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Kyoto University
ON THE CHARACTER OF SEEPAGE WATER
AND THEIR EFFECT ON THE STABILITY
OF EARTH EMBANKMENTS

BY

Kōichi Akai

KYOTO UNIVERSITY, KYOTO, JAPAN
Errata

Page 12, line 2, for \( \tau_3 = \left( \frac{K_1}{\pi} - \frac{\pi}{4} K \right) \tan \theta \cdot \gamma_s h \). read \( \tau_2 = \frac{1}{2} \left( \frac{K_1}{\pi} - \frac{\pi}{4} K \right) \tan \theta \cdot \gamma_s h \).

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  \item \ref{4}, line 4 from the foot,
  \item 20, line 6,
  \item 30, \ref{21},
  \item 36, line 2 from the foot,
  \item 38, \ref{10} from the foot,
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On the Character of Seepage Water and Their Effect on the Stability of Earth Embankments

By

Kōichi Akai

Contents

Synopsis

1. Introduction ................................................................. 2
2. Experiments on Seepage Failure of Embankments Using Sand Models ......................................................... 2
3. Theoretical Analysis of the Mechanism of Partial Failure of Embankment Slopes ........................................ 9
4. Preventive Methods of Seepage Failure .................................................. 13
5. Earth Dam Failure Caused by Rapid-Drawdown of the Reservoir ................................................................. 17
6. Conclusion .............................................................................. 24

Acknowledgments

References

Appendix .................................................................................... 26
Synopsis

Throughout the experimental studies of percolating flow using sand models, it has been cleared that the local failure near the upper portion of the surface of seepage is predominant, in the water-retaining embankment which consists of cohesionless materials. Performing the theoretical investigations concerning these experimental results, a proposed equation which gives the critical hydraulic gradient is deduced for this kind of failure, whereby the mechanism of the actual failure of embankments can be analysed. Further experimental research has been succeeded towards the disaster preventive methods of embankment design, and a reasonable criterion can be given adequately. Moreover an earth dam failure caused by rapid-drawdown of the reservoir has been analysed through the site investigations and the laboratory tests.

1. Introduction

The main function of the water-retaining embankments such as earth dams and river levees consists in establishing the effect of retaining water, making the quantity of leakage through these earth structures to be little as possible. There exists more consistent criterion, however, that any structural failure must not be allowed due to the seepage water through them. If a fracture of failure should once appear in earth fills, even though it seems very local fracture initially, it would gradually spread over the whole structure to cause a serious damage, making the cost of restoration to be much expensive.

In principle, two kinds of analytical consideration must be required in order to prevent the above structural failure; (1) the character of seepage water through embankments and (2) the mechanical behaviour of soil constituting the earth structures under the action of this seepage water. In contrast with many investigations which have ever been performed for the former item, too little endeavour has been made for the latter. Thus the author tries his investigation in the light of this latter sense in this paper, for the effect of seepage water on the stability of earth embankments.

2. Experiments on Seepage Failure of Embankments
   Using Sand Models

(1) Experimental apparatus and construction of embankments.
As the experimental apparatus of seepage flow, a steel channel of 400 cm length, 25 cm width, 50 cm height, shown in Photo. 1 has been used. One of the side walls is made of a glass plate of 6 mm thickness, marked cross lines of 5 cm distance together, for facilitating the observations of the behaviour of seepage water and the process of embankment failure. At the upstream end and the downstream end of the channel, several overflow pipes are set to maintain constant water levels arbitrarily, by opening out the arbitrary one of stop cocks. Determinations of the phreatic line and the flow lines in embankments have been performed by use of the Uranin solution whose specific weight equals nearly unity as the tracing dye. Besides, in order to draw the exact flow net, ten glass piezometric tubes (inner diameter 6 mm) are raised from the channel bottom, showing the distribution of pore pressures along the embankment base. A proposed method to find the exact position of phreatic line will be described later.

A uniform sand whose size distribution curve is shown in Fig. 1 has been used as the embankment material, and tamped layer by layer of 2.5 cm thickness in the channel to obtain a uniform dry.

![Photo. 1 Experimental apparatus.](image)

![Fig. 1 Grain-size curves of sand used for the model embankment and the filter-drain.](image)
density in whole experiments. The physical properties of this sand are as follows; specific gravity: 2.61, dry density: 1.59 g/cm³, submerged density: 1.96 g/cm³, void ratio: 0.642, permeability: 3.11×10⁻¹ cm/sec and angle of internal friction in submerged state: 30°27' (by triaxial compression test).

The geometry of the embankment for which failure experiments have been performed is as follows; height: 35 cm, width of crest: 10 cm, upstream slope: 1:2.5, downstream slope: 1:2.5 (called as the standard section), 1:3 and 1:2, respectively. In these series of experiments, the speed of rising water level at the upstream slope has been controlled very slowly, so as to not remain any air bubble within the submerged part of embankments.

