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ON THE THERMOELASTICITY IN THE SEMI-INFINITE ELASTIC SOLID

BY

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(Communicated by Prof. K. Sassa)

Abstract

When the spheroidal or spherical region of material of α larger than that of the surroundings, which is embedded in a semi-infinite elastic body, is heated, there appears the thermal stress. The displacements at the free boundary and the stresses round the thermal origin in such a problem of thermal elasticity are obtained, introducing the displacement function ϕ, and transforming the equations of equilibrium so that the results of potential theory and theory of centres of dilatation may be applied. Thus, the state of the thermal origin is estimated from the observed deformation of the free surface. For example, the dimension of the magmatic reservoir at the Volcano Aso is estimated at ca. 1 km from the observed crustal movement which may result from its expansion and contraction.

Nomenclature

The following nomenclature is used in the paper :

u, v, w : cartesian components of displacement
e_{ij} : strain (i, $j=x$, y, z)
e : dilatation
E : Young's modulus
μ : rigidity
σ : Poission's ratio
T_{ij} : stress (i, $j=x$, y, z)
ϕ : displacement function
α : coefficient of linear thermal expansion
α_i : coefficient of linear thermal expansion inside thermal region
α_e : coefficient of linear thermal expansion outside thermal region
T : change of temperature

θ_1 : inclination of spheroidal thermal origin.

§1.

When the temperature in an elastic body is not uniform, or the temperature in the elastic body of the non-uniform distribution of coefficient of thermal expansion changes uniformly, there appears a state of stress. Such a stress is called the thermal stress. In the case of the inclusion of material of the coefficient of thermal expansion larger than that of the surroundings in the earth's crust, heated by the convective currents of magma through the fissures, there appear the thermal stresses as the results of the increases of temperature and coefficient of thermal expansion, and we observe the deformation at the earth's surface. Particularly, if the temperature of the thermal origin is a little lower than the transition point of the heated material, the increase of coefficient of thermal expansion is remarkable. For example, the coefficient of thermal expansion of quartz increases rapidly near 573°C of α-β transition as shown in Fig. 1[1].

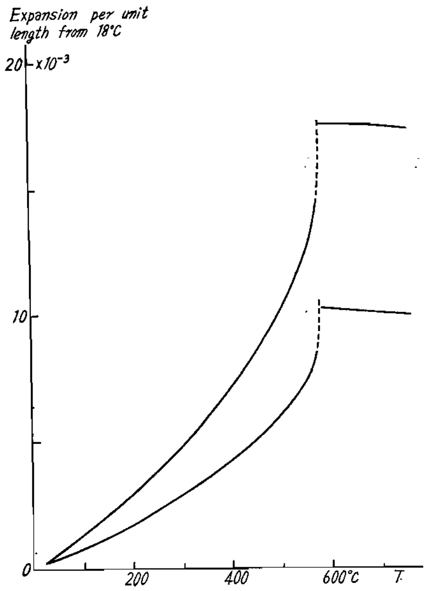


Fig. 1 Thermal expansion of quartz (after Jay).

In this paper, we calculate the thermal stresses round the thermal origin and the surface displacements which result from the increases of temperature and coefficient of thermal expansion in the spheroidal (or spherical) thermal origin of α_s larger than α_e in the elastically uniform semi-infinite solid.

§2

When there appears the change of temperature (x, y, z) in the infinite and free elastic solid which has the uniform elastic constants and the non-uniform linear coefficient of thermal expansion $\alpha(x, y, z)$, the free thermal expansion of every volume element is constrained partially by the surrounding material, and a state of thermal stress ensues. The difference between the actual strain and the free expansion $\int_0^T \alpha dT$ is related to the stress through

Hook's law. As the changes of elastic constants which result from that of temperature are very small, they can be neglected, but as that of coefficient of thermal expansion is fairly large, it is considered. As a uniform change of temperature of a small volume element does not create any angular distortion of the element, the shear stresses are unaffected by the term $\int_0^T \alpha dT$, that is, $T_{xy}=2\mu e_{xy}, \ldots\ldots$ However, the normal stresses are determined by the following equations.

$$T_{xx}=2\mu\left[\frac{\sigma}{1-2\sigma}e+e_{xx}-\frac{1+\sigma}{1-2\sigma}\int_0^T \alpha dT\right] \tag{1}$$

cyclic.

Then, the three equations of equilibrium take the form.

$$\frac{\partial e}{\partial x}+(1-2\sigma)\nabla^2 u=2(1+\sigma)\frac{\partial}{\partial x}\int_0^T \alpha dT \tag{2}$$

cyclic.

In order to solute the equation (2), the displacement function ϕ is introduced[2)]

$$\frac{\partial\phi}{\partial x}=u, \quad \frac{\partial\phi}{\partial y}=v, \quad \frac{\partial\phi}{\partial z}=w.$$

The equation (2) now can be written

$$\frac{\partial}{\partial x}\left[(1-\sigma)\nabla^2\phi-(1+\sigma)\int_0^T \alpha dT\right]=0.$$

Then equation (2) is all satisfied when

$$\nabla^2\phi=\frac{1+\sigma}{1-\sigma}\int_0^T \alpha dT \tag{3}$$

As the state of stress represented by this function ϕ ordinarily requires certain surface tractions at the boundary of solid, by means of the principle of superposition, a complementary stress function must be determined so as to satisfy the boundary conditions. This is only a problem of given boundary tractions in the ordinary theory of elasticity.

By means of equation (3), equation (1) can be written in the form

$$T_{xx}=2\mu\left[\frac{\partial^2\phi}{\partial x^2}-\frac{1+\sigma}{1-\sigma}\int_0^T \alpha d\,T\right]. \tag{4}$$

Equation (3) is of the same form as Poisson equation $\nabla^2 V=-4\pi\rho$ in the potential theory and a particular integral is given by the Newtonian potential of a distribution of material of the density $-(1+\sigma)\int_0^T \alpha dT/4\pi(1-\sigma)$, namely,[3]

$$\phi=-\frac{1+\sigma}{4\pi(1-\sigma)}\int\frac{\left[\int_0^T\alpha dT\right]}{r'}d\xi d\eta d\zeta \tag{5}$$

$$r'^2=(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2.$$

This potential ϕ represents the complete solution for the infinite solid when $T=0$ outside the heated origin.

