

DISASTER PREVENTION RESEARCH INSTITUTE

BULLETIN No. 33

MARCH, 1960

BASIC STUDIES ON HYDRAULIC PERFORMANCES  
OF OVERFLOW SPILLWAYS AND  
DIVERSION WEIRS

BY

TOJIRO ISHIHARA, YOSHIAKI IWASA AND KAZUNE IHDA



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# Basic Studies on Hydraulic Performances of Overflow Spillways and Diversion Weirs

by

Tojiro ISHIHARA, Yoshiaki IWASA and Kazune IHDA

## Synopsis

This paper describes the theoretical characteristics of round crested weirs which are one of controlling devices for released discharge from a reservoir or a main stream and the verification to the theory by the experimental research, as the basic study on the hydraulic performances of spillways and other similar structures observed in hydraulic works. The head-discharge relationship of a round crested weir is theoretically estimated through the hydraulic characteristics of control section mathematically obtained by the geometric properties of the basic dynamic equation, and the theory described herein can also be applied to the control structures of overflow spillways. The experimental verification indicates a good agreement with the theoretical evaluation on hydraulic performances of the weir. Some contributions to design procedures for controlling structures are also presented.

## 1. Introductory Statement

Almost all hydraulic works in the gravity projects involve many controlling structures for the flow. Overflow spillways, one of typical examples, provide the controlled release of surplus water in excess of the reservoir capacity. Diversion weirs and side channel spillways also function to discharge the regulated flow from a reservoir or a main stream to a branch channel. The importance of a proper design of these structures is not overemphasized. Improperly designed controlling structures can not serve their own function to the hydraulic purposes originally predicted. Furthermore, many failures of these structures and their appurtenances result from the improper design.

The analysis on the hydraulic performances of controlling structures thus becomes most fundamental and important to make adequate design of hydraulic structures like spillways and diversion works, though the general

scheme of structural design is always influenced by the economical, geological and topographical requirements.

Apart from other requirements occurred in practical designs, the problems will be restricted only to the hydraulic elements. The basic hydraulic requirements for structural design are 1) to correctly estimate the rate of released water with a very simple method and furthermore to increase the discharge efficiency of structure, and 2) to minimize the damage of structure itself and the future maintenance cost. Apparently, the former requirement is closely related to the flow behaviours which are the main subject to the present study and the latter to the pressure distribution along the solid boundary. As in usual design procedures of overflow spillways and similar structures, the shape of the solid boundary is commonly conformed to that of the lower nappe of free flow of a sharp crested weir, which evidently satisfies the pressure requirement. Although a large number of extensive studies have been made by many hydraulic engineers<sup>1)</sup> and institutions, with the advances in hydraulic research, the complete analytical conclusion on the shape of the nappe of stream flow has not yet been obtained, as it is not subject to exact mathematical description for the physics of weir flow. In some studies, it is treated as the projectile problem of a fluid particle. However, no reliable conclusion is not derived. For many years, therefore, the hydraulic performances of the controlling structures such as the establishment of relationship between head and discharge and the pressure evaluation along the solid boundary have been exclusively confirmed by the experimental investigations of model study in the preliminary design stage, and many empirical relations were obtained for particular shapes of boundary geometry. The unified analysis and the general treatment are not established.

On the other hand, when the solid boundary is of type of a round crested weir, the hydraulic behaviours of the flow will be expressible in mathematical form, because the flow is carried along the solid boundary of a weir, so that the performance of the structures to the released water is also hydraulically established. The round crested weir will be defined as a type of weir with solid and curved boundary which may guide the flow. As a special type of the weir, the circular weir with a constant boundary in radius is the simplest example. If the curved section of the weir is connected by a long discharge carrier, the hydraulic behaviours of flow will indicate those over a spillway, and in this case, the curved section is practically

called as a control structure of spillway elements, as it will surely become a channel control for all rates of released discharge from a reservoir, if not submerged by the elevation of tail-water. The hydraulics of round crested weir, therefore, is considered as a basis of hydraulic characteristics in control structures of spillways and diversion works.

Although the original analysis had been already seen in the work of J. Boussinesq<sup>2)</sup>, the analytical treatment for hydraulics of curved flow over a round crested weir was initiated by C. Jaeger<sup>3)</sup> in 1939. With the use of the velocity distribution in curved potential flow experimentally obtained by C. Fawer<sup>4)</sup>, the head-discharge relationship as one of the most significant element for hydraulic performances of control structures was obtained by a parametric expression of the curvature of solid boundary to the total upstream head and confirmed by experimental data obtained at universities of Darmstadt, Lausanne, Munich and others. The basic concept of Jaeger's analysis is essentially derived by the critical depth theory of Bélanger and Böss or the generalized theory for critical regime of curved flow. Without a rigorous mathematical proof, he used the critical depth theory for this case of the flow over a round weir. Of course, the control section will be observed in the flow, but the proof that the weir serves as a control structure for all rates of released discharges is particularly necessary.

In the present discussion of analytical treatment in hydraulics of a round crested weir, as one of basic forms of controlling structures, the hydraulic characteristics of flows over a round crested weir are first concerned. Theoretical analysis starts from the potential flow theory as the engineering approximation. This approximation will be confirmed by the systematic experimental investigations conducted at the Hydraulics Laboratory, Engineering Research Institute, Kyoto University. It is also verified that the weir serves as a control device for the flow and therefore the head-discharge relationship is uniquely determined, if the channel geometry in shape of boundary is given. Other hydraulic characteristics are then evaluated by computing the basic equation up- and downstream from the control section. The experimental verification to the theoretical analysis of flows over a circular weir is also described. Furthermore, the theory treated herein is approximately used to the hydraulic performances of overflow spillways, and some real contributions to give useful information for design procedures will be described, when the hydraulic structures are planned.

## 2. Theoretical Analysis of Flows over Round Crested Weir

### (1) Basic hydraulic characteristics of flows over round crested weir

The length of curved solid boundary is commonly very short, and thus the flow behaviours are rapidly changed within a short distance, so that the ignorance of shear influence and the assumption that the flow is irrotational will be possible in engineering approximation. The theoretical analysis starts from the assumption on irrotationality of flow and negligence of surface resistance.

Taking the  $x$ -axis along the boundary and the  $y$ -axis normal from the boundary, the irrotational water flow under the action of gravity is expressed by the following equations of motion, continuity and irrotationality<sup>6)</sup>.

$$u \frac{\partial u}{\partial x} + \left(1 + \frac{y}{R}\right) v \frac{\partial u}{\partial y} + \frac{uv}{R} = g \sin \theta \left(1 + \frac{y}{R}\right) - \frac{1}{\rho} \frac{\partial p}{\partial x}, \quad (1)$$

$$u \frac{\partial v}{\partial x} + \left(1 + \frac{y}{R}\right) v \frac{\partial v}{\partial y} - \frac{u^2}{R} = -g \cos \theta \left(1 + \frac{y}{R}\right) - \frac{1}{\rho} \frac{\partial p}{\partial y} \left(1 + \frac{y}{R}\right), \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial}{\partial y} \left\{ \left(1 + \frac{y}{R}\right) v \right\} = 0, \quad (3)$$

and

$$\frac{\partial v}{\partial x} - \frac{\partial}{\partial y} \left\{ \left(1 + \frac{y}{R}\right) u \right\} = 0, \quad (4)$$

in which  $u, v$  : velocity components in  $x$ - and  $y$ -directions,  $p$  : local pressure,  $R$  : local radius of curvature of boundary,  $g$  : acceleration of gravity, and  $\theta$  : local inclination angle.

In above equations, the complete solutions under given conditions can not be obtained, and therefore, some approximate treatments, which will be sufficiently made for engineering aspects, must be made.

