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**ANALYSIS WITH AN APPLICATION TO
ASEISMIC DESIGN OF BRIDGE PIERS**

BY

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Abstract

Response of a bridge pier on an elasto-plastic foundation subjected to idealized, transient ground motions have been investigated. The bridge pier has been idealized as a one-degree-of-freedom system undergoing a rocking motion relative to the ground, and the restoring force characteristic of the system has been assumed to be of bi-linear, hysteretic type.

Analysis was carried out by using an electronic analog computer equipped with a non-linear backlash element to simulate the specific vibration system. Accuracy of the response curves thus obtained was checked by means of the Phase-plane-delta method, and was found to be satisfactory.

Maximum-amplitude spectra of the bridge pier, as well as the energy dissipation due to the hysteresis of the system's restoring force characteristic, for the several types of ground motions, were interesting from the view-point of aseismic design of bridge piers.

Introduction

The purpose of the following study is to clarify dynamic response of a bridge pier resulting from transitory ground motions resembling those encountered in actual earthquakes but of highly idealized type.

Earthquake resistant design of bridge piers practically bases on the criteria which check only for ground accelerations determined from the past earthquake records, regardless of natural periods of vibration of the

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pier-and-foundation systems and of other important factors. It has been well noticed that a reasonable design criterion would necessarily be predicated on a basis of dynamic response spectra of bridge piers for probable earthquakes.

The chief characteristic of a bridge pier is that rocking motion about a horizontal axis is coupled with the translations. A translational ground motion induces rolling of the pier as well as translation. When the ground is elasto-plastic, the bridge pier oscillates in such a complicated manner that the motion of the pier is governed by a non-linear differential equation.

Mathematical analysis of the problem presents great difficulties because of irregularity in ground motions and of complexity of mechanical property of the foundation, and therefore some alternative means would have to be taken so as to analyze the situation with less difficulties.

Approximate solutions of non-linear differential equations of motion have been obtained so far by means of either step-by-step numerical integration or graphical construction; sometimes laboratory experimentation with mechanical models has been attempted to portray clearly our ideas of the general problem of dynamic behavior of structures.

In the present paper, the rocking motions of the bridge pier subjected to idealized, transient ground motions have been analyzed by employing an electronic analog computer equipped with a backlash element, which enables us to deal with responses of a vibration system if it has a bilinear, hysteretic restoring force characteristic. The computer belongs to the Department of Electronic Engineering and is located in the Electronics Laboratory at Kyoto University.

Attention has been given to the maximum displacement spectra, together with spectra of energy dissipation in the oscillating system, obtained from the time-displacement curves of the system under the action of ground motions.

Fundamental Equation of Motion of a Bridge Pier

Let us consider a bridge pier on an elasto-plastic foundation subjected to horizontal translations. Since the deformation in the pier itself during vibration is small in comparison with the deformation of the foundation

and effects of the gravitational force as a result of rolling motion of the pier are assumed to be negligible, the pier can be regarded as a rigid body of two degrees of freedom; namely, if a translational ground motion occurs in the direction parallel to the principal axis of the cross-section of the pier, motion of the pier will be described in terms of the horizontal displacement of any point in the pier and a rotation about the horizontal axis which passes through the point.

In case of a pier on an elastic foundation, it would be feasible to find a point in the pier where the relative, translational displacement disappears and the rotational displacement prevails — the “instantaneous center of rotation” in the relative coordinates. Dr. Mononobe has suggested in this case that such a point does not change its location for different magnitudes of rotational displacement of the pier (Ref. 1).

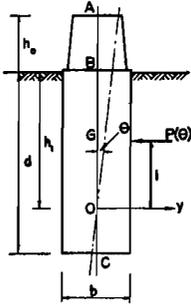


Fig. 1 Schematic Diagram of a Bridge Pier.

Also, for a presumably elasto-plastic foundation, the result of our experiment on a bridge pier model in a small size has led us to confirm that there certainly exists a stationary point in the pier where no relative displacements take place during vibration between the point and the ground motion.