As the general equation (2) implies $\frac{\partial T}{\partial x}$, the validity of any solution at a surface of temperature distribution requires examinations. However its validity is shown by potential theory.[2]

In order to relate the above solution to the ordinary elastic theory, the nucleus of thermoelastic strain is defined as follows. The formula (5) and the definition of ϕ shows that if a change of temperature of volume element $d\tau$ in the infinite body is, that of the remainder being zero, the displacement is the gradient of

$$-\frac{(1+\sigma)d\tau}{4\pi(1-\sigma)r'}\int_0^T\alpha dT$$

This is simply the singularity known in the ordinary theory of elasticity as the centre of dilatation[4], and (6) may be called its strength.

$$\frac{S}{4\pi}=\frac{1+\sigma}{4\pi(1-\sigma)}\int_0^T\alpha dT \tag{6}$$

Namely, the effect of heating is the same as that of a distribution of centres of dilatation of this strength $S/4\pi$ in an unheated body.

§3

Then we can obtain the stress distribution of a heated and bounded elastic body, if the formula for the same distribution $S(\xi,\ \eta,\ \zeta)/4\pi$ of centres of dilatation within the boundary is known.

In order to solute the problem of the distribution of centres of dilatation corresponding to that of the rise of temperature in the spheroidal (or spherical) region of a_t embedded in semi-infinite elastic body, firstly, we require the solution (displacement) (7) for the centre of dilatation of the strength $S/4\pi$[5]

$$\boldsymbol{u}=-\frac{S}{4\pi}\left[\nabla\left(\frac{1}{R_1}\right)+\nabla_2\left(\frac{1}{R_2}\right)\right]$$

$$\nabla_2=(3-4\sigma)\nabla+2\nabla z\frac{\partial}{\partial z}-4(1-\sigma)\boldsymbol{k}\nabla^2 z \tag{7}$$

$$R_1^2=(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2,\ R_2^2=(x-\xi)^2+(y-\eta)^2+(z+\zeta)^2,$$

, where the $x-y$ coordinate axes are laid on the free surface and z-axis is directed downward. If $S(\xi,\ \eta,\ \zeta)$ distributes in the region V_1 in $z\geqslant 0$,

$$\begin{aligned} u&=\int_{V_1}u_0 d\tau=-\frac{1}{4\pi}\left(\nabla\int_{V_1}\frac{S}{R_1}d\tau+\nabla_2\int_{V_1}\frac{S}{R_2}d\tau\right)\\ &=-\frac{1}{4\pi}\left(\nabla\int_{V_1}\frac{S}{R_1}d\tau+\nabla_2\int_{V_2}\frac{S}{R_1}d\tau\right)=\nabla\phi_1+\nabla_2\phi_2 \end{aligned} \tag{8}$$

$$\phi_1=-\frac{1}{4\pi}\int_{V_1}\frac{S}{R_1}d\tau \qquad \phi_2=-\frac{1}{4\pi}\int_{V_2}\frac{S}{R_1}d\tau\ ,$$

, or writing down their conponents

$$\begin{aligned} u&=\frac{\partial\phi_1}{\partial x}+(3-4\sigma)\frac{\partial\phi_2}{\partial x}+2z\frac{\partial^2\phi_2}{\partial x\partial z}\\ v&=\frac{\partial\phi_1}{\partial y}+(3-4\sigma)\frac{\partial\phi_2}{\partial y}+2z\frac{\partial^2\phi_2}{\partial y\partial z}\\ w&=\frac{\partial\phi_1}{\partial z}+(-3+4\sigma)\frac{\partial\phi_2}{\partial z}+2z\frac{\partial^2\phi_2}{\partial z^2} \end{aligned} \tag{9}$$

, where V_2 is the image of V_1 in the plane $z=0$ and ϕ_2 is simply the reflection transformation of ϕ_1 in the plane $z=0$.

Then, when the potentials ϕ_1 and ϕ_2 for a distribution of $S(\xi,\ \eta,\ \zeta)$ are known, we can obtain the displacements $(u,\ v,\ w)$ by means of differentiation of ϕ_1 and ϕ_2. When the spheroid and spheres are adopted as V_1,

spheroid

$$\frac{x^2\cos^2\theta_1+x(z-d)\sin 2\theta_1+(z-d)^2\sin^2\theta_1}{a^2}+\frac{y^2}{b^2}$$
$$+\frac{x^2\sin^2\theta_1-x(z-d)\sin 2\theta_1+(z-d)^2\cos^2\theta_1}{c^2}=1$$

two spheres

$$(x+b_1)^2+y^2+(z-d_1)^2=a^2, \qquad (x+b_2)^2+y^2+(z-d_2)^2=a^2, \tag{10}$$

from the known results for their potential[3], ϕ outside the thermal origin are

prolate spheroid $\qquad a>b=c$

$$\begin{aligned} \phi_i=-\frac{Sac^2}{4(a^2-c^2)}\Bigg[&\log\frac{\sqrt{a^2+q_i}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_i}}\left(2\sqrt{a^2-c^2}+\frac{y^2+Z_i-2X_i}{\sqrt{a^2-c^2}}\right)\\ &+\frac{2X_i}{\sqrt{a^2+q_i}}-\frac{(y^2+Z_i)\sqrt{a^2+q_i}}{c^2+q_i}\Bigg] \end{aligned} \tag{11}$$

$$q_t=\frac{1}{2}\left[r_t^2-a^2-c^2+\sqrt{(r_t^2+a^2-c^2)^2-4(a^2-c^2)X_t}\right],$$

oblate spheroid $\qquad a=b>c$

$$\phi_t=-\frac{Sa^2c}{4(a^2-c^2)}\left[\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_t}}\left(2\sqrt{a^2-c^2}-\frac{y^2+X_t-2Z_t}{\sqrt{a^2-c^2}}\right)\right.$$
$$\left.+\frac{(y^2+X_t)\sqrt{c^2+q_t}}{a^2+q_t}-\frac{2Z_t}{\sqrt{c^2+q_t}}\right]$$
$$q_t=\frac{1}{2}\left[r_t^2-a^2-c^2+\sqrt{(r_t^2-a^2+c^2)^2+4(a^2-c^2)Z_t}\right] \tag{12}$$