(2) Character of seepage water through embankments.

Fig. 2 shows the flow-net in the standard section of earth embankment on the impervious base, when the upstream water level is at 25 cm height in the model. Among two phreatic lines shown in Fig. 2(a), the full line is drawn experimentally and the dotted one is the graphical solution proposed by A. Casagrande. Fig. 2(b) is the distribution of velocity potential by the numerical calculation using the relaxation method. Although the flow-net obtained by the experiment using sand models coincides with any one by other various procedures, such as the graphical method, the analytical method and so on, the experimental method using sand models can let us know the existence of the capillary zone above the theoretical phreatic line, as shown in Fig. 2(a).
When the dye is poured into the point of intrusion (the beginning of phreatic line) at the upstream slope in the author's experiments, a part of dye solution traces along the path of seepage flow due to gravity, and the other suddenly rises at this point, reaching the height of the so-called capillary height. The strip-shaped capillary zone moves down towards the downstream slope along the upper side of the phreatic line. But the dye poured into the point just below the point of intrusion does not reach the capillary zone, but follows along the lower side of the phreatic line as the gravity flow. As the Uranin has little diffusibility and does not colour the sand grains, it is considered very favourable as a tracing dye. At the highest part of the capillary zone, the dye does not move but is in the state of repose, and so it is supposed that the dye remains in the pores between solid grains as the adhesive moisture at this height. The capillary water below this height flows as already mentioned, and the velocity distribution in vertical section is approximately equal to parabolic shape, shown in Fig. 3; the maximum velocity appears on the phreatic line.

On the other hand, the velocity distribution in the gravity zone below the phreatic line coincides with that obtained from the flow-net, which is drawn correctly by accumulating squares; the distance between two adjacent equipotential lines is inversely proportional to the hydraulic gradient, and the distance between two adjacent flow lines is inversely proportional to the percolating velocity. Thus, the percolating velocity is very slow near the embankment base and shows maximum on the phreatic line, in the adjacent part to the
upstream slope. In the adjacent part to the surface of seepage, however, the velocity distribution in vertical section becomes approximately uniform, as no evident difference appears between the largeress of squares constituting the flow-net, in either part adjacent to the phreatic line or the embankment base. Moreover, as the squares are generally small in this downstream region, it is supposed that the seepage flow has larger velocity here than in the upstream region. The precise description will be given later respecting the effect of velocity distribution on the seepage failure of the downstream slope of earth embankments.

(3) Seepage failure of the downstream slope.

It has been known through the experiments that, when the water level at the upstream slope rises to a certain critical height, a local seepage failure appears at the downstream slope in any section of embankments. As the water level rises upwards, the local failure spreads over successively, enlarging its scale more seriously. Though it has been ever understood that there exists more or less unsteady character in the seepage failure of earth embankments in principle, more complex problem would occur by including the time-depending factor in the analysis of the mechanism of this type of failure. Consequently in the author’s experiments, the experimental procedure has been performed to transform this unsteady problem into the quasi-steady one, keeping the rising speed of the upstream water level and the observing time of the behaviour of seepage flow very slowly.

The result of observing the seepage failure of embank-
ments which have three kinds of section shows that there are two different kinds of type, illustrated in Fig. 4. It is very outstanding that these are the local failures caused by scouring or sliding at the downstream slope below the point of exudation (the end of phreatic line), and not the general failures along the large sliding surface as considered in conventional analyses.

i) **Scouring failure**  This kind of failure has happened in the case of flatter downstream slope as 1:3 (Fig. 4(a)) or 1:2.5 (Fig. 4(b)). When the water level at the upstream slope of the embankment reaches a certain critical height, a single sand grain at the point of exudation cannot stay at the original position and is pushed on the downstream slope by the scouring action. The single grain thus pushed on the downstream slope is soon washed away downwards, as the thin-sheet flow of the percolating capillary water is already flowing down along the surface of slope. Then the adjacent grain supported by the ever-existing grain loses its support and is moved to the place where the supporting grain has ever existed. The same process will be repeated on, as illustrated in Fig. 5(a), (b). Such a scouring process spreads over the upper part of the downstream slope and also inside of the embankment, unless the upstream water level falls down below the critical height, and a trigonometric knotch is formed in some extent at the point of exudation. Then the thin sheet of seepage water grows on the bottom of this knotch and the water exudes at this portion (Fig. 5(c)), thus causing the piping phenomenon which enlarges the scouring action very much (Fig. 5(d), Photo. 2).

ii) **Sliding failure**  In the case of steeper downstream slope as 1:2 (Fig. 4(c)), it has been known that the sliding failure occurs along a small arc whose upper end starts from the point of exudation and lower end reaches the downstream toe. This type of failure is not the progressive one as the
scouring failure described above, but is more or less general slide along the downstream slope. Considering the whole figure of the embankment, however, this is still no more than the local failure. As the upstream water level increases even a little height and the corresponding upward movement of the point of exudation goes on, the area of sliding surface increases and the front spreads over the downstream side far from the embankment toe.

