surface of the Earth effects the interferences between the waves reflected from the surface and the vibration of the reservoir, we neglect the effect of the surface for the small proportion of the reflecting waves returning to the reservoir.

Assuming the reservoir to be the spherical cavity of radius a, it is easily shown that the displacement components, which are denoted by suffix "i" in the reservoir and "o" in the elastic medium, are as follows:

$$u_0 = \Big[-\frac{A_{mn}}{h_0^2}\frac{d}{dr}\Big\{\frac{H^2{}_{n+1/2}(h_0 r)}{\sqrt{r}}\Big\} P_n{}^m(\cos\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\phi$$
$$-\frac{n(n+1)}{mk_0^2} C_{mn}\frac{H^2{}_{n+1/2}(k_0 r)}{r^{3/2}} P_n{}^m(\cos\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\phi\Big] e^{ipt}$$
$$v_0 = \Big(-\frac{A_{mn}H^2{}_{n+1/2}(h_0 r)}{h_0^2\, r^{3/2}}\frac{dP_n{}^m(\cos\theta)}{d\theta}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\phi$$
$$-\frac{C_{mn}}{mk_0^2}\frac{1}{r}\frac{d}{dr}\Big\{\sqrt{r}\, H^2{}_{n+1/2}(k_0 r)\Big\}\frac{d}{d\theta}P_n{}^m(\cos\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\phi\Big) e^{ipt} \qquad (3)$$
$$w_0 = \Big(\frac{mA_{mn}}{h_0^2}\frac{H^2{}_{n+1/2}(h_0 r)}{r^{3/2}}\frac{P_n{}^m(\cos\theta)}{\sin\theta}\begin{Bmatrix}\sin\\ -\cos\end{Bmatrix} m\phi$$
$$+\frac{C_{mn}}{k_0^2}\frac{1}{r}\frac{d}{dr}\Big\{\sqrt{r}\, H^2{}_{n+1/2}(k_0 r)\Big\}\frac{P_n{}^m(\cos\theta)}{\sin\theta}\begin{Bmatrix}\sin\\ -\cos\end{Bmatrix} m\phi\Big) e^{ipt},$$

$$u_i = -\frac{B_{mn}}{h_i^2}\frac{d}{dr}\Big\{\frac{J_{n+1/2}(h_i r)}{\sqrt{r}}\Big\} P_n{}^m(\cos\theta)\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\phi e^{ipt}$$
$$v_i = -\frac{B_{mn}}{h_i^2}\frac{J_{n+1/2}(h_i r)}{r^{3/2}}\frac{dP_n{}^m(\cos\theta)}{d\theta}\begin{Bmatrix}\cos\\ \sin\end{Bmatrix} m\phi e^{ipt} \qquad (4)$$
$$w_i = \frac{mB_{mn}}{h_i^2}\frac{J_{n+1/2}(h_i r)}{r^{3/2}}\frac{P_n{}^m(\cos\theta)}{\sin\theta}\begin{Bmatrix}\sin\\ -\cos\end{Bmatrix} m\phi e^{ipt}$$

We adopt the solution, $H^2_{n+1/2}(h_0 r)=J_{n+1/2}(h_0 r)+iN_{n+1/2}(h_0 r)$ expressing the divergent wave in the surrounding elastic medium and $J_{n+1/2}(h_t r)$ having the finite value at origin, in the reservoir.

As the vibration of the reservoir arises from the explosion of gas-riched magma in the reservoir, the wave of $m=0$ and $n=0$ generates principally and there appears the mode of vibration which has the same direction as the propagation of wave. The tension wave which is observed only at one station seems to appear on account of deviation from the sphere of the form of the reservoir.

The boundary conditions at the reservoir are given as follows,

$$\left(\lambda_0 \Delta_0+2\mu_0\frac{\partial u_0}{\partial r}\right)_{r=a}=(\lambda_t\Delta_t)_{r=a} \qquad (5)$$

$$(u_0)_{r=a}=(u_t)_{r=a}.$$

Introducing the general solutions (3) and (4) into the above boundary conditions

$$\lambda_0 A_0\frac{H^2{}_{1/2}(h_0 a)}{\sqrt{a}}+2\mu_0\frac{d}{da}\left(-\frac{A_0}{h_0^2}\frac{d}{da}\left(\frac{H^2{}_{1/2}(h_0 a)}{\sqrt{a}}\right)\right)=\lambda_t B_0\frac{J_{1/2}(h_t a)}{\sqrt{a}}$$

$$\frac{A_0}{h_0^2}\frac{d}{da}\left(\frac{H^2{}_{1/2}(h_0 a)}{\sqrt{a}}\right)=\frac{B_0}{h_t^2}\frac{d}{da}\left(\frac{J_{1/2}(h_t a)}{\sqrt{a}}\right) \qquad (6)$$