$$X_t=x^2\cos^2\theta_t+xz_t\sin2\theta_t+z_t^2\sin^2\theta_t,\qquad Z_t=x^2\sin^2\theta_t-xz_t\sin2\theta_t+z_t^2\cos^2\theta_t$$
$$r_t^2=x^2+y^2+z_t^2,\qquad z_1=z-d,\qquad z_2=z+d,\qquad \theta_1=-\theta_2.$$

two spheres

$$\phi_t=-a^3S\left(\frac{1}{3R_{t1}}+\frac{1}{3R_{t2}}\right) \tag{13}$$

$$R_{11}^2=(x+b)^2+y^2+(z-d_1)^2,\quad R_{12}^2=(x-b)^2+y^2+(z-d_2)^2,$$
$$R_{21}^2=(x+b)^2+y^2+(z+d_1)^2,\quad R_{22}^2=(x-b)^2+y^2+(z+d_2)^2.$$

Inserting (11), (12), (13) into (8), we obtain

$$\begin{aligned}u&=-\gamma[A_1+(3-4\sigma)A_2+2zA]\\ v&=-\gamma[B_1+(3-4\sigma)B_2+2zB]\\ w&=-\gamma[C_1+(-3+4\sigma)C_2+2zC]\end{aligned} \tag{14}$$

prolate spheroid

$$\gamma=\frac{Sac^2}{4(a^2-c^2)}$$
$$A_t=\frac{2x(\sin^2\theta_t-2\cos^2\theta_t)-3z_t\sin2\theta_t}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_t}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_t}}$$
$$+\frac{4x\cos^2\theta_t+2z_t\sin2\theta_t}{\sqrt{a^2+q_t}}-\frac{(2x\sin^2\theta_t-z_t\sin2\theta_t)\sqrt{a^2+q_t}}{c^2+q_t}$$
$$A=-\frac{3\sin2\theta_2}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_2}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_2}}+\frac{2\sin2\theta_2}{\sqrt{a^2+q_2}}+\frac{\sin2\theta_2\sqrt{a^2+q_2}}{c^2+q_2}$$
$$-\frac{q_{2z}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{\frac{z_2\sin2\theta_2(a^2-c^2)}{(a^2+q_2)(c^2+q_2)}-\frac{2x\cos^2\theta_2}{a^2+q_2}-\frac{2x\sin^2\theta_2}{c^2+q_2}\right\}, \tag{15}$$

$$B_t=2y\left\{\frac{1}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_t}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_t}}-\frac{\sqrt{a^2+q_t}}{c^2+q_t}\right\}$$

$$B=\frac{2yq_{2y}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)^2},$$

$$C_i=\frac{-3x\sin2\theta_i+2z_i(\cos^2\theta_i-2\sin^2\theta_i)}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_i}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_i}}$$

$$+\frac{2x\sin2\theta_i+4z_i\sin^2\theta_i}{\sqrt{a^2+q_i}}-\frac{(-x\sin2\theta_i+2z_i\cos^2\theta_i)\sqrt{a^2+q_i}}{c^2+q_i}$$

$$C=\frac{2\cos^2\theta_2-4\sin^2\theta_2}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_2}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_2}}+\frac{4\sin^2\theta_2}{\sqrt{a^2+q_2}}-\frac{2\cos^2\theta_2\sqrt{a^2+q_2}}{c^2+q_2}$$

$$-\frac{(a^2-c^2)q_{2z}}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{\frac{x\sin2\theta_2(a^2-c^2)}{(a^2+q_2)(c^2+q_2)}-\frac{2z_2\sin^2\theta_2}{a^2+q_2}-\frac{2z_2\cos^2\theta_2}{c^2+q_2}\right\}$$

oblate spheroid

$$\gamma=\frac{Sa^2c}{4(a^2-c^2)}$$

$$A_i=-\frac{2x(\cos^2\theta_i-2\sin^2\theta_i)+3z_i\sin2\theta_i}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_i}}$$

$$-\frac{4x\sin^2\theta_i-2z_i\sin2\theta'}{\sqrt{c^2+q_i}}+\frac{(2x\cos^2\theta_i+z_i\sin2\theta_i)\sqrt{c^2+q_i}}{a^2+q_i}$$

$$A=-\frac{3\sin2\theta_2}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_2}}+\frac{2\sin2\theta_2}{\sqrt{c^2+q_2}}+\frac{\sin2\theta_2\sqrt{c^2+q_2}}{a^2+q_2}$$

$$+\frac{q_{2x}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{\frac{2x\cos^2\theta_2+z_2\sin2\theta_2}{a^2+q_2}+\frac{2x\sin^2\theta_2-z_2\sin2\theta_2}{c^2+q_2}\right\}$$

$$B_i=2y\left\{-\frac{1}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_i}}+\frac{\sqrt{c^2+q_i}}{a^2+q_i}\right\}$$

$$B=-\frac{2yq_{2z}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}, \tag{16}$$

$$C_i=-\frac{3x\sin2\theta_i+2z_i(\sin^2\theta_i-2\cos^2\theta_i)}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_i}}$$

$$+\frac{2x\sin2\theta_i-4z_i\cos^2\theta_i}{\sqrt{c^2+q_i}}+\frac{(x\sin2\theta_i+2z_i\sin^2\theta_i)\sqrt{c^2+q_i}}{a^2+q_i}$$

$$C=\frac{4\cos^2\theta_2-2\sin^2\theta_2}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_2}}-\frac{4\cos^2\theta_2}{\sqrt{c^2+q_2}}+\frac{2\sin^2\theta_2\sqrt{c^2+q_2}}{a^2+q_2}$$

$$+\frac{q_{2z}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{\frac{x\sin2\theta_2+2z_2\sin^2\theta_2}{a^2+q_2}+\frac{-x\sin2\theta_2+2z_2\cos^2\theta_2}{c^2+q_2}\right\}.$$