The determination of the phreatic line is one of the most important problems in the observation of the above-mentioned embankment failure. The phreatic line is generally defined as a curved line whereupon the pressure is equal to the atmospheric pressure. Therefore if we take the atmospheric pressure as the standard, pore pressures must be zero on the phreatic line. Though it is rather easy to find out the phreatic line experimentally, as in the electric analogue method or Hele-Shaw method, the distinction between the gravity flow and the capillary flow is very ambiguous in sand models, since there exists the capillary zone above the phreatic line even when the dye solution is used. The method using glass piezometric tubes does not directly determine the position of phreatic line unless the seepage flow has only a horizontal velocity component everywhere, but indicates pore pressures at the ends of tubes located upon the embankment base. Briefly speaking, the determination of the position of phreatic line can be performed by tracing locus of the points whereupon the indicating pressure shows zero magnitude, if we can fortunately stand the piezometric tubes on themselves. The necessity of such a trial determination coincides in principle with the trial treatment in the electric analogue method and the trial calculation in the relaxation method.
Then the author has adopted the measuring method of the shape of phreatic line trially, by using an L-shaped capillary tube. The capillary tube has the inner diameter of 1 mm, the stem of which is graduated. This is used after the calibration at the free water level, for the exploration of phreatic line. The determination of the exact position of phreatic line has been able to perform as follows; when the leg of tube is inserted below the phreatic line the water level in the tube rises above the zero-height of the graduated scale, and when the leg is above the phreatic line the water level falls below due to the negative pressure of capillary flow. In the following article, a proposed method of stability analysis for this kind of embankment failure is described from these experimental results.

3. Theoretical Analysis of the Mechanism of Partial Failure of Embankment Slopes

The theoretical studies ever presented for the failure of downstream slope of earth embankments due to seepage water have been performed by Bernatzik\textsuperscript{3)} and Haefeli\textsuperscript{4\textsuperscript{1}}. But they do not coincide with the actual phenomena of failure described in 2., as they have not been established after the experimental studies. The reason is that, in the former theory there exist some contradictions statically, since Bernatzik did not consider the geometric characters such as diameter or shape of the grain particle, when he discussed about the statical equilibrium of a single grain on the slope, in addition to including the factor in respect of the pressure gradient of seepage flow very ambiguously. The latter is a theory of the parallel seepage flow, on the other hand, and Haefeli did not consider the equilibrium of a single grain on the slope, but of a certain mass of soil element, which is apart from the above-mentioned behaviour of failure. Moreover the buoyancy is merely considered as the seepage pressure in

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Fig. 6 Equilibrium of a single sand grain on the surface of seepage.
the latter theory. Thus according to these theories, the stability slope becomes minimum at the toe of downstream slope, where theintersectional angle of the stream line and the slope is maximum. Therefore an approach of slope failure must begin from this portion, which contradicts the experimental results described in the preceding article. The author deduces a theoretical equation of slope failure, starting from the statical equilibrium of a single grain on the downstream slope, for the scouring failure described in 2. (3) i) from the result of his model experiments.

Let a small spheric sand grain on the surface of seepage be considered as Fig. 6, whose radius is \( r \), unit weight is \( \gamma_s \) and the unit weight of water is \( \gamma_w \). The forces acting on this small grain are as follows:

1) Own weight \( G = \frac{4}{3} \pi r^3 \gamma_s \)

2) Hydrostatic pressure (Buoyancy) \( F = \frac{4}{3} \pi r^3 \gamma_w \)

3) Seepage pressure \( P = \frac{1}{c} \pi r^3 p \)

where, \( c \) is the coefficient depending on the shape of grain and the largeness of pore opening, and \( p \) is the seepage pressure per unit area acting on the projective plane perpendicular to the stream line. Let it be taken the unit weight of the submerged sand grain as \( \gamma' = \gamma_s - \gamma_w \), the slope angle of the stream line \( \phi \), the force parallel to the stream line \( S \) and the force perpendicular to the stream line \( N \). Then,

\[
S = \frac{4}{3} \pi r^3 \sin \phi \cdot \gamma' + \frac{1}{c} \pi r^3 p,
\]

\[
N = \frac{4}{3} \pi r^3 \cos \phi \cdot \gamma'.
\]

If we take \( \phi_s \) as the angle of internal friction of submerged sand, the following equation holds:

\[
S \leq N \tan \phi_s.
\]

Putting Eq. (1) into Eq. (2), we obtain:

\[
\frac{p}{r} \leq \frac{4}{3} c \gamma' \cos \phi (\tan \phi_s - \tan \phi).
\]