The magnitudes of  $y$  and  $v$  are assumed small compared with those of  $x$  and  $u$ , because the coordinate system is so selected as to be consistent with the actual flow direction, and then the first approximations of Eqs. (1)~(4) become

$$u_1 \frac{\partial u_1}{\partial x} + \left(1 + \frac{y}{R}\right) v \frac{\partial u_1}{\partial y} = g \sin \theta \left(1 + \frac{y}{R}\right) - \frac{1}{\rho} \frac{\partial p_1}{\partial x}, \quad (5)$$

$$- \frac{1}{\rho} \frac{\partial p_1}{\partial y} = g \cos \theta - \frac{u_1^2}{R+y}, \quad (6)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial}{\partial y} \left\{ \left( 1 + \frac{y}{R} \right) v \right\} = 0, \quad (7)$$

and

$$\frac{\partial}{\partial y} \left\{ \left( 1 + \frac{y}{R} \right) u_1 \right\} = 0, \quad (8)$$

in which the subscript 1 indicates the value in first approximation.

The velocity distribution of  $u_1$  is then obtained, by once integrating Eq. (8) with respect to  $y$  and using the constancy of discharge throughout the whole zone of flow, as

$$u_1 = \frac{q}{\log\left(1 + \frac{h}{R}\right)} \cdot \frac{1}{R+y}, \quad (9)$$

in which  $h$  is the depth of flow. Eq. (9) indicates evidently the constancy of the product of velocity component and radius of curvature of a stream line, so that the bottom velocity  $u_{1b}$  and the surface velocity  $u_{1s}$  are

$$u_{1b} \cdot R = u_{1s}(R+h) = u_1(R+y). \quad (10)$$

The second approximation of velocity component,  $u_2$ , is obtained by the following procedure. Inserting Eq. (9) into Eq. (3), and integrating Eq. (3) with respect to  $y$ , the velocity of  $y$ -direction,  $v$ , becomes

$$\frac{v}{u_1} = \frac{R^2 \frac{d}{dx} (h/R)}{(R+h) \log\left(1 + \frac{h}{R}\right)} \cdot \log\left(1 + \frac{y}{R}\right) + \left(\frac{dR}{dx}\right) \left(\frac{y}{R+y}\right). \quad (11)$$

When the solid boundary is of constant shape in curvature, Eq. (11) becomes

$$\frac{v}{u_1} = \frac{\left(\frac{dh}{dx}\right)}{\left(1 + \frac{h}{R}\right) \log\left(1 + \frac{h}{R}\right)} \cdot \log\left(1 + \frac{y}{R}\right). \quad (12)$$

Putting Eq. (11) into Eq. (4), and again integrating with respect to  $y$ , the second approximation of  $u$  becomes

$$\begin{aligned} \frac{u_2}{u_1} = & 1 - \frac{F(x, h)}{2\left(1 + \frac{h}{R}\right)} \left[ \frac{1}{3} - \frac{\left\{ \log\left(1 + \frac{y}{R}\right) \right\}^2}{\left\{ \log\left(1 + \frac{h}{R}\right) \right\}^2} \right] \\ & + R \left( \frac{d^2 R}{dx^2} \right) \left[ \frac{\frac{h}{R} \log\left(1 + \frac{h}{R}\right)}{6\left(1 + \frac{h}{R}\right)} - \frac{\frac{h}{R}}{\left(1 + \frac{h}{R}\right) \log\left(1 + \frac{h}{R}\right)} \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{(\log R)^2}{2 \log\left(1 + \frac{h}{R}\right)} - \frac{1}{2} (\log R)^2 \log\left(1 + \frac{h}{R}\right) + \frac{1}{1 + \frac{y}{R}} \\
& + \log R \cdot \log\left(1 + \frac{y}{R}\right) - \frac{\frac{h}{R}}{2\left(1 + \frac{h}{R}\right) \log\left(1 + \frac{h}{R}\right)} \left\{ \log\left(1 + \frac{y}{R}\right) \right\}^2 \\
& + \varphi\left(\left(\frac{dR}{dx}\right)\left(\frac{dh}{dx}\right), \left(\frac{dR}{dx}\right)^2\right), \tag{13}
\end{aligned}$$

in which

$$F(x, h) = R \left(\frac{d^2h}{dx^2}\right) \log\left(1 + \frac{h}{R}\right) - \frac{\left(\frac{dh}{dx}\right)^2 \left\{2 + \log\left(1 + \frac{h}{R}\right)\right\}}{\left(1 + \frac{h}{R}\right)},$$

and

$\varphi\left(\left(\frac{dR}{dx}\right)\left(\frac{dh}{dx}\right), \left(\frac{dR}{dx}\right)^2\right)$  is the higher term depending on the square of  $(dR/dx)$  and the product of  $(dR/dx)$  and  $(dh/dx)$ . If the solid boundary of constant shape in local radius, the last two terms in Eq. (13) are evidently zero.

The pressure distribution is determined by integrating Eq. (6) with respect to  $y$  from  $y$  to  $h$  as the first approximation and by Eq. (2) as the second approximation. The first approximation of pressure distribution is

$$\frac{p_1}{\rho g} = \cos \theta (h - y) - \left(\frac{u_1 v^2 R^2}{2g}\right) \left\{ \frac{1}{(R+y)^2} - \frac{1}{(R+h)^2} \right\}. \tag{14}$$

The second term in the right indicates evidently the influence of curved flow.

Usual one-dimensional methods of hydraulic analysis are the energy and the momentum approaches for a single tube as a bulk quantity. In the present study, the analysis will be derived by the mechanical energy conservation law. When the distributions of velocity and pressure in the  $y$ -direction are obtained, the total head of the flow from the weir crest is approximately calculated, and by the use of Eqs. (9), (10) and (14), the first approximation for head is



$$H_0 = \left( \frac{1}{q} \right) \cdot \int_0^h \left( \frac{u_1^2}{2g} \right) + \left( \frac{p_1}{\rho g} \right) + y \cos \theta - z \Big\} u dy = h \cos \theta + \frac{u_1^2}{2g} - z, \quad (15)$$

in which  $z$  indicates the vertical distance from the weir crest to a point under consideration. Eq. (15) is evidently the Bernoulli equation for non-viscous flows. The surface and bottom velocities in first approximation are expressible as, in terms of the head, the depth, and the geometric quantities,

$$u_{1s} = \sqrt{2g(H_0 + z - h \cos \theta)}, \quad (16)$$

and

$$u_{1b} = \left( 1 + \frac{h}{R} \right) \sqrt{2g(H_0 + z - h \cos \theta)}. \quad (17)$$

The hydraulic characteristics of flows over a round crested weir can be determined by analyzing the geometric properties of the surface profile equation of the flow, which is derived by differentiating Eq. (15) with respect to  $x$ . The resulting equation for surface profiles of water is

$$\frac{dh}{dx} = \frac{f_1(x, h)}{f_2(x, h)}, \quad (18)$$

in which

$$f_1(x, h) = \sin \theta \left\{ 1 + h \left( \frac{d\theta}{dx} \right) \right\} + \frac{\frac{q^2}{g} \left( \frac{dR}{dx} \right) \left\{ \log \left( 1 + \frac{h}{R} \right) - \frac{h}{R} \right\}}{(R+h)^3 \left\{ \log \left( 1 + \frac{h}{R} \right) \right\}^3},$$

and

$$f_2(x, h) = \cos \theta - \frac{\frac{q^2}{g} \left\{ 1 + \log \left( 1 + \frac{h}{R} \right) \right\}}{(R+h)^3 \left\{ \log \left( 1 + \frac{h}{R} \right) \right\}^3},$$

as  $(dz/dx) = \sin \theta$ .

When a definite rate of discharge is carried through a round crested weir of definite geometry, all the hydraulic characteristics are completely described by the mathematical properties of Eq. (18). The round crested weir designed for controlling structures of flows like the discharge measurement device must be a channel control in the whole range of discharges

requested by the design purpose, so that Eq. (18) involves the singular point classified as a saddle point by the transitional behaviours of steady flows<sup>6,7)</sup>. If the saddle point is not involved, the weir will be generally classified as a channel transition in hydraulics of open channel flows, and therefore the hydraulic characteristics of the flow are influenced by up- and downstream conditions, depending on the flow regime. The discharge metering, for example, can be made by a double water-level measurement, as done in the Venturi flumes.