two spheres

$$\gamma=\frac{Sa^3}{3},$$

$$A_t=-\left\{\frac{x+b}{R_{t1}^3}+\frac{x-b}{R_{t2}^3}\right\}\qquad A=3\left\{\frac{(x+b)(z+d_1)}{R_{21}^5}+\frac{(x-b)(z+d_2)}{R_{12}^5}\right\},$$

$$B_t=-y\left\{\frac{1}{R_{t1}^3}+\frac{1}{R_{t2}^3}\right\}\qquad B=3y\left\{\frac{z+d_1}{R_{21}^5}+\frac{z+d_2}{R_{22}^5}\right\},$$

$$C_1=-\left\{\frac{z-d_1}{R_{11}^3}+\frac{z-d_2}{R_{12}^3}\right\}\qquad C_2=-\left\{\frac{z+d_1}{R_{21}^3}+\frac{z+d_2}{R_{22}^3}\right\}$$

$$C=-\left\{\frac{1}{R_{21}^5}+\frac{1}{R_{22}^5}-\frac{3(z+d_1)^2}{R_{21}^5}-\frac{3(z+d_2)^2}{R_{22}^5}\right\}\tag{17}$$

, where q_{tx}, q_{ty}, q_{tz} are the derivates of q_t with respect to x, y, z.

Inserting the strains which are derived by differentiation of (14), (15), (16), (17) into (1), the stresses are as follows

$$T_{xx}=2\mu\gamma\left(D_1+(3-4\sigma)D_2+2zD-4\sigma F_2-\frac{1+\sigma}{1-2\sigma}\int_0^T adT\right)$$

$$T_{yy}=2\mu\gamma\left(E_1+(3-4\sigma)E_2+2zE-4\sigma F_2-\frac{1+\sigma}{1-2\sigma}\int_0^T adT\right)$$

$$T_{zz}=2\mu\gamma\left(F_1-F_2+2zF-\frac{1+\sigma}{1-2\sigma}\int_0^T adT\right)$$

$$T_{xy}=\mu\gamma[G_1+(3-4\sigma]G_2+2zG]$$

$$T_{yz}=\mu\gamma[H_1+H_2+2zH]$$

$$T_{zx}=\mu\gamma[I_1+I_2+2zI].\tag{18}$$

prolate spheroid

$$D_t=\frac{4\cos^2\theta_t-2\sin^2\theta_t}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_t}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_t}}-\frac{4\cos^2\theta_t}{\sqrt{a^2+q_t}}+\frac{2\sin^2\theta_t\sqrt{a^2+q_t}}{c^2+q_t}$$
$$+\frac{q_{tx}(a^2-c^2)}{\sqrt{a^2+q_t}(c^2+q_t)}\left\{\frac{(a^2-c^2)z_t\sin2\theta_t}{(a^2+q_t)(c^2+q_t)}-\frac{2x\sin^2\theta_t}{c^2+q_t}-\frac{2x\cos^2\theta_t}{a^2+q_t}\right\}$$

$$D=\frac{q_{2x}\sin2\theta_2(a^2-c^2)}{(a^2+q_2)^{3/2}(c^2+q_2)^2}-\frac{2q_{2z}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{\frac{\sin^2\theta_2}{c^2+q_2}+\frac{\cos^2\theta_2}{a^2+q_2}\right\}$$
$$+\frac{q_{2zx}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{\frac{z_2\sin2\theta_2(a^2-c^2)}{(a^2+q_2)(c^2+q_2)}-\frac{2x\sin^2\theta_2}{c^2+q_2}-\frac{2x\cos^2\theta_2}{a^2+q_2}\right\}$$
$$+\frac{q_{2x}q_{2z}(a^2-c^2)}{(a^2+q_2)^{3/2}(c^2+q_2)^2}\left\{-\frac{z_2\sin2\theta_2(a^2-c^2)(2a^2+3c^2/2+7q_2/2)}{(a^2+q_2)(c^2+q_2)}\right.$$
$$\left.+\frac{2x\sin^2\theta_2(2a^2+c^2/2+5q_2/2)}{c^2+q_2}+\frac{2x\cos^2\theta_2(a^2+3c^2/2+5q_2/2)}{a^2+q_2}\right\},$$

$$E_t=-\frac{2}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_t}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_t}}+\frac{2\sqrt{a^2+q_t}}{c^2+q_t}+\frac{2yq_{ty}(a^2-c^2)}{\sqrt{a^2+q_t}(c^2+q_t)^2}$$

$$E=-\frac{2(q_{2z}+yq_{2zy})(a^2-c^2)}{\sqrt{a^2+q_2}(a^2+q_2)^2}+\frac{yq_{2y}q_{2z}(a^2-c^2)(4a^2+c^2+5q_2)}{(a^2+q_2)^{3/2}(c^2+q_2)^3}$$

$$\begin{aligned}F_1=&\frac{4\sin^2\theta_1-2\cos^2\theta_1}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_1}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_1}}-\frac{4\sin^2\theta_1}{\sqrt{a^2+q_1}}+\frac{2\cos^2\theta_1\sqrt{a^2+q_1}}{c^2+q_1}\\&+\frac{q_{1z}(a^2-c^2)}{\sqrt{a^2+q_1}(c^2+q_1)}\left\{\frac{x\sin2\theta_1(a^2-c^2)}{(a^2+q_1)(c^2+q_1)}-\frac{2z_1\sin^2\theta_1}{a^2+q_1}-\frac{2z_1\cos^2\theta_1}{c^2+q_1}\right\}\end{aligned}$$

$$\begin{aligned}F=&-\frac{4q_{2z}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{\frac{\sin^2\theta_2}{a^2+q_2}+\frac{\cos^2\theta_2}{c^2+q_2}\right\}+\frac{q_{2zz}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\\&\times\left\{\frac{x\sin2\theta_2(a^2-c^2)}{(a^2+q_2)(c^2+q_2)}-\frac{2z_2\sin^2\theta_2}{a^2+q_2}-\frac{2z_2\cos^2\theta}{c^2+q_2}\right\}\\&+\frac{q_{2z}^2(a^2-c^2)}{(a^2+q_2)^{3/2}(c^2+q_2)^2}\left\{-\frac{x\sin2\theta_2(a^2-c^2)(2a^2+3c^2/2+7q_2/2)}{(a^2+q_2)(c^2+q_2)}\right.\\&\left.+\frac{2z_2\sin^2\theta_2(a^2+3c^2/2+5q_2/2)}{a^2+q_2}+\frac{2z_2\cos^2\theta_2(2a^2+c^2/2+5q_2/2)}{c^2+q_2}\right\}\end{aligned}$$