Thus, the motion of the pier will be of one-degree-of-freedom and will be described by the following equation in the relative coordinates, with the origin located at the point O, shown in Fig. 1, which is supposed to follow a prescribed ground motion.

$$I_0 \frac{d^2\theta}{dt^2} + lP(\theta) = -Ml \frac{d^2y_G}{dt^2} \quad (1)$$

where M = total mass of bridge pier,
 I_0 = moment of inertia of bridge pier about point O,
 θ = angular displacement of bridge pier, and
 y_G = ground displacement.

The function, $lP(\theta)$, in the whole represents the restoring moment, or restoring couple, resulted from the reaction of the foundation, with l being a distance from the origin to the center of gravity of the pier, G .

Consequently, $P(\theta)$ can be interpreted as a resultant force of the

reaction distributed in such a pattern as idealized in Fig. 2. The non-linearity of the function and the value of yield point will be concerned mainly with the mechanical property of soil.

If we let $y = h_1\theta$, Eq. (1) may be written as

$$\frac{d^2y}{dt^2} + G(y) = -a \frac{d^2y_G}{dt^2} \quad (2)$$

where

$$G(y) = \frac{lh_1}{I_0} P(\theta) \quad \text{and} \quad a = \frac{Mlh_1}{I_0}$$

In the above setting up of the equation of motion, effects of viscosity or internal friction in soil, and friction between the pier and the foundation were neglected. Also, hydraulic effects of water stream on the motion of the pier-and-foundation system were not taken into consideration, since it has been attempted to evaluate clearly the effects of the damping, or energy loss per cycle of vibration, due to hysteresis of the restoring force characteristic.

Reconciliation of Restoring Force-distortion Characteristic Curve of the Foundation

The result of mono-axial compression tests on alluvium clay specimens indicates that the load-distortion diagram of the soil can be approximated as being of ideally elastic-plastic type (Fig. 3). Tilting of the pier will induce plastic deformation of the foundation in the neighborhood of the pier, so that the stress-strain relation for some part of the foundation in contact with the pier will be in the state of unrestricted plastic flow.

Under the assumption that the magnitude of yield stress of the foundation increases with the earth pressure which would be proportional to the depth of the part in question below the surface of the ground, it is possible to look for an implicit relationship between the angular displacement of the pier, θ , and the growth in the range of the plastic zone as

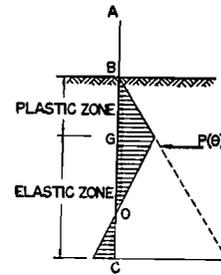


Fig. 2 Idealized Distribution of Reaction of the Ground.

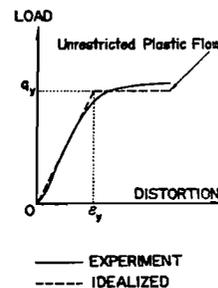


Fig. 3 Load-Distortion Characteristic of Soil.

shown in Fig. 2, in which a distribution of the reaction of the foundation is idealized for an angular displacement assumed.

As the angular displacement becomes twice as large, the reaction in the elastic zone will be doubled while the plastic zone increases its range. Thus, rough tendency of a curve indicating the $P(\theta) - \theta$ relationship, the implicit expression for the moment contribution of the distributed reactions for the increasing angular displacement, can be drawn by the area-moment method of finding $IP(\theta)$.

Moreover, data of measurements of the natural frequency and amplitude of a bridge pier prototype at a small amplitude vibration test are useful to determine the stiffness of the system, or the initial slope of the $P(\theta) - \theta$ diagram, together with the amplitude range of elastic, linear vibration.

A prototype of bridge piers chosen here is a pier to support a multi-spanned, railroad bridge with a span length of 32.1 meters and has the dimensions as listed in Table 1. By using the data, the quantities in Eq. (2) can be calculated as follows.

$$\frac{lh_1}{I_0} = 0.0043 \text{ m/ton} \cdot \text{sec}^2 \quad \text{and} \quad a = \frac{Mlh_1}{I_0} = 0.67 \quad (3)$$

Table. 1 Prototype of bridge pier.