At a singular point, the following relationships are evidently obtained :

$$\sin \theta_c \left\{ 1 + h_c \left( \frac{d\theta}{dx} \right)_c \right\} + \frac{q^2 \left( \frac{dR}{dx} \right)_c \left\{ \log \left( 1 + \frac{h_c}{R_c} \right) - \frac{h_c}{R_c} \right\}}{(R_c + h_c)^3 \left\{ \log \left( 1 + \frac{h_c}{R_c} \right) \right\}^3} = 0, \quad (19)$$

and

$$\cos \theta_c = \frac{\frac{q^2}{g} \left\{ 1 + \log \left( 1 + \frac{h_c}{R_c} \right) \right\}}{(R_c + h_c)^3 \left\{ \log \left( 1 + \frac{h_c}{R_c} \right) \right\}^3}, \quad (20)$$

where the subscript  $c$  indicates the values at the singular point. Eq. (19) describes the curve of normal flow and Eq. (20) the critical depth curve which is also known as the theorem of Bélanger-Böss. Solutions for  $x_c$  and  $h_c$  give the hydraulic characteristics of flows at the singular point as control section in open channel hydraulics, if  $x_c$  and  $h_c$  make the singular point a saddle point, and the relationship between head and discharge is also established.

As the simplest but the most fundamental case in hydraulic performances of round crested weirs, the weir of constant curvature like a circular and parabolic weirs will be concerned.

Eq. (18) is then simplified and it becomes

$$\frac{dh}{dx} = \frac{f_1(x, h)}{f_2(x, h)}, \quad (21)$$

where

$$f_1(x, h) = \sin \theta \left( 1 + \frac{h}{R} \right),$$

and

$$f_2(x, h) = \cos \theta - \frac{\frac{q^2}{g} \left\{ 1 + \log \left( 1 + \frac{h}{R} \right) \right\}}{(R+h)^3 \left\{ \log \left( 1 + \frac{h}{R} \right) \right\}^3}.$$

Transforming the variable from  $x$  to  $\theta$ , the new equation is of similar type of Eq. (21), and it is

$$\frac{dh}{d\theta} = \frac{f_1(\theta, h)}{f_2(\theta, h)}, \quad (22)$$

in which

$$f_1(\theta, h) = \sin \theta (R+h),$$

and

$$f_2(\theta, h) = \cos \theta - \frac{\frac{q^2}{g} \left\{ 1 + \log \left( 1 + \frac{h}{R} \right) \right\}}{(R+h)^3 \left\{ \log \left( 1 + \frac{h}{R} \right) \right\}^3}.$$

Evidently, the following relations are established at the singular point :

$$\sin \theta_c = 0,$$

and

$$\cos \theta_c = \frac{\frac{q^2}{g} \left\{ 1 + \log \left( 1 + \frac{h_c}{R} \right) \right\}}{(R+h_c)^3 \left\{ \log \left( 1 + \frac{h_c}{R} \right) \right\}^3}.$$

The first expression indicates the singular point must be located at the section of the weir crest, and the head-discharge relationship is uniquely determined by the second expression, if the singular point of Eq. (22) is mathematically verified as a saddle point in the geometric theory of ordinary differential equation.

Eq. (22) can be approximately expressed by the following relation, with the use of Taylor's theorem in the immediate vicinity of the saddle point.

$$\frac{dh'}{d\theta'} = \frac{b}{c} \cdot \frac{\theta'}{h'}, \quad (23)$$

in which  $h'$  and  $\theta'$  are new variables transformed by  $h = h_c + h'$  and  $\theta = 0 + \theta'$ , and  $b$  and  $c$  are respectively

$$b = \frac{u_{18} c^4}{g q^2} \left[ 1 + \log \left( 1 + \frac{h_c}{R} \right) + \left\{ \log \left( 1 + \frac{h_c}{R} \right) \right\}^2 \right] > 0,$$

and

$$c = R + h_c > 0.$$

Referring to the description<sup>(6,7)</sup> for general behaviours of transitional characteristics of steady flows, the characteristic equation of Eq. (23) has two real roots of opposite sign. The singular point is then classified as a saddle point and the flow changes its regime from tranquil to shooting at the section of the weir crest, which is a control section. Consequently, the estimation empirically made by many hydraulic engineers through their past experiences can be verified, and the theoretical evaluation for head-discharge relationship by a round crested weir becomes possible for all rates of discharges.

Eq. (20) is evidently the expression for head-discharge relationship to the flow over a round crested weir and it is known as the Böss theorem<sup>(6,7)</sup>. By means of the simultaneity theorem of Jaeger, it also represents the Bélanger equation for maximum discharge, and the direct proof of this theorem is easily made by differentiating Eq. (15) with respect to the depth for given total head.

The theoretical analysis described in the foregoing can bring the complete clarification for the hydraulic characteristics of flows over a round crested weir. In the immediate neighbourhood of the saddle point, the critical depth curve expressed by putting the denominator of the basic dynamic equation zero becomes horizontal, whereas the normal depth curve is always vertical. In the zone enclosed by two curves of critical depth and normal flow, the sign of  $(dh/dx)$  is apparently negative. With the use of the general theory for open channel flows, all the possible surface profiles of water are expressible. Fig. 1 illustrates a family of surface profiles of water passing over a round crested weir. When the weir is a type of uncontrolled structure, the transition curve illustrated in the figure becomes a surface profile of water, and the transition from a tranquil to shooting flow takes place at the weir crest. All other curves are physically observed when other control structures like a gate and a weir are located up- and downstream. Fig. 2 illustrates schematically the

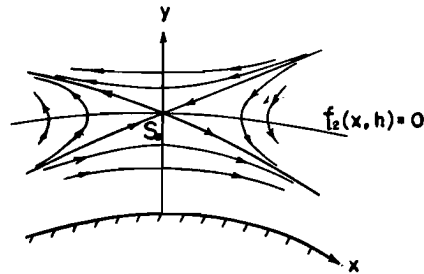


Fig. 1 All Possible Surface Profiles of Water Passing over Round Crested Weir

flow behaviours produced by other controlling structures. When the other structures like weirs and dams are located downstream, the surface profiles of water are indicated in Figs. 2-1 and 2-2. In the former figure, the flow over a round crested weir being under consideration is submerged, and the weir thus can not serve as a control, whereas in the latter case, the weir still serves as a control. However, the flow changes suddenly its regime from shooting to tranquil. On the other hand, when a gate is located upstream from the weir crest, the flow behaviours are illustrated in Figs. 2-3 and 2-4. Fig. 2-3 indicates the efflux of water from a gate,

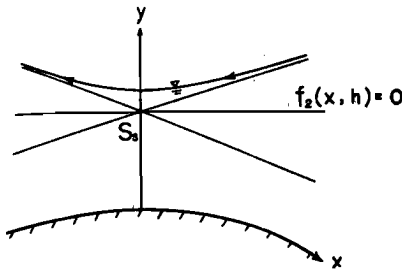


Fig. 2-1 Surface Profile of Water over Round Crested Weir, When Other Control Structure Is Constructed Downstream (Weir Becomes Transition)

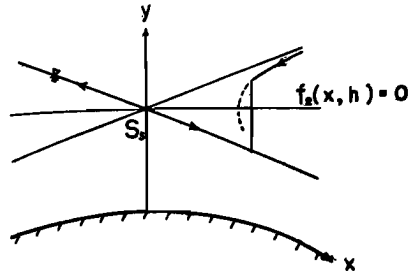


Fig. 2-2 Surface Profile of Water over Round Crested Weir, When Other Control Structure Is Constructed Downstream (Weir Is Still Control)

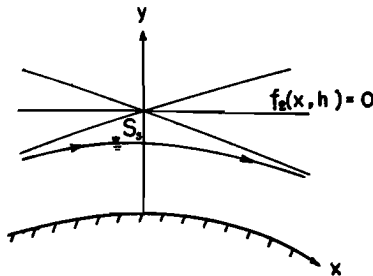


Fig. 2-3 Surface Profile of Water over Round Crested Weir, When Gate Is Upstream from Weir (Gate Becomes Control)

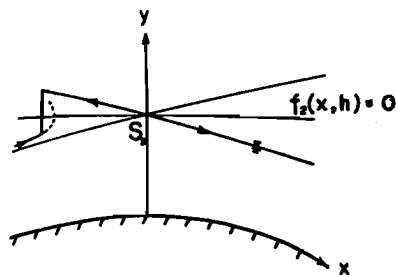


Fig. 2-4 Surface Profile of Water over Round Crested Weir, When Gate Is Upstream from Weir (Gate and Weir Become Controls)

in which the flow is shooting. In this case, the weir is not a channel control, and the gate is thus uniquely a control, because the water elevation is determined by the opening of the gate. When the gate is furthermore lifted to release more rates of discharge, the weir also becomes a

channel control, and the surface profile of water in this case is indicated in Fig. 2-4. Apparently, in this case, a hydraulic jump must be occurred upstream from the weir crest to change the flow regime from shooting to tranquil.