\( p/r \) in Eq. (3) is the average pressure gradient of seepage flow \( \bar{I}_p \) at the adjacent part to the surface of seepage, and there exists the relationship \( \bar{I}_p = I_{\gamma w} \) with the average hydraulic gradient \( I \). Thus Eq. (3) becomes:
\[ i \leq \frac{4}{3} c \frac{r'}{r_w} \cos \phi (\tan \phi - \tan \theta). \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (4) \]

Eq. (4) gives the relationship between the slope angle of the stream line \( \phi \) and the average hydraulic gradient in the adjacent part to the downstream slope \( i \). The scouring failure of the slope cannot occur during the inequality in Eq. (4) is held. When the equality in Eq. (4) is established in result of increasing value of \( i \), the slope is at the limit of equilibrium of failure. \( i \) at this instance is called the critical hydraulic gradient \( i_c \).

The behaviour of the change of value in the right side of Eq. (4) with the angle \( \theta \) from zero to the slope angle \( \psi \) is as follows; the maximum is at the toe where \( \psi = 0 \), and decreases with increasing value of \( \phi \). On the other hand, the hydraulic gradient \( i \) in the left side of Eq. (4) is almost constant inspite of the situation, from the measurement of seepage velocity and the shape of flow-net, as described in 2. (2). Thus according to the theoretical consideration described here, it is proved that a single sand grain begins to move at the highest part of the surface of seepage (i.e. at the point of exudation), which causes the following scouring failure. The critical hydraulic gradient at this point can be written in the next equation, transforming Eq. (4):

\[ i_c^* = (i_c)_{\psi=\theta} = \frac{4c}{3} \frac{r'}{r_w} \cos \theta (\tan \phi - \tan \theta). \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (5) \]

Fig. 7 shows the correlation between \( \psi, \theta \) and \( i_c^* \) in Eq. (5).

The coefficient \( c \) in Eq. (4), as described before, depends upon the shape of grain and the largeness of pore opening, which must be determined experimentally. Let it be put \( c = 1/\alpha \beta \), where \( \alpha \) is the shape factor and \( \beta \) is the pore factor. Since \( \alpha \) is considered as the factor which reduces the pore pressure by the shape of sand grain, it can be generally assumed as \( \alpha < 1 \). On the other hand, \( \beta \) is the factor depending on the effect of the real velocity of seepage flow to the apparent one; namely if we take \( n \) as the porosity \( (0 < n < 1) \), the real velocity
is $1/n$ times larger than the apparent. Thus it becomes $\beta > 1$.

Comparing Eq. (4) with the experimental results, let it be taken $c = 0.6$. The experimental constants are as follows; $\gamma_s = 2.61 \text{ g/cm}^3$, $\gamma_w = 1 \text{ g/cm}^3$, $\varphi_s = 30^\circ 27'$ and the downstream slope of embankments is $1:2$, $1:2.5$ and $1:3$, respectively. The comparison of the critical hydraulic gradient is shown in Table 1.

<table>
<thead>
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<th>Downstream slope</th>
<th>1 : 2</th>
<th>1 : 2.5</th>
<th>1 : 3</th>
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<tr>
<td>Critical hydraulic gradient</td>
<td>Calculated</td>
<td>0.10</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>Experimental</td>
<td>0.18</td>
<td>0.23</td>
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It is found from Table 1 that the accordance is pretty good except in the case of slope of $1:2$. Therefore $c = 0.6$ can be used for the embankment in this experiment as the coefficient in Eq. (4).

If let it be assumed $c = 0.75$ and $i_c = \sin \theta$ for the seepage flow parallel to the downstream slope, we obtain from Eq. (4):

$$\tan \theta = \frac{\gamma'}{\gamma' + 1} \tan \varphi_s$$

This coincides formally with the equation which gives the stability slope represented by Bernatzik. In actual case, however, it is not correct to use Eq. (6) in the stability computation of downstream slope of embankments, as the seepage flow parallel to the slope cannot occur at the adjacent part to the surface of seepage and the coefficient $c = 0.75$ is not always adopted.

Next, the consideration must be made to the inaccuracy between the critical hydraulic gradient calculated from Eq. (4) and from the experimental results, in the case of downstream slope of $1:2$. As is described in 2. (3) ii), the local sliding has occurred in this case, in place of the scouring failure as is seen in the case of flatter slopes. Performing the stability analysis by the slip circle method as shown in Fig. 4(c) from the shape of the observed sliding surface, the factor of safety against sliding becomes 0.49. Owing to the lack of the factor of safety, it is understood that the sliding failure has occurred before the scouring failure. The factor of safety against sliding is 0.96 and 1.18 for the case of downstream slope $1:2.5$ and $1:3$, respectively.
There must be the stability consideration, therefore, whether the local failure of the downstream slope due to seepage water appears in the form of scouring or sliding.