$$G_1=-\frac{4yq_{1x}(a^2-c^2)}{\sqrt{a^2+q_1}(c^2+q_1)^2},$$

$$G=\frac{4(a^2-c^2)y}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{-q_{2xz}+\frac{q_{2x}q_{2z}(a^2+4c^2+5q_2)}{2(a^2+q_2)(c^2+q_2)^2}\right\},$$

$$H_1=-\frac{4yq_{1z}(a^2-c^2)}{\sqrt{a^2+q_1}(c^2+q_1)^2}$$

$$H=\frac{4y(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{-q_{2zz}+\frac{q_{2z}^2(a^2+4c^2+5q_2)}{2(a^2+q_2)(c^2+q_2)^2}\right\},$$

$$\begin{aligned}I_1=&\frac{6\sin2\theta_1}{\sqrt{a^2-c^2}}\log\frac{\sqrt{a^2+q_1}+\sqrt{a^2-c^2}}{\sqrt{c^2+q_1}}-\frac{2\sin2\theta_1(a^2+2c^2+3q_1)}{\sqrt{a^2+q_1}(c^2+q_1)}\\&+\frac{2q_{1z}(a^2-c^2)}{\sqrt{a^2+q_1}(c^2+q_1)}\left\{\frac{z_1\sin2\theta_1(a^2-c^2)}{(a^2+q_1)(c^2+q_1)}-\frac{2x\sin^2\theta_1}{c^2+q_1}-\frac{2x\cos^2\theta_1}{a^2+q_1}\right\}\end{aligned}$$

$$\begin{aligned}I=&\frac{4q_{2z}\sin2\theta_2(a^2-c^2)^2}{(a^2+q_2)^{3/2}(c^2+q_2)^2}+\frac{2q_{2zz}(a^2-c^2)}{\sqrt{a^2+q_2}(c^2+q_2)}\left\{\frac{z_2\sin2\theta_2(a^2-c^2)}{(a^2+q_2)(c^2+q_2)}\right.\\&\left.-\frac{2x\cos^2\theta_2}{a^2+q_2}-\frac{2x\sin^2\theta_2}{c^2+q_2}\right\}+\frac{2q_{2z}^2(a^2-c^2)}{(a^2+q_2)^{3/2}(c^2+q_2)^2}\\&\times\left\{-\frac{z_2\sin2\theta_2(a^2-c^2)(2a^2+3c^2/2+7q_2/2)}{(a^2+q_2)(c^2+q_2)}+\frac{2x\sin^2\theta_2(2a^2+c^2/2+5q_2/2)}{c^2+q_2}\right.\\&\left.+\frac{2x\cos^2\theta_2(a^2+3c^2/2+5q_2/2)}{a^2+q_2}\right\}.\end{aligned}$$

oblate spheroid

$$\begin{aligned}D_1=&\frac{2\cos^2\theta_1-4\sin^2\theta_1}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{c^2+q_1}+\frac{4\sin^2\theta_1}{\sqrt{c^2+q_1}}-\frac{2\cos^2\theta_1\sqrt{c^2+q_1}}{a^2+q_1}\\&-\frac{q_{1x}(a^2-c^2)}{(a^2+q_1)\sqrt{c^2+q_1}}\left\{\frac{2x\cos^2\theta_1+z_1\sin2\theta_1}{a^2+q_1}+\frac{2x\sin^2\theta_1-z_1\sin2\theta_1}{c^2+q_1}\right\}\end{aligned}$$

$$D=-\frac{q_{2z}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{\frac{2\cos^2\theta_2+\sin2\theta_2}{a^2+q_2}+\frac{2\sin^2\theta_2-\sin2\theta_2}{c^2+q_2}\right\}$$

$$-\frac{q_{2xz}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{\frac{2x\cos^2\theta_2+z_2\sin2\theta_2}{a^2+q_2}+\frac{2x\sin^2\theta_2-z_2\sin2\theta_2}{c^2+q_2}\right\}$$

$$+\frac{q_{2x}q_{2z}(a^2-c^2)}{(a^2+q_2)^2(c^2+q_2)^{3/2}}\left\{\frac{(2x\cos^2\theta_2+z_2\sin2\theta_2)(a^2/2+2c^2+5q_2/2)}{a^2+q_2}\right.$$

$$\left.+\frac{(3a^2/2+c^2+5q_2/2)(2x\sin^2\theta_2-z_2\sin2\theta_2)}{c^2+q_2}\right\},$$

$$E_l=\frac{2}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_l}}-\frac{2\sqrt{c^2+q_l}}{a^2+q_l}-\frac{2yq_{ly}(a^2-c^2)}{(a^2+q_l)^2\sqrt{c^2+q_l}} \tag{20}$$

$$E=-\frac{2(q_{2z}+yq_{2zy})(a^2-c^2)}{(a^2+q_2)^2\sqrt{c^2+q_2}}+\frac{2yq_{2y}q_{2z}(a^2-c^2)(a^2/2+2c^2+5q_2/2)}{(a^2+q_2)^3(c^2+q_2)^{3/2}},$$

$$F_l=\frac{2\sin^2\theta_l-4\cos^2\theta_l}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_l}}+\frac{4\cos^2\theta_l}{\sqrt{c^2+q_l}}-\frac{2\sin^2\theta_l\sqrt{c^2+q_l}}{a^2+q_l}$$

$$-\frac{q_{lz}(a^2-c^2)}{(a^2+q_l)\sqrt{c^2+q_l}}\left\{\frac{x\sin2\theta_l+2z_l\sin^2\theta_l}{a^2+q_l}+\frac{2z_l\cos^2\theta_l-x\sin2\theta_l}{c^2+q_l}\right\}$$