### (2) Head-discharge relationship of round crested weir

One of the most important relationship for hydraulic performances of control structures is to predict correctly the head-discharge relationship of the structure. This relationship is commonly described in terms of the upstream head and the depth, which is known as the weir formula. The description of weir formula is then used in two ways of

$$q = \frac{2\sqrt{2g}}{3} CH_0^{3/2} , \quad (24)$$

and

$$q = \frac{2\sqrt{2g}}{3} C_a h_0^{3/2} , \quad (25)$$

in which  $q$  : discharge per unit width,  $h_0$  : upstream depth from the weir crest, and  $C$  and  $C_a$  : discharge coefficients for total head and overflow depth, respectively.

The first treatment concerns with the hydraulic behaviours of  $C$  as functions of discharge and channel geometry. In the same manner as done by Jaeger, the following dimensionless parameters are introduced :

$$h_e = KH_0, \quad \text{and} \quad R = \lambda H_0.$$

Inserting above parameters into Eqs. (20) and (24) and eliminating  $q$  with the use of the equation of Bernoulli,  $C$  can be expressed in terms of dimensionless parameters of  $K$  and  $\lambda$ , as follows.

$$2(1-K) \left\{ 1 + \log \left( 1 + \frac{K}{\lambda} \right) \right\} = (\lambda + K) \log \left( 1 + \frac{K}{\lambda} \right) , \quad (26)$$

and

$$C = \frac{3}{2} (1-K)^{1/2} (\lambda + K) \log \left( 1 + \frac{K}{\lambda} \right) . \quad (27)$$

When the ratio of radius of curvature to head is given, the discharge coefficient can be calculated by the trial and cut method through Eqs. (26) and (27). Fig. 3 indicates the theoretical curve obtained by the present approach, as the irrotational theory, with the theoretical curve of Jaeger<sup>8)</sup>

and the empirical curve obtained at the University of Lausanne. The discharge coefficient increases rapidly with the decrease of radius of curva-

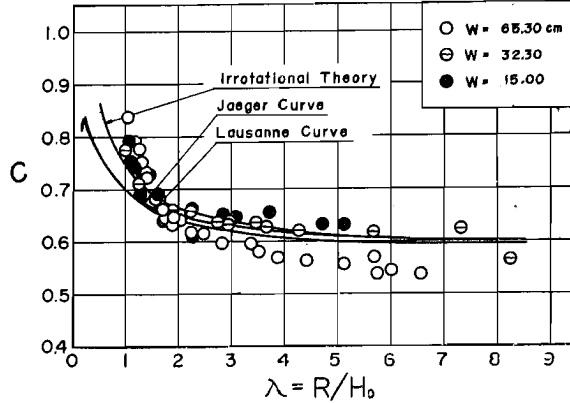


Fig. 3 Discharge Coefficient  $C$  for Total Head

ture for constant heads and the increase of upstream total head for given channel geometry, whereas it gradually tends to 0.577, when the value of  $(R/H_0)$  becomes very large, which results in that the flow is assumed parallel-streamlined, as seen in many hydraulic literatures.

The curve of Jaeger is also calculated by the use of Bélanger theorem for invicid flows, using the velocity profile experimentally obtained by C. Fawer<sup>4)</sup>, being in a form of

$$\frac{u}{u_b} = \left\{ \frac{R}{(R+2y)} \right\}^{1/2} \quad (28)$$

For large values of  $\lambda$ ,  $C$  also tends to 0.577, which is of equal value derived by the irrotational treatment, whereas for small values of  $\lambda$ ,  $C$  in the Jaeger curve becomes less than that derived herein. The Lausanne curve represents the empirical relationship obtained by the experimental data at the University of Lausanne and it is

$$C = \frac{3}{2} \left( 0.385 + \frac{0.085}{\lambda} - \frac{0.010}{\lambda^2} \right) \quad (29)$$

Eq. (29) has a maximum value of  $C$ , 0.849, at  $\lambda = 4/17$  and with the increase of head,  $C$  diminishes rapidly.

Hydraulic behaviours of the discharge coefficient for upstream depth  $C_a$  will be next concerned. When the energy loss between a point where the water-level measurement will be made and the section of weir crest is

ignored, the energy equation can be practically expressed by

$$H_0 = h_0 + \frac{q^2}{2g(W+h_0)^2}, \quad (30)$$

in which  $W$  is the height of weir from channel bed to weir crest. The relationship between  $C$  and  $C_a$  is established through Eqs. (24) and (25), which is

$$C_a = C \left( \frac{H_0}{h_0} \right)^{2/3}, \quad (31)$$

where  $(H_0/h_0)$  can be evaluated by

$$\frac{H_0}{h_0} = 1 + \frac{4}{9} C^2 \left( \frac{H_0}{h_0} \right)^3 \left( \frac{h_0}{h_0 + W} \right)^2, \quad (32)$$

so that the ratio of  $C_a$  to  $C$  is connected through the ratio of  $h_0$  to  $W$ , as follows.

$$C_a = C \left\{ 1 + \frac{4}{9} C^2 \left( \frac{h_0}{h_0 + W} \right)^2 \right\}^{1/2}. \quad (33)$$

Consequently, the discharge coefficient for upstream head is expressed as a function of  $(h_0/W)$  or  $h_0/(h_0 + W)$  for given values of the discharge coefficient for total head. However, referring to the behaviours of  $C$ ,  $C_a$  must be indicated as two parametric expression of  $(R/H_0)$  and  $(h_0/W)$  or  $h_0/(h_0 + W)$ , and therefore the graphical illustration of  $C_a$  becomes complicated compared with that of  $C$ . Of course, the total head  $H_0$  can not be directly measured, but for relatively high weirs, the head will be practically equivalent to the upstream depth. In this case, the discharge coefficient for upstream head can be illustrated by a single parametric value of  $(R/h_0)$ .

### (3) Pressure requirement for round crested weir

Another important requirement, when the round crested weir and other similar control structures are designed, is the pressure distribution along the solid boundary, though other factors related to hydraulic designs, structural vibration caused by the running water and so on, must be considered. If the local bed pressure decreases suddenly, the cavitation phenomenon will be possibly formed, and therefore the solid boundary must be constructed so smooth that the bed pressure is not lowered to the vapour pressure. The flow velocity over a round crested weir will be gradually increased after passing the weir crest, so that the pressure also will decrease.



and therefore, when the zero pressure condition is once obtained, longer solid boundary becomes inversely improper from the point view of structural maintenance. The limiting condition of zero pressure over a round crested weir is theoretically estimated by the following procedure.