4. Preventive Methods of Seepage Failure

It has been cleared from the experimental and the theoretical considerations described in 2. and 3., that the seepage failure of the embankment which consists of cohesionless materials occurs in the form of scouring a single grain at the point of exudation or local sliding at the surface of seepage. In order to maintain the stability of embankment against these seepage failures, the method of decreasing pore pressures due to seepage flow must be considered. The author has tried experimental studies for two methods of preventing the seepage failure as follows.

(1) Arrangement of berm on the downstream slope (Photo. 3).

There have been many controversies regarding the advantage of berm. Under the same amount of soil, the increase of the width of embankment has

![Photo. 3 Arrangement of berm on the downstream slope.](image)

the effect of prolonging the stream line of seepage water and of lowering the height of the point of exudation on the stability of embankment. On the other hand, however, the danger of sliding failure also increases due to the percolation of rainfall from the berm.

The experiments have been performed for three types shown in Fig. 8 (a), (b), (c). Fig. 8(a) corresponds to the case of setting a berm at the point of exudation of capillary seepage, Fig. 8(b) to at the point of exudation.
of gravity seepage and Fig. 8(c) to more downwards than in Fig. 8(b), on the downstream slope of 1:3. In these types of embankment, flow-nets have been entirely coincident with that of the simple slope of 1:3, and the scour-

![Flow-net diagrams](image)

Fig. 8 Arrangement of berm on the downstream slope.

...ing failure has appeared only in the case of Fig. 8(c). But in Fig. 8(b), the capillary water has appeared once above the berm, and intruded again into the embankment from the berm. At this state, even if the upstream water level rises a little, the failure comes on in the state of Fig. 8(c). In conclusion the berm should be set at the height of downstream slope high enough to the capillary rise of seepage water, after drawing the flow-net of simple slope. It must be noted, however, that since the capillary rise is almost equal both in the small model embankment such as in Fig. 8 and in the large actual one for the same embankment material, the capillary flow in the latter can almost be negligible in the stability computation.

(2) Arrangement of filter-drain at the downstream toe (Photo. 4).

The arrangement of filter-drain at the downstream toe of embankment has the effect that the stream line does not intersect to the downstream slope. The larger the capacity of filter-drain is, the more effect comes on. The capacity of filter-drain, however, cannot be so large, because the function of
embankments such as earth dams and river levees consists in water-proofing. Therefore it must be determined by the comparative relation between the permissible seepage amount and the stability of earth embankments. If the grain size of sand or gravel used in the filter-drain is not suitable, the permeability decreases by using for long period, until the function becomes insufficient comparing with that just after construction. As the relationship of grain size between the embankment material and the filter-drain has been already obtained experimentally, the author has performed his experiments for the capacity of filter-drain, the amount of seepage water and the stability of embankments. The permeability of coarse sand used as the filter-drain is 1.56 cm/sec, the angle of internal friction is 36°0' and the grain-size distribution is plotted in Fig. 1.
First, the behaviour of slope failure due to seepage flow has been observed for two types of embankment having the downstream slope of 1:2.5, shown in Fig. 9. In Fig. 9(a), owing to the insufficiency of the capacity of filter-drain, the failure has occurred at the point of exudation, when the upstream water level has reached 30 cm. On the other hand in Fig. 9(b), the embankment has kept its original stability even when the water level has reached as high as 34 cm, which is almost full storage for the embankment having the height of 35 cm, as the phreatic line has not intersected to the downstream slope. It has been cleared by this experiment that there exists very close relationship between the capacity of filter-drain and the stability of embankment. Then the correlation between the amount of seepage water and the occurrence of slope failure has been examined, when the different length of filter-drain has been used, for the embankment having the upstream slope.

![Fig. 10. Correlation between the capacity of filter-drain and the amount of seepage water.](image-url)
of 1:2.5 and the downstream slope of 1:2, 1:2.5 and 1:3, respectively. Fig. 10 shows their results, whose abscissa is the ratio of the length of filter-drain to the embankment base \( l/L \), whose ordinate is the amount of seepage water \( Q \), and whose parameter is the ratio of the water level at the upstream slope to the embankment height \( h/H \). Breaks of curves in the figure show the seepage failure owing to the insufficiency of the capacity of filter-drain. It is known from this figure that the amount of seepage water somewhat increases with the increase of \( l/L \), and if we permit the increase of the amount of seepage water as this, it is considered that the length of filter-drain is necessary to have 1/10 times that of embankment base, in order to keep the embankment safely.

From the above experiments, it has been known that the arrangement of berm at the suitable height on the downstream slope makes the embankment construction economically, and the arrangement of filter-drain at the downstream toe makes the embankment safely. By the use of these two methods at the same time, it is possible to construct the earth embankment economically and safely.