$$F=-\frac{4q_{2z}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{\frac{\sin^2\theta_2}{a^2+q_2}+\frac{\cos^2\theta_2}{c^2+q_2}\right\}-\frac{q_{2zz}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}$$

$$\times\left\{\frac{x\sin2\theta_2+2z_2\sin^2\theta_2}{a^2+q_2}+\frac{-x\sin2\theta_2+2z_2\cos^2\theta_2}{c^2+q_2}\right\}+\frac{q_{2z}{}^2(a^2-c^2)}{(a^2+q_2)^2(c^2+q_2)^{3/2}}$$

$$\times\left\{\frac{x\sin2\theta_2+2z_2\sin^2\theta_2)(a^2/2+2c^2+5q_2/2)}{a^2+q_2}\right.$$

$$\left.+\frac{(-x\sin2\theta_2+2z_2\cos^2\theta_2)(3a^2/2+c^2+5q_2/2)}{c^2+q_2}\right\},$$

$$G_l=-\frac{4yq_{lx}(a^2-c^2)}{(a^2+q_l)^2\sqrt{c^2+q_l}},$$

$$G=\frac{4y(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{-q_{2xz}+\frac{q_{2x}q_{2z}(a^2/2+2c^2+5q_2/2)}{(a^2+q_2)^2(c^2+q_2)}\right\},$$

$$H_l=\frac{-4yq_{lz}(a^2-c^2)}{(a^2+q_l)^2\sqrt{c^2+q_l}},$$

$$H=\frac{4y(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{-q_{2zz}+\frac{q_{2z}{}^2(a^2/2+2c^2+5q_2/2)}{(a^2+q_2)^2(c^2+q_2)}\right\},$$

$$I_l=\frac{6\sin2\theta_l}{\sqrt{a^2-c^2}}\tan^{-1}\frac{\sqrt{a^2-c^2}}{\sqrt{c^2+q_l}}-\frac{4\sin2\theta_l(a^2+c^2/2+3q_l/2)}{(a^2+q_l)\sqrt{c^2+q_l}}$$

$$-\frac{2q_{lz}(a^2-c^2)}{(a^2+q_l)\sqrt{c^2+q_l}}\left\{\frac{2x\cos^2\theta_l+z_l\sin2\theta_l}{a^2+q_l}+\frac{2x\sin^2\theta_l-z_l\sin2\theta_l}{c^2+q_l}\right\}$$

$$I=\frac{4q_{2z}\sin2\theta_2(a^2-c^2)^2}{(a^2+q_2)^2(c^2+q_2)^{3/2}}-\frac{2q_{2zz}(a^2-c^2)}{(a^2+q_2)\sqrt{c^2+q_2}}\left\{\frac{2x\cos^2\theta_2+z_2\sin2\theta_2}{a^2+q_2}\right.$$

$$+\frac{2x\sin^2\theta_2-z_2\sin2\theta_2}{c^2+q_2}\Big\}+\frac{2q_{2z}{}^2(a^2-c^2)}{(a^2+q_2)(c^2+q_2)^{3/2}}$$

$$\times\Big\{\frac{(2x\cos^2\theta_2+z_2\sin2\theta_2)(a^2/2+2c^2+5q_2/2)}{a^2+q_2}$$

$$+\frac{(2x\sin^2\theta_2-z_2\sin2\theta_2)(3a^2/2+c^2+5q_2/2)}{c^2+q_2}\Big\}.$$

two spheres

$$D_t=\frac{1}{R_{t1}{}^3}+\frac{1}{R_{t2}{}^3}-\frac{3(x+b)^2}{R_{t1}{}^5}-\frac{3(x-b)^2}{R_{t2}{}^5}$$

$$D=-\frac{3(z+d_1)}{R_{21}{}^5}-\frac{3(z+d_2)}{R_{22}{}^5}+\frac{15(x+b)^2(z+d_1)}{R_{21}{}^7}+\frac{15(x-b)^2(z+d_2)}{R_{22}{}^7},$$

$$E_t=\frac{1}{R_{t1}{}^3}+\frac{1}{R_{t2}{}^3}-\frac{3y^2}{R_{t1}{}^5}-\frac{3y^2}{R_{t2}{}^5}$$

$$E=-\frac{3(z+d_1)}{R_{21}{}^5}-\frac{3(z+d_2)}{R_{22}{}^5}+\frac{15y^2(z+d_1)}{R_{21}{}^7}+\frac{15y^2(z+d_2)}{R_{22}{}^7}$$

$$F_1=\frac{1}{R_{11}{}^3}+\frac{1}{R_{12}{}^3}-\frac{3(z-d_1)^2}{R_{11}{}^5}-\frac{3(z-d_2)^2}{R_{12}{}^5}$$

$$F_2=\frac{1}{R_{21}{}^3}+\frac{1}{R_{22}{}^3}-\frac{3(z+d_1)^2}{R_{21}{}^5}-\frac{3(z+d_2)^2}{R_{22}{}^5}$$

$$F=-\frac{9(z+d_1)}{R_{21}{}^5}-\frac{9(z+d_2)}{R_{22}{}^5}+\frac{15(z+d_1)^3}{R_{21}{}^7}+\frac{15(z+d_2)^3}{R_{22}{}^7},$$

$$G_t=-\frac{6y(x+b)}{R_{t1}{}^5}-\frac{6y(x-b)}{R_{t2}{}^5}$$

$$G=\frac{30y(x+b)(z+d_1)}{R_{21}{}^7}+\frac{30y(x-b)(z+d_2)}{R_{22}{}^7},$$

$$H_1=-\frac{6y(z-d_1)}{R_{11}{}^5}-\frac{6y(z-d_2)}{R_{12}{}^5}\qquad H_2=-\frac{6y(z+d_1)}{R_{21}{}^5}-\frac{6y(z+d_2)}{R_{22}{}^5}$$

$$H=-\frac{6yz}{R_{21}{}^5}-\frac{6yz}{R_{22}{}^5}+\frac{30y(z+d_1)^2}{R_{21}}+\frac{30y(z+d_2)^2}{R_{22}{}^7}.$$

$$I_1=-\frac{6(x+b)(z-d_1)}{R_{11}{}^5}-\frac{6(x-b)(z-d_2)}{R_{12}{}^5}$$

$$I_2=-\frac{6(x+b)(z+d_1)}{R_{21}{}^5}-\frac{6(x-b)(z+d_2)}{R_{22}{}^5}$$

$$I=-\frac{6(x+b)}{R_{21}{}^5}-\frac{6(x-b)}{R_{22}{}^5}+\frac{30(x+b)(z+d_1)^2}{R_{21}{}^7}+\frac{30(x-b)(z+d_2)^2}{R_{22}{}^7},$$

§4

When

spheroid : $a=1$, $c=2$, $d=3$, $\theta_1=-\pi/4$, $\sigma=1/4$

two spheres : $a=1$, $b=1$, $d_1=2$, $d_2=4$, $\sigma=1/4$

as shown in Fig. 2, the results of calculations of u, v, w at the free surface for the three kinds of the above thermal origin are shown in Fig. 3~6.