At a point where the bed pressure becomes zero, the following relationship in terms of the discharge, the radius of curvature, and the depth is derived by Eq. (14).

$$2g \cos \theta \cdot R^2(R+h)^2 \left\{ \log \left( 1 + \frac{h}{R} \right) \right\}^2 = (2R+h)q^2. \quad (34)$$

Again introducing the dimensionless parameters of  $(R/H_0) = \lambda$ ,  $(h_c/H_0) = K$ ,  $(h/H_0) = K'$  and  $(z/H_0) = \int_0^\theta \lambda \sin \theta \cdot d\theta$ , eliminating  $\cos \theta$  with the use of the Bernoulli equation, the relationship of depth to total head at the section where the bed pressure becomes zero,  $K'$ , to  $\lambda$  and  $K$  is obtained. In the case of constant curvature, the resulting equation is

$$\frac{(3\lambda^2 + 3\lambda K' + K'^2)}{(\lambda + K')^2 \left\{ \log \left( 1 + \frac{K'}{\lambda} \right) \right\}^2} = \frac{9\lambda^2(1+\lambda)}{4C^2} = \frac{2\lambda^2(1+\lambda) \left\{ 1 + \log \left( 1 + \frac{K}{\lambda} \right) \right\}}{(\lambda + K)^3 \left\{ \log \left( 1 + \frac{K}{\lambda} \right) \right\}^3}. \quad (35)$$

In the above equation, the right hand term is apparently known by given channel geometry and the head over a weir crest, as already described, so

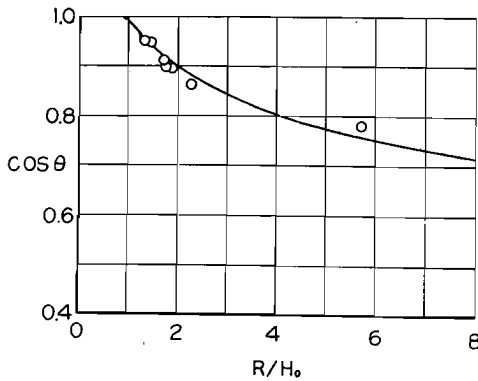


Fig. 4 Location of Zero Pressure for Round Crested Weir

that the critical value of water depth for zero pressure is explicitly calculated by given values of  $\lambda$ . Fig. 4 indicates the theoretical relationship between the local angle of inclination of solid boundary and the ratio of  $R$  to  $H_0$ . This figure gives one of the basic information to hydraulic design procedures of round crested weirs and other similar structures. Especially when

the overflow spillway is designed, the discharge carrier as one of hydraulic elements of spillways must be connected to the control structure of spillway until the critical condition for pressure requirement is attained.

### 3. Experimental Verification to Theoretical Analysis of Flow Behaviours

The research program to verify experimentally the theoretical analysis on hydraulic behaviours of curved flows has been conducted at the Hydraulics Laboratory, Engineering Research Institute, Kyoto University. The model used in the research work is a circular weir, which is 30 cm in width and 15 cm in radius. The solid boundary is made of thin brass plate. The discharge rate per unit width varies from 0 to 1000 cm<sup>3</sup>/sec/cm.

#### (1) Velocity distribution of curved flows

With the use of the model weir already described, a large number of experimental runs for the measurement of local velocities by the Pitot-tube have been made. Typical examples of test runs are listed in Tables 1 and 2. As evidently seen in the tables, it will be understood that the product of  $u$  and  $(R+y)$  is practically constant, and consequently the basic assumption for constancy of  $u(R+y)$  is also verified. This indication was also proved for all runs of experiments. Values, however, in the immediate vicinity of solid boundary are less than those in the upper zone of flow, which

Table 1 Velocity Profiles of Flows over Round Crested Weir ( $q=313.3\text{cm}^3/\text{sec}/\text{cm}$ )

Dis- tance $y$ cm	X=15.0 cm			X=20.0 cm			X=25.0 cm		
	$u_1$ cm/sec	$u$ cm/sec	$u(R+y)$	$u_1$ cm/sec	$u$ cm/sec	$u(R+y)$	$u_1$ cm/sec	$u$ cm/sec	$u(R+y)$
0.12	137.1	133.8	2023	106.6	102.5	1550	77.6	73.7	1114
0.32	135.3	132.7	2033	105.2	101.8	1560	76.6	74.5	1141
0.52	133.6	131.7	2044	103.8	101.3	1572	75.6	73.5	1141
0.72	131.9	131.0	2059	102.5	99.9	1570	74.7	73.0	1147
0.92	130.2	130.2	2073	101.2	99.6	1586	73.7	72.2	1147
1.12	128.6	129.2	2083	99.9	99.1	1597	72.8	71.3	1149
1.32	127.0	128.2	2092	98.7	98.3	1604	—	—	—
1.52	125.5	127.1	2100	97.5	97.1	1604	—	—	—
1.62	—	—	—	—	—	—	70.6	70.3	1168
1.72	124.0	125.2	2093	96.4	96.8	1618	—	—	—
1.92	122.5	124.2	2101	95.2	95.7	1619	—	—	—
2.12	121.1	123.9	2121	94.1	94.8	1623	68.6	69.3	1186
2.32	119.7	121.2	2099	93.0	94.1	1630	—	—	—
2.52	—	—	—	92.0	93.2	1633	—	—	—
2.62	—	—	—	—	—	—	66.6	66.8	1177
2.72	—	—	—	90.9	92.4	1637	—	—	—
2.92	—	—	—	89.9	92.1	1650	—	—	—
3.12	—	—	—	—	—	—	64.8	65.7	1190
3.62	—	—	—	—	—	—	63.0	65.0	1210
3.92	—	—	—	—	—	—	62.0	63.4	1199

X : distance along the solid boundary from the downstream edge. The diameter of model weir is 30 cm, and X at the weir crest is 23.56 cm.

Table 2 Velocity Profiles of Flows over Round Crested Weir ( $q=502.7 \text{ cm}^3/\text{sec}/\text{cm}$ )

Dis- tance $y$ cm	X=15.0 cm			X=17.5 cm			X=20.0 cm		
	$u_1$ cm/sec	$u$ cm/sec	$u(R+y)$	$u_1$ cm/sec	$u$ cm/sec	$u(R+y)$	$u_1$ cm/sec	$u$ cm/sec	$u(R+y)$
0.12	150.8	145.2	2195	137.9	130.6	1975	122.4	114.2	1727
0.32	148.9	143.1	2195	136.1	129.1	1978	120.8	113.7	1742
0.52	147.0	141.4	2196	134.3	129.5	2010	119.3	112.0	1739
0.72	145.1	140.7	2215	132.6	128.3	2020	117.8	111.1	1748
0.92	143.3	140.0	2230	131.0	127.2	2030	116.3	110.2	1755
1.12	141.5	139.3	2245	129.3	127.5	2055	114.9	109.8	1770
1.32	139.8	138.9	2268	127.7	124.8	2038	113.4	108.9	1778
1.52	138.1	137.9	2280	126.2	125.2	2070	112.1	108.4	1791
1.72	136.4	137.5	2300	124.7	125.2	2093	110.7	108.0	1808
1.92	134.8	136.8	2315	123.4	124.8	2110	109.4	107.5	1820
2.12	133.2	136.8	2343	121.8	124.4	2130	108.3	107.1	1834
2.32	131.7	136.1	2359	120.4	123.2	2130	106.9	107.1	1855
2.52	130.2	136.1	2383	119.0	122.4	2145	105.7	106.2	1860
2.72	128.7	134.3	2380	117.7	122.4	2170	104.5	106.2	1882
2.92	127.3	132.4	2375	116.3	121.6	2180	103.3	106.2	1903
3.12	125.9	130.6	2368	115.0	119.6	2168	102.2	105.2	1908
3.32	—	—	—	113.8	120.0	2198	101.1	104.3	1908
3.52	—	—	—	112.6	118.4	2195	100.0	104.3	1912
3.72	—	—	—	111.4	116.7	2185	—	—	—
3.92	—	—	—	—	—	—	97.9	102.9	1948
4.32	—	—	—	—	—	—	95.8	102.4	1980

results from the surface resistance of the brass plate to the flow and from the ignorance of secondary influence of vertical velocity. The latter influence can be easily explained in the following. For the circular weir being under present consideration, the velocity ratio of the first approximation to the second one is expressed as follows, by Eq. (13).

$$\frac{u_2}{u_1} = 1 - \frac{F(x, h)}{2\left(1 + \frac{h}{R}\right)} \left[ \frac{1}{3} - \frac{\left\{\log\left(1 + \frac{y}{R}\right)\right\}^2}{\left\{\log\left(1 + \frac{h}{R}\right)\right\}^2} \right], \quad (36)$$

in which  $F(x, h)$  will be possibly positive, because the second term in  $F(x, h)$ , depending on the square of surface gradient of water, is very small compared with the first term.