Symbols	Dimensions
h_0	7.3 m
h_1	19.5 m
d	24.0 m
l	11.9 m
a : Thickness of Well	3.6 m
b : Breadth of Well	8.3 m
A : Cross-section Area	27.1 m ²
W_0 : Carrying Load	354 ton
W_1 : Dead Load	1,163 ton
W : Total Weight	1,517 ton
M : Mass	155 ton \cdot sec ² /m
I_0 : Moment of Inertia	54,000 ton \cdot m \cdot sec ²

A field measurement for the period at a small amplitude steady forced vibration has given the value of $T_0 = 0.15$ sec., the natural period of vibration of the pier-and-foundation prototype, in the direction normal to the

bridge axis. Hence, the initial slope of the $P(\theta) - \theta$ curve for the prototype becomes

$$\tan \beta = \frac{4\pi^2}{T_0^2} \cdot \frac{I_0}{l} = 8.0 \times 10^6 \text{ ton/rad} \quad (4)$$

The $P(\theta) - \theta$ curve thus reconciled for the oscillating system is shown in Fig. 4. This curve of softening type will correspond to the one-way deformation of the system with increasing θ , and this may again be approximated into a bi-linear characteristic within an appropriately small range of θ .

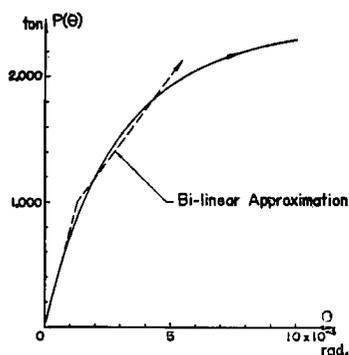


Fig. 4 Restoring Force-Displacement Curve Reconciled, and its Bi-linear Approximation.

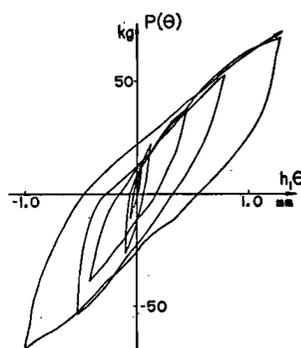


Fig. 5 Load-Distortion Hysteresis Curve of a Bridge Pier Model.

However, the restoring force-displacement diagram of the system generally shows a certain type of hysteresis for alternating displacements. An experiment on a pier-and-foundation model (Ref. 2) has furnished a typical load-displacement curve under a cyclic loading in the horizontal direction (Fig. 5). The above discussion thus leads us to assume a hysteretic, bi-linear type as shown in Fig. 6, for the restoring force-displacement characteristic inherent to the pier-and-foundation system.

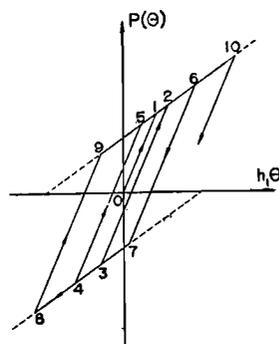


Fig. 6 Idealized Restoring Force-Displacement Characteristic.

Earthquake Response of the Bridge Pier Obtained by an Analog Computer

The mechanical system considered is like that of an equivalent mass and a stiffness member having a bi-linear, hysteretic restoring force. The restoring force is proportional to displacement up to the yield point and then increases with a less-steep slope for increasing displacement. When the velocity changes sign the restoring force-displacement characteristic decreases along a line parallel to the initial linear portion.

The hysteresis loop is thus composed of linear segments as shown in Fig. 6. When such a system is acted upon by a large impulse from the earthquake the stiffness member is forced beyond the elastic range into plastic range and the system now oscillates about a new equilibrium position. Further impulse may shift this equilibrium position to result the "permanent set" being increased or decreased, and therefore the magnitude of the restoring force at the yield point varies with the equilibrium position.

This mechanical system whose motions are governed by the differential equation, Eq. (2), has been treated by setting up an electrical analogy by employing a non-linear, backlash element equipped in an electronic analog computer of slow-speed, indirect type. Equation (2) can be integrated directly and the velocity, \dot{y} , together with the displacement, y , of the oscillating system are obtained as

$$\dot{y} = -p^2 \int_0^t \{ a\ddot{y}_0/p^2 + g(y) \} dt \quad (5)$$

and

$$y = -p^2 \int_0^t \int_0^t \{ a\ddot{y}_0/p^2 + g(y) \} dt \cdot dt \quad (6)$$

in which $G(y) = p^2 g(y)$ with $g(y) = y - k(y)$,

$k(y)$ = linearity difference which may be expressed in terms of a polynomial function of y higher than the second order, and

p^2 = circular natural frequency of the system for a small amplitude vibration.