5. Earth Dam Failure Caused by Rapid-Drawdown of the Reservoir

(1) General description.

When the rapid-drawdown of the reservoir happens by any unexpected reasons, the upstream slope of earth dam becomes unstable, because of the decrease of total normal stresses and the insufficient decrease of the pore pressure that should be accompanied with that of the former. The author

Fig. 11 Cross section of Aiai-ike Dam.
has ever tried some theoretical studies with respect to this earth dam failure, introducing the residual strength represented in terms of the overall pore pressure coefficient $B$. In the following, an example of the stability analysis of actual failure recently happened is reported in some detail.

Aiai-ike Dam, which is the name of damaged earth dam for irrigation, was situated at the north-east of Himeji City in Japan. As is seen in Fig. 11, the geometry of the dam is as follows: dam height: 13.00 m, width of crest: 4.5 m, upstream slope: 1:1 (upper half) and 1:2 (lower half), downstream slope: 1:2 and length of dam: 200 m. The upper half of the upstream slope was made of the masonry facing, through joints of which the storing water could percolate into the dam, and the berm of 1 m width was arranged at the middle height of the downstream slope. Materials used in dam body were clay loam and sandy clay loam whose size distribution curves are shown in Fig. 12 (Nos. in Fig. 12 correspond to sampling situations shown in Fig. 11.). The foundation of the earth dam consisted of tertiary liparite. The
maximum amount of water storage was about 350,000 m$^3$, and the maximum discharge flowing out from the spillway, whose length was 20 m and overflowing depth was 0.7 m, was designed as 20.8 m$^3$/sec so as to drain the rainfall of 58 mm in an hour. The free-board under the crest of earth dam to that of spillway was about 2 m, and three drain holes were set with sluice gates along the upstream slope near the center of dam length for irrigation (Photo. 5).

In the early morning of July 18, 1957, the amount of rainfall reached about 50 mm for a few hour at the dam site. Three sluice gates along the upstream slope, however, were not opened considering the capacity of the spillway. As the result the water level at the upstream slope was raised, until water flowed out over the crest of spillway, showing the depth of 15~20 cm, which was the first chance since the beginning of dam operation. But soon after the full reservoir, the upper portion of the downstream slope was washed back over the length of 48 m near the dam center, which caused a complete failure of the dam (Photo. 6).

As soon as the decrease of the storage level accompanied with the failure initially occurred, the upstream slope sloughed away into the pond, leaving the width of crest of 2 m wide, over the length of 37 m in the left side of dam length (Photo. 7.). In this article the author mainly treats the analysis of this latter failure caused by the rapid-drawdown of the reservoir, though more brief discussion will be given additionally to the former due to the full
reservoir.

(2) Principles of stability analysis.

Fig. 13 shows the Mohr’s stress circles and the residual strengths at any point in the earth-embankment defined by the authors before and after drawdown of the reservoir. From this figure the position of stress circle and the

![Fig. 13 Mohr's stress circles and residual strengths in the embankment before and after drawdown.](image)

Table 2 Relation between stresses, pore pressure and residual strength in the earth embankment before and after drawdown.

<table>
<thead>
<tr>
<th></th>
<th>Before drawdown</th>
<th>Just after drawdown</th>
<th>After lapse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total stresses</td>
<td>$\sigma_{10}$, $\sigma_{30}$</td>
<td>$\sigma_{1}$, $\sigma_{3}$</td>
<td>$\sigma_{1}$, $\sigma_{3}$</td>
</tr>
<tr>
<td>Pore pressure</td>
<td>$\omega_0$</td>
<td>$\omega$</td>
<td>$\omega \rightarrow 0$</td>
</tr>
<tr>
<td>Effective stresses</td>
<td>$\sigma_{10}'$, $\sigma_{30}'$</td>
<td>$\sigma_{1}'$, $\sigma_{3}'$</td>
<td>$\sigma_{1}' \rightarrow \sigma_{1}$, $\sigma_{3}' \rightarrow \sigma_{3}$</td>
</tr>
<tr>
<td>Residual strength</td>
<td>$R_0$</td>
<td>$R (&lt; R_0)$</td>
<td>$R \rightarrow R_f$</td>
</tr>
</tbody>
</table>
residual strength just after drawdown and in the following period are cleared, referring Table 2.

As is shown in Fig. 13 and Table 2, at the instance of rapid-drawdown, it is known that the residual strength in the embankment decreases, so that the structure approaches to the dangerous state.