Putting  $y=h$ , the resulting equation indicates the velocity ratio at the free surface, which is

$$\frac{u_{2s}}{u_{1s}} = 1 + \frac{F(x, h)}{3\left(1 + \frac{h}{R}\right)}, \quad (37)$$

Apparently,  $u_{2s}$  is greater in magnitude than  $u_{1s}$ . On the other hand, at the solid boundary, Eq. (36) becomes

$$\frac{u_{2b}}{u_{1b}} = 1 - \frac{F(x, h)}{6\left(1 + \frac{h}{R}\right)}, \quad (38)$$

so that  $u_{2b} < u_{1b}$ .

Consequently, the value of  $u_1$  is smaller than that of  $u_2$  at the free surface, whereas the reverse indication is seen at the solid boundary. Experimental values listed in Tables 1 and 2 prove the above theoretical prediction.

The less validity of constancy in  $u(R+y)$  will be seen in the upstream reach far from the downstream end, and it results from the negligence of secondary influences based on the vertical velocity indicated in Eq. (11). Because the value of surface gradient of water becomes quite large in the upstream reservoir zone, as the  $x$ -axis is taken along the solid boundary. In performing practical hydraulic design of these structures, therefore, the approaching channel must be determined by further theoretical and experimental knowledge.

## (2) Surface profiles of released water from a reservoir and bed pressure distribution

As already described in the previous section, all the possible surface profiles of water are classified by the saddle point located at the section of weir crest. When the flow is released from a reservoir, no other control structures are commonly located, which means the released water flow is free. Consequently, the transition curve schematically illustrated in Fig. 1 is observed in the experimental runs.

The theoretical curve of transition, as a surface profile of water, for a definite rate of released discharge can be computed by the general theory of steady transitional flows<sup>6,7)</sup> through the numerical analysis. The computation must be started from the saddle point to both directions of up- and downstream with the definite value of initial surface gradient,  $(dh/dx) = -\sqrt{c/b}$ . Figs. 5-1 and -2 indicate the theoretically calculated curves for  $q=313.3 \text{ cm}^3/\text{sec}/\text{cm}$  and  $q=502.7 \text{ cm}^3/\text{sec}/\text{cm}$ , respectively. In the same figures, observed depth of water is also plotted, and it is easily understood that the experimental data can prove the theoretical prediction for surface profiles of water and thus the foregoing procedure of analysis and rather surprising is that a close agreement is obtained between experimental data and theoretical values in the middle and downstream weir sections.

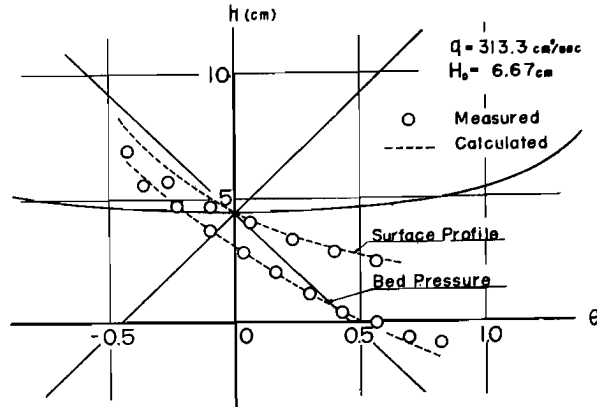


Fig. 5-1 Surface Profiles of Water and Bed Pressure along Circular Weir ( $313.3 \text{ cm}^3/\text{sec/cm}$ )

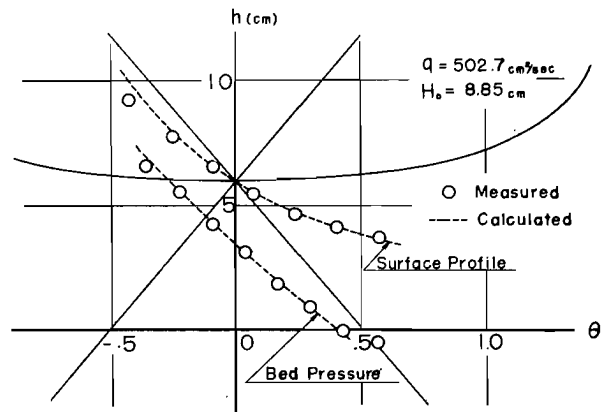


Fig. 5-2 Surface Profiles of Water and Bed Pressure along Circular Weir ( $502.7 \text{ cm}^3/\text{sec/cm}$ )

In relation to the location of saddle point predicted uniquely by the theoretical approach, Fig. 6 indicates the theoretical relationship between the released discharge and the critical depth at the saddle point, with the plots of observed data for the present circular weir. It is evident that the experimental results confirm the theoretical prediction on hydraulic behaviours at the saddle point.

The theoretical values for bed pressure are also calculated by Eq. (14), putting  $y=0$ , and the results computed are indicated in Figs. 5-1 and -2

with the experimental data of bed pressure distribution, measured by the manometers. A close agreement between the theory and the experiments is also evident.

As seen in Figs. 5-1 and -2, however, the calculated curves become greater in their magnitude than the observed data in the upstream reach far from the weir crest. It results from the negligence of secondary influence due to the vertical component of

velocity and thus the present analysis is essentially of first approximation. The validity of the analysis described herein will be then limited in the weir section except the approaching channel, as shortly cited in the previous subsection.

#### 4. Application of Theory Described to Hydraulic Performances of Normal and Side Channel Spillways

The spillways are designed to provide controlled release of surplus water in excess of the reservoir capacity and convey it to the downstream channel or other watercourses. Among various types of spillways classified by the structural forms, the problem is limited to the normal and side channel spillways of uncontrolled crests. Although almost all spillways are equipped by the crest gates, the hydraulic performance of spillways is classified as uncontrolled, when the gate is fully open. The uncontrolled crest of spillway is evidently one of typical examples of control structures in open channel hydraulics, so that the analytical treatment described herein will be applied to make the hydraulic performances of these spillways clear.

Usually, the crest shape of overflow spillways are designed to fit the lower nappe of free flow. The establishment of the trajectory in a mathe-

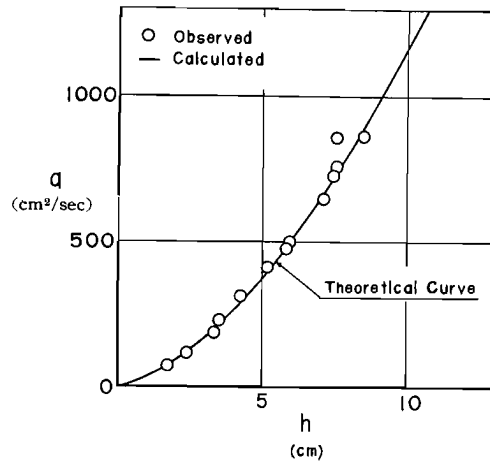


Fig. 6 Relationship between Discharge and Critical Depth at the Saddle Point

mathematical form thus becomes of basic significance, and many hydraulic engineers and institutions have eagerly performed to obtain the comprehensive knowledge of the nappe flow. Among these research works, the methods of pursuing the trajectory of a water particle from a sharp crested weir, made by U. S. Bureau of Reclamation<sup>9)</sup>, by J. Hinds, W. P. Creager and J. D. Justin<sup>10)</sup>, by A. T. Ippen<sup>11)</sup> and by F. W. Blaisdell<sup>12)</sup> and the empirical equation for standard crest shapes obtained at the Waterway Experiment Station, U. S. Corps of Engineers are famous. However, the nappe problem is not still subject to mathematical analysis. In reality, the crest shape will be designed by a combined form of several mathematical expressions so that the boundary form becomes quite similar to the lower nappe of free flow. The suitable crest shape must be designed to fit all the hydraulic requirements, avoidance of negative pressure, increase of hydraulic efficiency for discharge, economy and so on. Nevertheless, the usual design procedures are made only to satisfy the pressure requirement. The discharge requirement was thus considered of secondary significance, and commonly the discharge characteristics for particular spillways are estimated by the scale model test in the laboratory design stage. The results experimentally obtained are expressed in terms of the discharge coefficient  $C$  for the total upstream head from the spillway crest and  $C$  varies between 0.655 and 0.730 at maximum discharge and 0.505 at minimum discharge with poor entrances. Fig. 7 indicates some