The block diagram of the electronic analog computer circuit is shown in Fig. 7 (a). In the figure, letters, P , I , and S , represent potentiometers,

integrating amplifiers and a summing amplifier, respectively. In Fig. 7(b), a typical circuit of the backlash element is shown. This is a circuit to establish the relationship between the input voltage X_1 and the output voltage X_0 , as demonstrated in the figure, by utilizing a characteristic of the diode component to control the function of the integrating amplifier in the circuit.

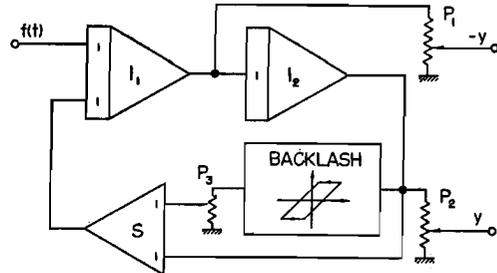


Fig. 7(a) Electronic Analog Computer Circuit.

When the analog computer circuit is driven with $a\ddot{y}_0/p^2$, or the function $f(t)$, as a change in the input voltage, variation in voltage at the outputs, $-y$ and y , are resulted. The changes in y and y with respect to time are recorded on a roll of paper in an inkwriting oscillograph.

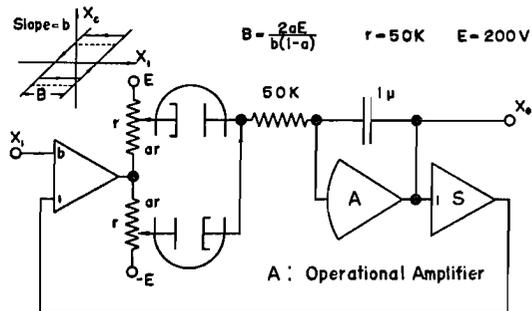


Fig. 7(b) Backlash Element Circuit.

If the ground displacement y_0 may be assumed as of a simple analytic form such as sinusoidal, rectangular, or exponential shapes, some suitable electric circuits can easily take care of the form. Of course, responses of the pier to complicated earthquake motions recorded by seismometer can be analyzed whenever the records are converted into a corresponding electric quantity by some means as that adopted by Bycroft, Murphy and Brown (Ref. 3).

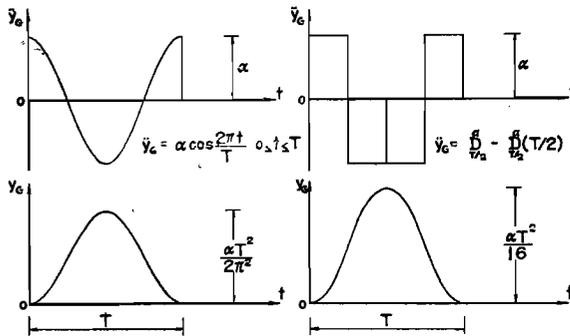
Ground Motions Assumed :

Since there are not enough strong-motion seismograph records available at present to employ them in analyzing their effects on structures, a comprehensive investigation on responses of structures to all possible patterns of earthquake motion will not be feasible although it is greatly desirable

and indispensable.

An earthquake motion consists of minor tremors, of relatively short duration, impulses followed by free vibrations as well as of forced vibration with all their transients superimposed. Usually, there are no regularities or common patterns observed in the ground motion records at different locations even if they were obtained simultaneously. However, if we confine our attention to structures located at a reasonable distance from the earthquake center, it may well be noted that a small number of cycles of large-amplitude, transient ground vibrations predominate and that they seem to play an important role to cause much damages to the structures.

The simplest type of ground motion resembling the significant part of an actual earthquake motion will be with one degree of freedom and will be of the one that the ground translates to a distance and back within a



(a) Versed-sine Pulse (b) Quadratic Pulse
 Fig. 8 Ground Motions Assumed for the Analysis.

finite period, as shown in Fig. 8, where two kinds of idealization of ground displacement are made. These are a full-cycle versed-sine pulse and a quadratic pulse, both with the same maximum acceleration value, α , and of a duration T . Consequently, the maxi-

imum ground displacement for the versed-sine pulse will be $\alpha T^2/2\pi^2$ and that for the quadratic pulse will be equal to $\alpha T^2/16$, about 23 percent larger than the former.