From the comparision of the case of prolate spheroid with that of oblate spheroid, both their states of displacements are resemble and the ratio of their quantities is nearly equal to that of their occupying

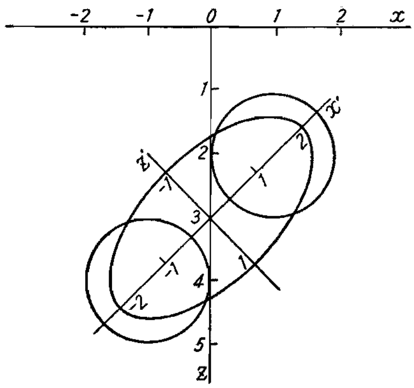


Fig. 2.

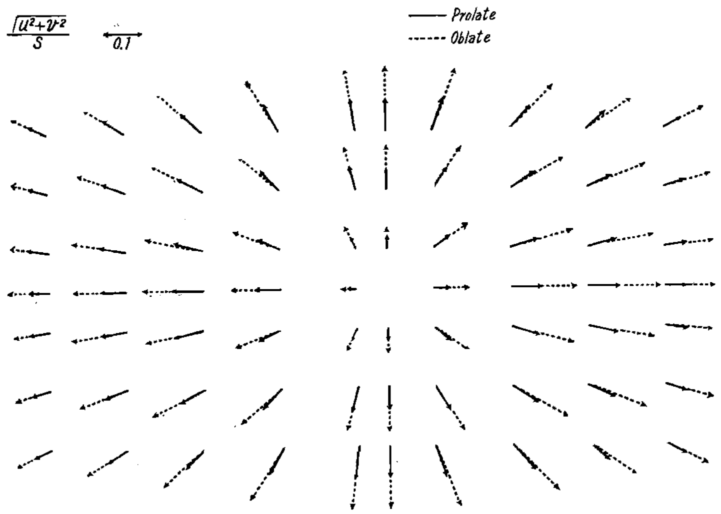


Fig. 3. Horizontal displacement for spheroid.

region 1 : 2. The ratio of the mean slope of the left side to that of the right is about 3 : 4. It may be difficult that we estimate the inclination of the thermal origin from the difference between the slopes of both sides, as it is small even at $\theta_1 = -\pi/4$. This tendency appears also in the case of two spheres, which occupy nearly the same place as the

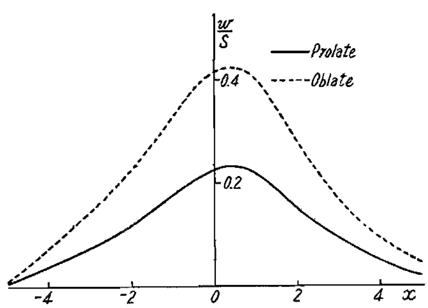


Fig. 4 Vertical dislaacement for sheroid on $y=0$.

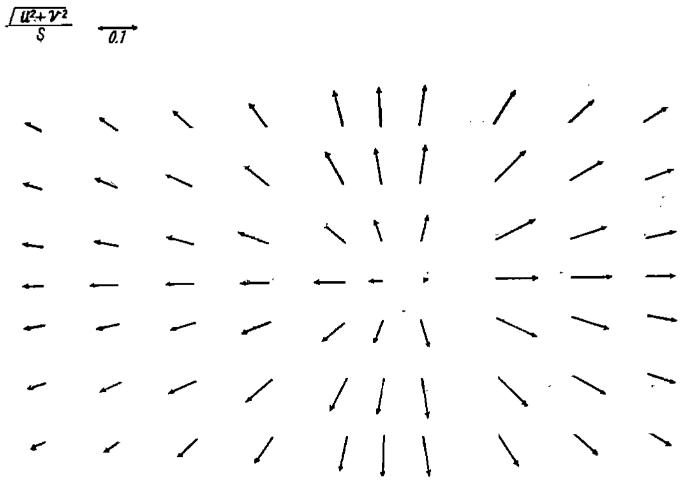


Fig. 5 Horizontal displacement for two spheres

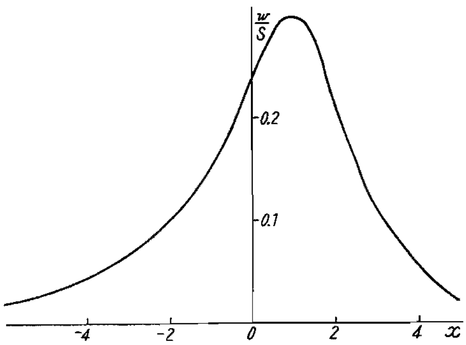


Fig. 6 Vertical displacement for two spheres.

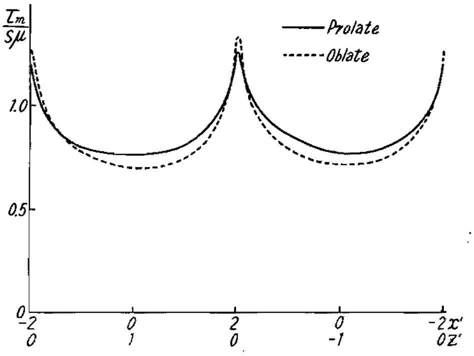


Fig. 7 τ_m for sheroid

that of spheroids.