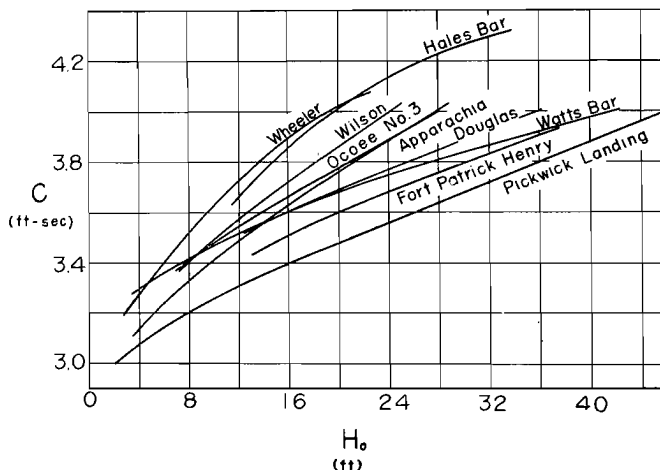


Fig. 7 Discharge Characteristics of Normal Spillways at TVA

of examples for  $C$  in ft-sec unit obtained by K. W. Kirkpatrick<sup>13)</sup> through various model studies of overflow spillways of TVA under the condition of free flow.

The TVA crests all fairly closely approximate the standard curve for the upstream spillway face to a point somewhere downstream from the crest which was determined by the position of the gate seal. Below this latter point, the crest shape was modified to fit the trajectory of jet issuing from the gate when set at a small opening. Among dams listed in the figure, two pairs of these, the Ocoee No. 3-Apalachia set and the Douglas-Watts Bar set have crest shapes that are identical within the pair. Apparently, it is seen that the head-discharge relationship for various shapes of overflow spillways is dependent on their geometric shape and no unified relationship for  $C$  can not be found.

However, in all cases, the spillway crest as the control structure of spillway elements must be a channel control, at which all the hydraulic characteristics of released water can be uniquely predicted through the general transitional theory for open channel flows. The analytical treatment described in the foregoing to estimate the hydraulic performances of the structure will be also applied to the present problem. As the local radius of curvature of the control structure of spillways is spatially variable, so the calculation procedure must be essentially made through the numerical analysis of the basic dynamic equation with the introduction of surface resistance. If doing so, however, the location of singular point, which will be located near the spillway crest, also varies depending on the crest shape and the discharge of released water, so that the tremendous amount of labours to perform the calculation is still needed.

As the first approximation to estimate the hydraulic performances of control structures of spillways, the surface resistance of the boundary will be assumed practically negligible. Under this assumption, the treatment for analysis becomes simpler and the method described in the foregoing will be easily applied without any modification. The results calculated will be possibly prove the validity of the treatment as did for the case of round crested weirs. In the present case, the TVA dams of constant radius of curvature at the downstream spillway surface, Wilson, Wheeler and Fickwick Landing dams, are concerned to verify the above assumption and indicate the hydraulic performances in a first approximate expression.



Table 3 lists the results of analysis computed by the foregoing method of analysis. Column (2) represents the values of the radius of curvature at the downstream face. If the present approximation will strictly applied to the subject,  $R$  must be the local value at the spillway crest. However,

Table 3 Relationship between Actual Value of  $R$  and Estimated Value by Means of Irrotational Curved Flow Theory for TVA Dams of Constant  $R$

(1) Dam	(2) $R_{ac}$ (ft)	(3) $H_0$ (ft)	(4) $C$ (ft-sec)	(5) $C$ (m-sec)	(6) $R/H$	(7) $R_{est}$
Pickwick Landing	45.50	12.0	3.31	0.618	4.25	51.0
		16.0	3.39	0.634	3.20	51.2
		20.0	3.47	0.650	2.46	49.2
		24.0	3.55	0.664	2.00	48.0
		28.0	3.64	0.680	1.65	46.2
		32.0	3.72	0.695	1.45	46.4
		36.0	3.80	0.711	1.26	45.4
		40.0	3.89	0.727	1.13	45.2
Wilson	21.30	8.0	3.41	0.638	3.02	34.1
		12.0	3.57	0.668	1.88	32.6
		16.0	3.72	0.696	1.44	23.1
		20.0	3.86	0.722	1.17	23.4
		24.0	3.99	0.747	0.98	23.5
Wheeler	16.00	4.0	3.28	0.612	4.85	19.4
		8.0	3.53	0.660	2.11	17.9
		12.0	3.73	0.697	1.43	17.2
		16.0	3.89	0.727	1.12	17.9
		20.0	4.01	0.749	0.96	19.2

in usual design procedures, the spillway shape is expressed by combined curves connected at the crest to fit only the pressure requirement, and consequently the local value of  $R$  is commonly discontinuous at the crest. In view of consideration to the basic dynamic equation, the location of singular point will possibly be at the downstream face near the spillway crest, so that the value of  $R$  in Column (2) is represented by the value at the downstream face. Columns (3) and (4) are the results calculated by Fig. 7 in ft-sec unit. Discharge coefficients  $C$  in Column (5) which is defined by Eq. (24) in the foregoing are transformed to values in m-sec unit, so that with the use of the relationships of Eqs. (26) and (27) ( $R/H_0$ ) is also calculated, and finally the value of  $R$  is estimated for various heads of upstream. It is rather surprising that the estimated values of  $R$  made by the above approximate procedure agree with the actual value of  $R$  at these dams. It, therefore, indicates that the flow near the crest of spillway

is approximated by a curvilinear motion. The discharge characteristics of flows at the overflow spillways of constant radius of curvature can be predicted in terms of the value of  $(R/H_0)$ , without conducting model experiments. Fig. 8 also is the graphical representation of the head-discharge relationship, and it is seen that the application of the present treatment to make the hydraulic performances of control structures of spillways clear will become possible.

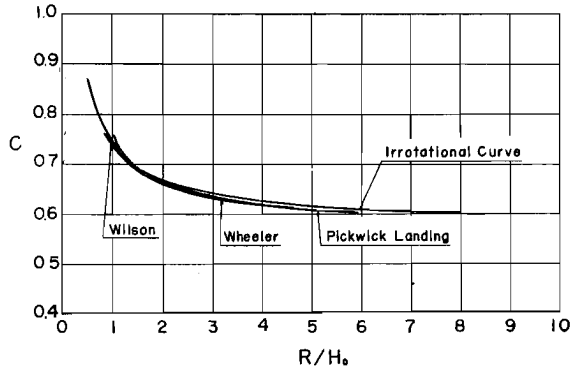


Fig. 8 Discharge Coefficient of Free Flow at TVA Dams of Constant  $R$

The same approxi-

mate treatment will be applied to other dams of variable curvatures at TVA projects, Fort Patrick Henry, Hales, Apalachia and Ocoee No. 3 Dams. Among these dams, the dimensions are identical at the Apalachia and Ocoee No. 3 Dams. The local curvatures are variable, in this case, so that the tremendous numerical calculations are needed to find the complete hydraulic performances of spillways. The comparison of actual values of  $R$  at these dams to the estimated values made by the foregoing approximate analysis will be indicated, and the results is in the following table. Evidently seen in the table, values of  $R$  estimated by the approximate treatment are nearly constant for a wide range of upstream heads, whereas the local curvatures change from point to point in these dams. Of special evidence is that the estimation indicates the actual value within the limit of engineering purpose and the validity of the present treatment for determination of hydraulic performances of overflow spillway will be expected.