We are chiefly concerned with the effects of variation in the ground acceleration and the ground-motion duration relative to the natural period of vibration of the bridge pier, T_0 . Therefore, responses were obtained for the two types of idealized pulses with the duration ranging from 0.075 to 0.30 sec at five different values of ground acceleration. The maximum ground acceleration values were in the range from 0.16 g to 0.64 g, with g being the acceleration of gravity, provided that the elastic range of $P(\theta)$ for the stiffness member of the pier-and-foundation system is up to $\pm 1,000$ tons,

as illustrated in Fig. 4.

Results of Computation :

By the electrical analogy method, a total of 225 sets of response curves for the bridge pier subjected to the idealized ground motions have been obtained within a relatively short time. An example of the response curves is shown in Fig. 9, in which the change in the ground acceleration with respect to time is recorded on the top. The displacement-time, as well as the velocity-time, curves for the oscillating system subjected to the ground motion are shown in the middle and bottom of the figure, respectively.

The maximum distortion, Y_M , or the maximum of full swing of the system during vibration was interested, and hence, in Figs. 10 and 11, the response was plotted against T/T_0 , ratio of the duration of the ground pulse to the natural period of vibration of the system for five values of a chosen as a parameter. The values of a are presented in these figures in terms of the ratio of the maximum inertia force acting upon the mass of the mechanical system to the yield point restoring force of the stiffness member.

Namely, the case of $a=1$ corresponds to the maximum ground acceleration of 0.64 g, while $a=1/4$ is the case where the system is acted upon by the ground motion with the maximum acceleration of 0.16 g. We should note here that the amplitude of quadratic pulse is about 1.23 times as large as that of versed-sine pulse when they have the same value for the maximum acceleration.

When the oscillating system is subjected to such a large impulse from the earthquake the stiffness member is forced beyond the elastic range and the restoring force-distortion characteristic curve now shows hysteresis loops of parallelogram. The area enclosed in a hysteresis loop for a cycle of vibration measures the energy loss within the cycle, which has dissipa-

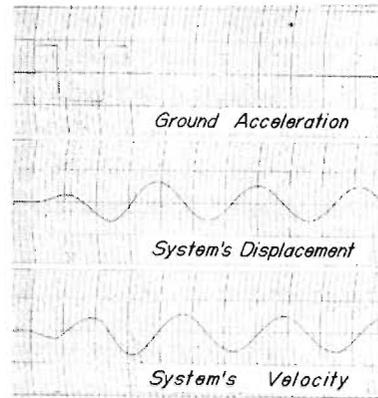


Fig. 9 Example of Response Curves Recorded on the Ink-writing Oscillograph Paper.

ted from the system. The restoring force-distortion curve was then drawn for each displacement-time response curve, and the area of the hysteresis loop for each cycle was determined by numerical integration.

It was noticed that for the ground pulses of one full-cycle with the duration T/T_0 ranging from 0.5 to 2.0 the energy dissipation occurred within the first three or four half-cycles of vibration in the forced-vibration era and, in many cases, almost all the energy dissipation had taken place until the first half-cycle of the free-vibration era.

Spectra of the total energy loss during vibration of the system are shown in Figs. 12 and 13, for a full-cycle versed-sine, as well as quadratic, ground pulses. Each spectrum is plotted against T/T_0 with α as a parameter.

Accuracy :

All components of the computer are accurate to within 0.2% and the overall linearity of the amplifiers is 0.5% in the slow-speed range from 0.1 to 1 cycle/sec. The behavior of the analog was checked by comparing the response curves with those obtained by the graphical "phase-plane" analysis (Ref. 4) under the same conditions. The results are shown in Fig. 14.

It may be mentioned that discrepancy in the curves at the corresponding maximum amplitudes is less than a few percent of either one of the maximum amplitudes, and that both methods of analysis are equally useful on the basis of engineering judgement, while the electrical analogy method is much more time saving.