We obtain the following equation determing the period by eliminating the coefficients A_0 and B_0 from these two equations,

$$\left\{\frac{1}{h_0^2}\frac{d}{da}\left(\frac{H^2{}_{1/2}(h_0 a)}{\sqrt{a}}\right)\right\}\left\{\lambda_t\frac{J_{1/2}(h_t a)}{\sqrt{a}}\right\}-\left\{\lambda_0\frac{H^2{}_{1/2}(h_0 a)}{\sqrt{a}}\right.$$

$$\left.+2\mu_0\frac{d}{da}\left(-\frac{1}{h_0^2}\frac{d}{da}\left(\frac{H^2{}_{1/2}(h_0 a)}{\sqrt{a}}\right)\right)\right\}\times\left\{\frac{1}{h_t^2}\frac{d}{da}\left(\frac{J_{1/2}(h_t a)}{\sqrt{a}}\right)\right\}=0. \qquad (7)$$

Using the expressions for the Bessel functions,[3]

$$\frac{(p_1+ip_2)^2}{\rho_0}\sin\left\{\frac{a(p_1+ip_2)}{v_{pt}}\right\}\left\{\frac{1}{a}+i\frac{(p_1+ip_2)}{v_{p0}}\right\}+\frac{1}{\rho_t}\left[\left(\frac{(p_1+ip_2)}{v_{pt}}\cos\left\{\frac{a(p_1+ip_2)}{v_{pt}}\right\}\right.\right.$$

$$\left.\left.-\frac{1}{a}\sin\left\{\frac{a(p_1+ip_2)}{v_{pt}}\right\}\right]\left((p_1+ip_2)^2-\frac{4v^2{}_{s0}}{a}\left\{\frac{1}{a}+i\frac{(p_1+ip_2)}{v_{p0}}\right\}\right)\right]=0 \qquad (8)$$

This system containing the reservoir damps with elapsion of time and p is the the complex number $p=p_1+ip_2$, of which the imaginary part p_2 expresses the damping factor.

Introducing the following terms into (8),

$$\sin\left\{\frac{a(p_1+ip_2)}{v_{pi}}\right\}=\sin\frac{ap_1}{v_{pi}}\cosh\frac{ap_2}{v_{pi}}+i\cos\frac{ap_1}{v_{pi}}\sinh\frac{ap_2}{v_{pi}}$$

$$\cos\left\{\frac{a(p_1+ip_2)}{v_{pi}}\right\}=\cos\frac{ap_1}{v_{pi}}\cosh\frac{ap_2}{v_{pi}}-i\sin\frac{ap_1}{v_{pi}}\sinh\frac{ap_2}{v_{pi}}$$

we obtain,

$$A\sin\frac{ap_1}{v_{pi}}\cosh\frac{ap_2}{v_{pi}}+B\cos\frac{ap_1}{v_{pi}}\sinh\frac{ap_2}{v_{pi}}+C\cos\frac{ap_1}{v_{pi}}\cosh\frac{ap_2}{v_{pi}}$$

$$+D\sin\frac{ap_1}{v_{pi}}\sinh\frac{ap_2}{v_{pi}}=0 \qquad (9)$$

$$B\sin\frac{ap_1}{v_{pi}}\cosh\frac{ap_2}{v_{pi}}-A\cos\frac{ap_1}{v_{pi}}\sinh\frac{ap_2}{v_{pi}}-D\cos\frac{ap_1}{v_{pi}}\cosh\frac{ap_2}{v_{pi}}$$

$$+C\sin\frac{ap_1}{v_{pi}}\sinh\frac{ap_2}{v_{pi}}=0 \qquad (10)$$

$$A=-(p_1^2-p_2^2)^2-\frac{4v^2_{s0}}{av_{p0}}p_2+\frac{4v^2_{s0}}{a^2}+\frac{\rho_i}{\rho_0}\left\{p_1^2-p_2^2-\frac{a}{v_{p0}}(3p_1^2p_2-p_2^3)\right\}$$

$$B=2p_1p_2-\frac{4v^2_{s0}}{av_{p0}}p_1+\frac{\rho_i}{\rho_0}\left\{-2p_1p_2-\frac{a}{v_{p0}}(p_1^3-3p_1p_2^2)\right\}$$

$$C=\frac{a}{v_{pi}}(p_1^3-3p_1p_2^2)+\frac{8v^2_{s0}}{v_{pi}v_{p0}}p_1p_2-\frac{4v_{s0}^2}{av_{pi}}p_1$$

$$D=\frac{a}{v_{pi}}(3p_1^2p_2-p_2^3)+\frac{4v^2_{s0}}{v_{pi}v_{p0}}(p_2^2-p_1^2)-\frac{4v_{s0}^2}{av_{pi}}p_2$$