In reality, for hydraulic design of crest shape in overflow spillways, the common procedures are to use the standard shape or some famous empirical curves like Creager and others, which can not be explicitly expressed in a mathematical form, so that local curvatures may become discontinuous at the spillway crest and the pressure requirement is only satisfied. For examples,  $R$  at the spillway crest in dams of Fort Patrick

Henry, Hales Bar, Apalachia and Ocoee No. 3 is suddenly changed as seen in Table 4, which will be seen in almost all cases of projects. If the continuous curve in curvature is used for the design stage of control structure, the head-discharge relationship of spillways will be theoretically

Table 4 Relationship between Actual  $R$  and Estimated  $R$  at TVA Dams of Variable Curvatures

(1) Dam	(2) Distance (ft)	(3) $R_{ac}$ (ft)	(4) $H_0$ (ft)	(5) $C$ (ft)	(6) $R_{est}$ (ft)	
Fort Patrick Henry	Upstream	-2	22.0	16.0	3.51	34.3
		-1	43.9	20.0	3.60	34.5
		0	$\infty$	24.0	3.68	34.6
	Downstream	0	0.0	28.0	3.76	34.6
		1	24.0	32.0	3.85	34.8
		2	27.7	36.0	3.91	34.8
		3	30.5			
		4	32.8			
Hales Bar	Upstream	-2	6.1	10.0	3.59	18.7
		-1	11.6	12.0	3.68	18.6
		0	$\infty$	16.0	3.86	18.4
	Downstream	0	0.0	20.0	4.01	18.6
		1	14.2	24.0	4.14	19.2
		2	16.6	28.0	4.25	20.2
		3	18.7	32.0	4.31	21.6
		4	20.6			
Apalachia	Upstream	-2	9.6	8.0	3.33	31.2
		-1	18.9	12.0	3.49	29.1
		0	$\infty$	16.0	3.63	31.6
	Downstream	0	34.3	20.0	2.77	26.4
		1	34.7	24.0	3.89	26.4
		2	35.3			
Ocoee No. 3	Upstream	-2	9.6	6.0	3.35	22.2
		-1	18.9	8.0	3.41	24.4
		0	$\infty$	12.0	3.55	24.4
	Downstream	0	34.3	16.0	3.67	25.1
		1	34.7	20.0	3.78	25.2
		2	35.3	24.0	3.88	25.9
	3	36.0				

predicted by means of the present method of analysis, and this requirement compatible with the pressure requirement is possibly needed for pertinent hydraulic design of spillway structures to ensure the hydraulic efficiency for discharges.

## 5. Some Comments to Design Criteria for Control Structures

A control structure is the class of transition structures in which the

transitional behaviours from a tranquil to shooting state or, in rarely cases, vice versa take place for all rates of carried discharges. The general theory on steady open channel flows<sup>7)</sup> describes that the hydraulic characteristics of flows in channel controls are exclusively determined by the theory of control section and therefore that the hydraulic performances of controls are completely evaluated, if the physics of flows are expressed by a complete mathematical form being in terms of the one-dimensional procedure of analysis. Control structures are used for various purposes in hydraulic works. Weirs, flumes, normal spillways and side channel spillways are typical examples of control structures, and these controlling devices are hydraulically designed by means of the general theory of transitional behaviours of open channel flows, as described in the foregoing. A gate is also an example of control structure, which can regulate the water stage and the discharge. However, it can not give a comprehensive information to the design procedure, if the hydraulic knowledge on the flow is remained at the present level, because the basic flow pattern is not subject to the one-dimensional expression in mathematical hydraulics.

Limiting the problem only to the hydraulic performances of weirs and spillways as the objects in rapidly varied flows, the analysis can be definitely made, as seen in the description of the preceding sections. The hydraulic design of controlling structures through the use of the knowledge of basic hydraulic performances of structures must be made to satisfy completely all the necessary requirements imposed by a particular project. In view of engineering aspects in hydraulics, the requirements for the control structures are mainly that

- i the discharge metering with a single water-level measurement can be simply and correctly made, and
- ii the sudden decrease in bed pressure must not be involved so as to minimize the damage of materials due to the cavitation phenomenon and the future repair and maintenance cost.

The former need can be completely solved by the use of the present analytical treatment, if the trouble in computation is out of question. In the preliminary design stage for the hydraulic projects, Fig. 3 becomes a substantial tool to the design problem. When the design discharge for the controlling structures is given, it provides the determination of the geometry and dimension of the structure. When the weir and spillway

geometries are given, the hydraulic performances to the discharge characteristics are approximately obtained. After determining the final design for these structures satisfied by the basic requirements through the successive trial method, all the hydraulic characteristics of flows over these structures are numerically solved by the foregoing method. The experimental study by the models will provide the more useful informations to design the structure and to evaluate the hydraulic characteristics of flows. However, several repetitions for tentative plans will be required, as the design procedure is always the try and cut method.

In parallel with the former requirement for hydraulic efficiency, the pressure requirement as the second one must be also investigated during the preliminary design stage. When the structural geometry is of constant shape in local curvature like a circular weir, Eq. (14) is useful to estimate the bed pressure distribution, and Fig. 4 also becomes a significant representation to determine the sufficient length of the solid boundary. Especially, when the control structure of spillways is designed, it must be connected by the discharge carrier until the bed pressure becomes zero. However, being different from the discharge characteristics described, the pressure can be sensitively influenced by the local condition in structural geometry, so that the experimental research study for the bed pressure distribution will be largely needed to obtain the supplemental information, if the basic flow pattern is not subject in a strict mathematical form as described in the foregoing.

Another important consideration to the hydraulic design procedure must be putted on the design of approaching channel to the control structure. As seen in the analysis of second-order solutions, the present theoretical analysis on the hydraulic performances of controlling structures indicates less validity for the upstream reservoir reach, so that the careful consideration will be required, if the connecting part from the approaching channel to the controlling structure is not of gradually varied type in geometry.

## Conclusion

This paper describes the hydraulic performances of controlling structures like round crested weirs, spillways and the like to the released flow

by the theoretical method of analysis obtained by the general theory of analysis provided by the general theory of steady open channel flows, and the verification is also made through the experimental works with the use of the model of particular type in shape. Both theoretical and experimental results are in a good agreement. With the aid of the results obtained, some contributions to hydraulic design procedures of controlling structures are also described.

Although the present study is still basic, the following conclusive statement through the analysis will be summarized.

1. The flow over a round crested weir will be practically assumed invicid and irrotational, and the hydraulic treatment for the flow can be made by the first order solution.

2. The round crested weir can serve as a control structure for all discharges released from a reservoir, so that all the hydraulic characteristics of flows are also uniquely determined by the theory of transitional behaviours.

3. Experimental verification to the theoretical analysis is made with sufficient accuracy. Especially, referring to the discharge coefficient, a good agreement between theoretical and experimental results is shown in the medium range of  $\lambda$ .

4. Hydraulic performances of overflow spillways are also approximated by the present analysis based on the irrotational theory, and the theoretical prediction to the performances is proved by the experimental data made by the model studies of TVA dams.

5. Hydraulic design of controlling structures, as round crested weirs, overflow spillways and other similar control structures, will be possible by means of the present analysis. However, problems to design approaching channels and to determine the precise distributions of bed pressure on the spillway crest are still unsolved, and the further study must be needed.

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Bulletin No. 33                      Published March, 1960

昭和 35 年 3 月 15 日 印刷

昭和 35 年 3 月 17 日 発行

編輯兼  
発行者 京都大学防災研究所

印刷者 山代多三郎

京都市上京区寺之内通小川西入

印刷所 山代印刷株式会社