Discussion of Results

The maximum distortion spectra in Fig. 10 were obtained for the highly idealized oscillating system when it was acted upon by the ground motions with different values of α and T/T_0 , but of versed-sine shape. The ordinate, Y_M , at a value of T/T_0 is nearly proportional to α if it is very small, for instance, if $\alpha=1/4$ or $3/8$. However, for a larger value of α , the spectra of Y_M is not proportional to α except the range of T/T_0 less than 0.8.

A pretty large amount of plastic deformation of the stiffness member

of the system can be observed when the value of α approaches to unity, while the spectrum for $\alpha=1/4$ falls almost entirely within the elasticity limit where no plastic deformation of the stiffness member occurs. It may well be seen that each spectrum curve has an apparent peak which is located at about 1.2 of T/T_0 when $\alpha=1/4$, and about 1.5 for $\alpha=1$. Namely, the peak changes its location toward larger values of T/T_0 if α becomes larger.

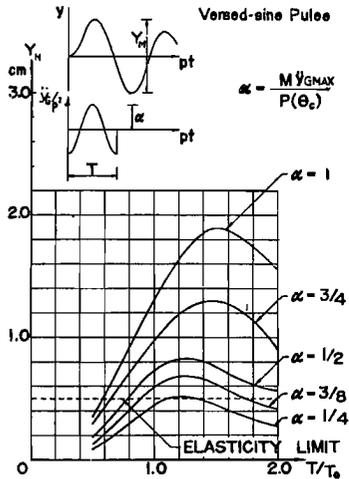


Fig. 10 Period-Amplitude Responses or Spectra of Maximum Distortion for the Full-cycle Versed-sine Pulse.

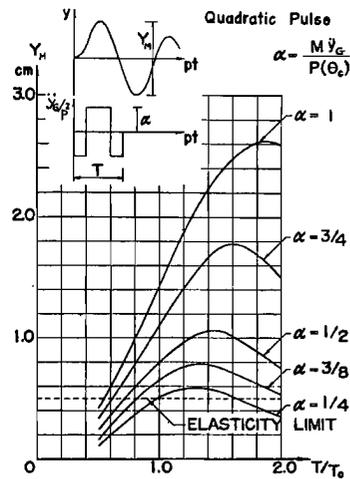


Fig. 11 Period-Amplitude Responses or Spectra of Maximum Distortion for the Full-cycle Quadratic Pulse.

Similar comments will be made on the results of analysis shown in Fig. 11, the case of quadratic ground pulses. However, the magnitude of Y_M in this case is different from that in Fig. 10 even for the corresponding values of α and T/T_0 . This is partly because of the fact that the amplitudes of the two types of idealized ground motions are not the same, that is to say, the quadratic pulse has the maximum displacement about 23% larger than that of the versed-sine pulse when they have the identical values for α and T . The spectra in Fig. 11 are as a whole more than 23% greater than the corresponding responses in Fig. 10. The deviation is more distinct whenever α is large.

Hence, it may be confirmed that deformation of the system, i.e., failure of a bridge pier, due to an earthquake is related not only to the absolute value of the ground acceleration but also to the shape and duration of the

ground motion as well as the mechanical property of the foundation. It seems to us that the response of structure to a ground motion will be more accurately predicted if we shall evaluate the effect or intensity of the ground motion in terms of either the maximum ground displacement or the maximum ground velocity, which are functions of both α and T , and therefore that reasonable criteria for aseismic design of bridge piers or other types of structures will have to base not on the maximum ground acceleration alone but on a composite quantity consisting of the acceleration value and time.

In addition, structural damping, or the non-linear hysteretic characteristic of the restoring force for the system, must be paid more attention. An interesting feature has been observed on the relationship between the total energy loss from the system and ground motions.

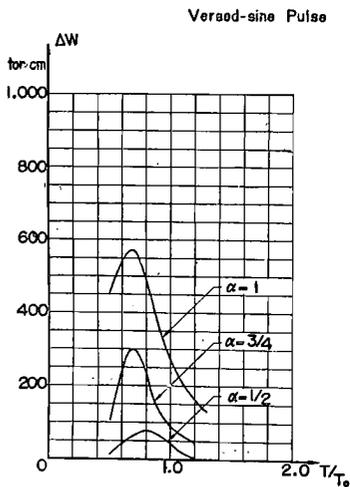


Fig. 12 Spectra of Total Energy-loss for the Full-cycle Versed-sine Pulse.