Using the expressions for the above stresses (18), (19), (20), (21), we calculate the maximum value of shear stress

$$\tau_m = \sqrt{(T_{zz} - T_{xx})^2 + 4T_{zx}^2}/2$$

in the plane $y = 0$ along the intersection between the plane $y = 0$ and the surface of the thermal origin of the described form. These results are shown in Fig. 7~8.

Their values are maximum about the minimums of the radius of curvature as we expect. But the difference between the value near the surface and that distant from there is small. For, toward the free surface, while the principal tension increases, the principal compression decreases.

As the thermal origin is compressed by the surrounding material, the shear stress is very small within it, and then any fracture will not occur first in it.

It is evident from the expressions that the displacement and the stress are respectively in proportion to SD and $S\mu$, where D is the dimension of the thermal origin. We will estimate their values of rocks. Examples of coefficient of thermal expansion of rocks are shown in the following table.[1]

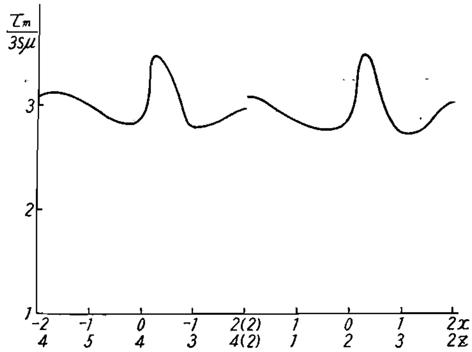


Fig. 8 τ_m for two spheres.

Table 1 Thermal Expansion of Rocks

Rock	a (ordinary temperature)
Granite	8×10^{-6} deg^{-1}
Basalt	5.4
Periclase	10
Andesites	7

While the values of coefficient of thermal expansion at high temperature (700°~1000°C) are ordinarily 2~4 times as much as the above, they decrease with pressure. But the effect of pressure is small and then those at about 700°C and 1000 atmosphere may be 2~3 times as much.[1] Taking the rise of temperature 10°C and a_{tm} $3\cdot10^{-5}$ which is the mean value in its interval, assuming that the variation of a_t is linear and that D is 1 km, SD is

$$SD=\frac{(1+\sigma)D}{1-\sigma}\int_0^T adT=\frac{5}{3}Da_{tm}T=\frac{5}{3}10^5\cdot3\cdot10^{-5}\cdot10=50$$

For example, the displacements w of prolate spheroid and oblate spheroid are ca. 11 cm and 20.5 cm.

Taking the rigidity of rock $3\cdot10^{11}$ dyne/cm^2 in the granite layer estimated from the seismic wave velocities, we obtain

$$S\mu=5\cdot10^{-4}3\cdot10^{11}=1.5\cdot10^8$$

For example, the maximum of τ_m for prolate spheroid is $2.3\cdot10^8$ dyne/cm^2.

Besides, in case that the temperature changes similarly also in the surrounding medium, we replace $a_{tm}T$ with $(a_{tm}-a_{em})T$.

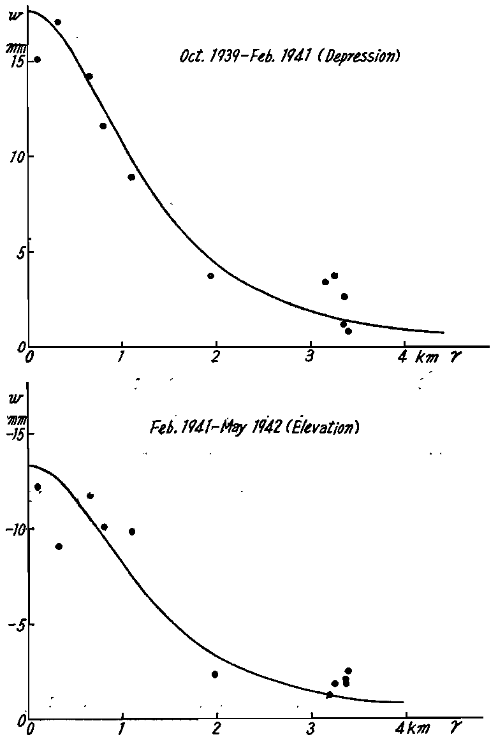
In the above discussion, the elastic constants outside and inside the thermal origin have been assumed to be the same.
But, in fact, they differ if coefficients of thermal expansion are not equal. Nevertheless, if the configurations of the thermal origin are the same, the states of deformation may be similar, in spite of the differences between the absolute quantities. Thus, we may estimate the depth of the thermal origin and etc. from the figure of deformation at the surface.

§5.

From the results of succesive levelings at Volcano Aso, the crustal movement is remarkable in the region of radius of order of 2km near the craters.[6] This fact suggests that there is the thermal origin (magmatic reservoir) under the earth's surface, corresponding to this phenomena. That the depth of the upper surface of the thermal origin is ca. 0.86 km has been estimated from the records of eruption earthquakes.[7] Now assuming the thermal origin the spherical form for simplicity, we estimate the dimension. The displacement w at the surface is

$$w=\frac{a^3S\cdot d}{3(r^2+d^2)^{3/2}} \qquad (23)$$

, where d is the depth of centre of the thermal origin. The point $=r$ where w is $1/\varepsilon$ of the maximum value at $r=0$, is



Fag. 9 Calculated curve of vertical dasplacement with cbserved results. (depth is 1.6km)

$$\frac{a^3Sd}{3(r^2+d^2)^{3/2}}\Big/\frac{a^3S}{3d^2}=\frac{1}{\varepsilon}$$

Then,

$$d = \frac{r}{\sqrt{\varepsilon^{2/3} - 1}} \tag{24}$$

Evidently, r is not related to the radius a. Taking $\varepsilon = 4$ which corresponds to the margin of the remarkable crustal movement, we obtain d = ca. 1.6 km and then estimate the radius of thermal origin at 0.7～0.8 km. The vertical displacements which are calculated by inserting d = 1.6km into (23) are shown in Fig. 9 with the observed results. Although the above inference is ambiguous in some respects, it may be said that the diameter is the order of 1km.

The writer wishes to express his hearty thanks Prof. K. Sassa for his instructions.

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