The intensity of the residual pore pressure varies with the geometry of the embankment, physical properties of the material and the degree of compaction. According to Skempton-Bishop, it is possible to express these factors by a unique term, defined as the overall pore pressure coefficient $\bar{B}$. Thus the residual pore pressure just after rapid-drawdown is given in the following form referring Fig. 14:

$$w = \gamma_w(h_0 + h_w(1 - \bar{B}) - h'). \quad (7)$$

For a typical element of the upstream portion of an earth dam, drawdown of the reservoir results in a decrease in both major and minor total principal stresses, accompanied by an increase in deviator stress. The pore pressure change under undrained conditions can therefore be obtained only from a test which reproduces these stress changes, in order to evaluate the magnitude of $\bar{B}$. The experimental method to determine $\bar{B}$ quantitatively has been shown precisely by Bishop-Henkel. As it needs very complex procedures with use of triaxial compression apparatus for long testing periods, the author assumes some reasonable value of $\bar{B}$, referring the test results represented by Bishop-Henkel, instead of performing the cumbersome experiments, in the following stability analysis of the earth dam under consideration.

In order to estimate the residual pore pressure after rapid-drawdown of the reservoir from Eq. (7), the potential drop $h'$ in Fig. 14 should be known at each situation in the earth embankment, drawing the flow-net of steady seepage flow at full reservoir. Using the residual pore pressure thus obtained, the stability analysis of the upstream slope can be performed. In the preceding paper the author has treated the stability problem as a progressive way, comparing the residual strength reserved in the embankment with the externally
applied forces and the internally acting pore pressures. In the present paper, however, the Swedish slip circle method is adopted in the stability analysis of Aiai-ike Dam, because of the simplicity in practical calculations.

(3) Physical properties of dam material.

The physical characters necessary for the stability computation of Aiai-ike Dam have been determined by the laboratory tests, such as specific gravity, Atterberg limits and mechanical analysis, using samples taken from the marked points in Fig. 11 at the dam site. The result is shown in Table 3 and Fig. 12.

<table>
<thead>
<tr>
<th>Table 3 Physical properties of dam material.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
</tr>
<tr>
<td>No. 1</td>
</tr>
<tr>
<td>No. 2</td>
</tr>
<tr>
<td>No. 3</td>
</tr>
<tr>
<td>No. 4</td>
</tr>
</tbody>
</table>

From these table and figure, it can be assumed that the earth dam was composed of almost homogeneous materials; this assumption much simplifies the stability computation.

A series of triaxial compression tests has been performed using specimens of 3.5 cm diameter and 8 cm height to determine the strength constants of the dam material. The specimens have been compacted in the mould so as to have the void ratios about 0.65 (corresponding to that of actual dam) before the triaxial tests, and then sheared under the consolidated-undrained conditions. The result of tests is as follows; the angle of internal friction \( \phi_r = 18^\circ 0' \) and the cohesion \( C = 0.1 \text{ kg/cm}^2 \).

(4) Stability computation.

It is supposed that the seepage flow through Aiai-ike Dam when the collapse occurred was almost near the steady state, according to the report spoken by witnesses that the first signs of seepage failure appeared at the upper portion of downstream slope. Hence the flow-net of steady seepage at full reservoir can be drawn using the Casagrande’s graphical method, from which the upstream portion is enlarged as shown in Fig. 15. In this figure, a circular segment upon the actual slide surface observed at the failed dam is
divided into eight sluices with vertical faces.

The factor of safety against sliding along the plane shown in Fig. 15 is computed for three cases; (a) at full reservoir, (b) just after partial drawdown to the junction of upstream slope and (c) just after integral drawdown to the upstream toe. The overall pore pressure coefficient \( B \) just after drawdown is adopted \( B = 1.0 \) (most conservative value), 1.1 and 1.2, referring to the experimental results performed by Bishop-Henkel. The result is tabulated in Table 4.

As an example for another type of sliding is considered such a case shown in the table.

**Table 4** Factor of safety against sliding of upstream slope.

<table>
<thead>
<tr>
<th>( B )</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.50</td>
<td>1.15</td>
<td>0.87</td>
</tr>
<tr>
<td>1.1</td>
<td>1.50</td>
<td>1.16</td>
<td>0.91</td>
</tr>
<tr>
<td>1.2</td>
<td>1.50</td>
<td>1.17</td>
<td>0.98</td>
</tr>
</tbody>
</table>

![Fig. 15 Stability analysis of upstream slope for rapid-drawdown (1).](image)

![Fig. 16 Stability analysis of upstream slope for rapid-drawdown (2).](image)
in Fig. 16, i.e. the toe of slide surface reaches the junction of upstream slope when the water surface falls to this level. The factor of safety against sliding in this case is 0.87, adopting $B=1.0$.

Above stability computation shows, in either cases, that the secondary failure due to rapid-drawdown of the reservoir accompanied with the primary seepage failure was caused by the existence of high residual pore pressure adjacent to the upstream slope of earth embankment. On the other hand, the seepage failure at the downstream slope, which was the essential cause of the earth dam failure, has been analysed by the slip circle method described in 2. (3) ii), as shown in Fig. 17. The minimum factor of safety in this case becomes equal to 0.95, from which the cause of this failure is concluded as the insufficiency of embankment section in its original design.