The velocities of p wave and s wave calculated from the records of the eruption earthqake on Nov. 22, 1932[1] are

$$v_{p0}=1.25\text{km}$$
$$v_{s0}=0.98\text{km},$$

and the sound velocity of the gas-riched magma calculated by K. Sassa from the records of the eruption earthquakes on March, 1933[1] is

$$v_{pi}=0.79\text{km}$$

As the density of the gas-rich magma in the reservoir is uncertain, it is assumed that the density is the tenth as much as that of the surrounding solid medium. Introducing these values into (9) and (10),

$$p_1=\frac{125000}{a}$$

$$p_2 = \frac{105000}{a}$$

Introducing the period 3.5∼7 seconds of the micro-tremor of the second kind into (11), we obtain as the radius of the reservoir

$$a = 696 \sim 1392 \text{ m}.$$

That the damping factor p_2 is large seems to correspond to the rapid decay of the observed micro-tremors of this kind. Although the calculated radius is rather large, in comparison with ca. 0.5 km radius which is estimated from the region of the remarkable crustal movement in the vicinity of the craters for the active period[4] and from the depth of the origin of the micro-tremor of this kind,[1] these results suggest that the vibration of the reservoir accompanied with the explosive motion of the gas-riched magma may be expected for the origin of the micro-tremor of the second kind.

The writer wishes to express his hearty thanks to Prof. K. Sassa for his instructions.

Reference

1) K. Sassa ; Memoirs of the College of Science Kyoto University Series A, Vol. XVIII, p. 255

2) K. Sezawa ; Shindogaku (1931) p. 147

3) E. T. Whittaker & G. N. Watson ; Modern Analysis (1927) p. 355

4) K. Yoshikawa ; Zisin, Vol. 7, p. 151.

Publications of the Disaster Prevention Research Institute

The Disaster Prevention Research Institute publishes reports of the research results in the form of bulletins. Publications not out of print may be obtained free of charge upon request to the Director, Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan.

Bulletins :

No. 1 On the Propagation of Flood Waves by Shoitiro Hayami, 1951.
No. 2 On the Effect of Sand Storm in Controlling the Mouth of the Kiku River by Tojiro Ishihara and Yuichi Iwagaki, 1952.
No. 3 Observation of Tidal Strain of the Earth (Part I) by Kenzo Sassa, Izuo Ozawa and Soji Yoshikawa. And Observation of Tidal Strain of the Earth by the Extensometer (Part II) by Izuo Ozawa, 1952.
No. 4 Earthquake Damages and Elastic Properties of the Ground by Ryo Tanabashi and Hatsuo Ishizaki, 1953.
No. 5 Some Studies on Beach Erosions by Shoitiro Hayami, Tojiro Ishihara and Yuichi Iwagaki, 1953.
No. 6 Study on Some Phenomena Foretelling the Occurrence of Destructive Earthquakes by Eiichi Nishimura, 1953.
No. 7 Vibration Problems of Skyscraper. Destructive Element of Seismic Waves for Structures by Ryo Tanabashi, Takuzi Kobori and Kiyoshi Kaneta, 1954.
No. 8 Studies on the Failure and the Settlement of Foundations by Sakurō Murayama, 1954.
No. 9 Experimental Studies on Meteorological Tsunamis Traveling up the Rivers and Canals in Osaka City by Shoitiro Hayami, Katsumasa Yano, Shohei Adachi and Hideaki Kunishi, 1955.
No.10 Fundamental Studies on the Runoff Analysis by Characteristics by Yuichi Iwagaki, 1955.
No.11 Fundamental Considerations on the Earthquake Resistant Properties of the Earth Dam by Motohiro Hatanaka, 1955.
No.12 The Effect of the Moisture Content on the Strength of an Alluvial Clay by Sakurō Murayama, Kōichi Akai and Tōru Shibata, 1955.
No.13 On Phenomena Forerunning Earthquakes by Kenzo Sassa and Eiichi Nishimura, 1956.
No.14 A Theoretical Study on Differential Settlements of Structures by Yoshitsura Yokoo and Kunio Yamagata, 1956.
No.15 Study on Elastic Strain of the Ground in Earth Tides by Izuo Ozawa, 1957.
No.16 Consideration on the Mechanism of Structural Cracking of Reinforced Concrete Buildings due to Concrete Shrinkage by Yoshitsura Yokoo and S. Tsunoda. 1957.
No.17 Onithe Stress Analysis and the Stability Computation of Earth Embankments by Kōchi Akai, 1957.
No.18 On the Numerical Solutions of Harmonic, Biharmonic and Similar Equations by the Difference Method not Through Successive Approximations by Hatsuo Ishizaki, 1957.
No.19 On the Application of the Unit Hydrograph Method to Runoff Andysis for Rivers in Japan by Tojiro Ishihara and Akiharu Kanamaru, 1958.
No.20 Analysis of statically Indeterminate Structures in the Ultimate State by Ryo Tanabashi, 1958.
No.21 The Propagation of Waves near Explosion and Fracture of Rock (I) by Soji Yoshikawa
No.22 On the Second Volcanic Micro-Tremor at the Volcano Aso by Michiyasu Shima

Bulletin No. 22　　Published March, 1958

昭和 33 年 3 月 20 日　印 刷
昭和 33 年 3 月 31 日　発 行

編輯兼 発行者　京都大学防災研究所
印刷者　山 代 多 三 郎
京都市上京区寺之内通小川西入
印刷所　山代印刷株式会社