![Fig. 17 Stability analysis of downstream slope for full reservoir.](image)

### 6. Conclusion

The conventional design of water-retaining embankment has depended on the experiences in many cases. Even when the influence of seepage water has been considered, the theories have not necessarily been in accordance with the actual phenomena of failure, owing to performing somewhat abstract calculation.

In this paper, many experimental studies performed by the author using sand models are described, and it has been cleared that the failure at the downstream slope of embankments appears in the form of local failure, which is distinguished into the scouring failure and the sliding failure, according to the steepness of the downstream slope. From the above result of experiments, the author has deduced a theoretical equation which gives the critical hydraulic
gradient for the local failure of downstream slope. Then as the preventive methods of these seepage failure, the experimental investigation has been succeeded against the arrangement of berm or filter-drain on the downstream slope, and a reasonable criterion on design of these arrangement has been obtained. For the embankment composed of cohesive materials, it is supposed that the mechanism of failure does not become local failure described here, but appears as general one. Synthetic stability analysis of those embankments will be studied on the other opportunity.

Besides, an earth dam failure caused by rapid-dawdown of the reservoir has been analysed in this paper, throughout the site investigations and the laboratory tests, and it has been known that the failure was caused by the existence of high residual pore pressure adjacent to the upstream slope of earth embankment.

Acknowledgments

The author earnestly wishes to express his appreciation to Profs. Katsumasa Yano and Sakurō Murayama for their constant instructions in performing this study. Thanks are also due to Mr. T. Jin-nouchi by whom the experiments and the computations were conducted under the author’s supervision.

References

Appendix : Character of Non-Steady Seepage Flow through Levees

The character of non-steady seepage flow through homogeneous embankments has been previously studied by Schmied*. His experiments were performed by measuring the change of water level in the piezometers standing at the base of embankments, during the external water level was gradually increasing. In the analysis of these experimental results, he assumed that the seepage flow followed Darcy's law and that a constant hydraulic gradient was held in a vertical plane. Thus he obtained:

\[ x = \left( \frac{h^2 - y^2}{\sqrt{h^2 - y^2}} \right) \sqrt{3/4} \left( \frac{a}{\mu} \right) t, \]

where \( x \) : horizontal distance from the upstream toe, \( y \) : water level in the piezometric tube at \( x \), \( h \) : external water level, \( t \) : percolating time, \( a \) and \( \mu \) : constants varying with embankment materials.

The results of his experiments show:

1. the amount of seepage water is proportional to the external water level and is inversely proportional to the width of embankment base, and
2. the time required till the seepage water reaches the downstream slope is proportional to the square of the width of embankment base and is inversely proportional to the external water level.

The time required till the seepage water reaches the downstream slope is one of the most essential factors which determine the factor of safety of the embankment, since the seepage failure begins after complete saturation.

The character of non-steady seepage flow through levees when the external water level rises or falls corresponding to the flood curve in the river is so complex that any satisfactory analytical investigation has not yet been established. The author has recently experimented for this problem using the same apparatus and embankment material as described in this paper. The shape of levee has the height: 40 cm, the width of crest: 27.5 cm and the both slope: 1:2.5 and the initial moisture conditions are dry or wet. The types of the flood curve are as follows; the duration period of flood is 1 hr, 2.5 hr and 5 hr and the time ratio of rise to fall of flood is 1:3 and 1:5 in trigonometric form, respectively.

* I. Schmied : Die Wasserbewegung in Dammkörpern, Julius Springer, Berlin (1928)
As an example, Fig. 18(a), (b) shows the position of seepage front in the case of flood duration of 1 hr and the time ratio of 1:3, for the dry or moist levee, respectively. Fig. 19 also shows the behaviour of the seepage front going along the base of dry levee, for the time ratio of 1:3 respecting three flood duration periods. The dotted line in this figure gives the position of seepage front when the external water level just reaches its maximum height. From this figure, it is known that the seepage front moves with almost constant speed until the external water level reaches its maximum height, and after then the velocity decreases parabolically as the external water level gradually falls. In the case of short flood duration, therefore, the seepage front sometimes does not reach the downstream slope of embankments. In the case of long flood duration, on the other hand, the speed of moving front is extremely accelerated near the downstream slope. This is considered because of the high suction force which is peculiar to the
preceding capillary water. As is seen from Fig. 18, the moving speed of
seepage front is pretty greater in the moist state than in the dry state. In
closing, it can be supposed that if a flood comes after the long-continued
rainfall which submerges the levee adequately moist, there exists the possibility
of occurring the non-steady seepage flow in the moist embankment, which
makes the movement of seepage front more speedy.
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