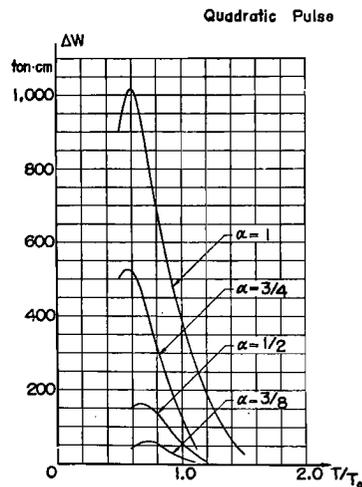


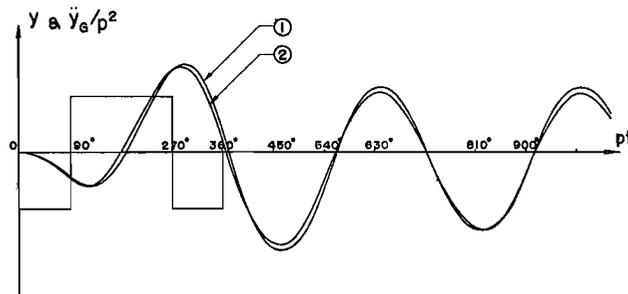
Fig. 13 Spectra of Total Energy-loss for the Full-cycle Quadratic Pulse.

In Figs. 12 and 13, it may commonly be noticed that the total dissipated-energy from the system during the free, as well as forced, vibration eras increases as α becomes large. The energy loss is entirely due to plastic deformation of the system, and hence, no energy loss is seen for the cases in which α is less than $1/4$.

A very distinct peak is found on every energy-loss spectrum curve, and it shifts the location mostly toward the smaller values of T/T_0 as α

increases. And the slope of each curve is exceedingly steep in the range of T/T_0 from 0.8 to 1.2. It can be said that the total energy loss does not grow up to any appreciable amount if ratio T/T_0 is more than 1.2, despite the fact that the maximum distortion spectra in this range show large plastic deformations.

This may be illustrated as follows. In the case where T/T_0 is small, a great deal of energy dissipation would take place within the first two or three half-cycles, namely, for the whole era of the forced-vibration; and due to the energy loss in the early stage of the phenomenon the subsequent swings of vibration do not develop so much. On the contrary, in case of T/T_0 ranging from 1.2 to 1.5, almost all energy dissipation occurs only in a half-cycle that corresponds to the maximum amplitude of the system.



Square Wave ① Analog Computer, ② Phase-plane-delta Method
 Fig. 14 Comparison of Response Curves Obtained by Means of the
 Phase-plane-delta Method and of the Electronic Analog Computer.

Conclusions

Earthquake response of a simplified oscillating system with a bi-linear, hysteretic restoring force characteristic has been made available by means of electrical analogy with satisfactory accuracy. Study on the response curves of the system to the idealized ground motions has indicated great importance of damping capacity as well as the ground-motion duration and acceleration, whereas only the maximum value of earthquake acceleration is being taken into consideration in the prevalent aseismic design criteria for bridge piers.

Further comprehensive research associated with strong motion seismograph records is herewith needed for more reasonable design criteria.

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Nomenclature

	Unit
$a = Mh_1/I_0$, constant,	(none)
Reduction coefficient for potentiometers	(none)
A = Operational amplifier	
b = Slope of input-output voltage characteristic for backlash element	(none)
$B = 2aE/b(1-a)$, width of constant-output zone in backlash element hysteresis	(volt)
d = Depth of bridge pier under ground	(m)
$\frac{D}{\tau^{1/2}}$ = Expression for acceleration doublet corresponding to quadratic, ground displacement pulse	(m/sec ²)
E = Bias volts on diodes	(volt)
$f(t)$ = Driving function for analog computer circuit	(volt)
g = Acceleration of gravity	(m/sec ²)
$g(y) = G(y)/p^2$	(m·rad ²)
$G(y)$ = Non-linear restoring force of oscillating system	(m/sec ²)
h_0 = Height of bridge pier above ground level	(m)

	Unit
h_1 = Distance from ground surface to stationary point or origin of coordinates for oscillating system	(m)
I = Integrating amplifier	
I_0 = Moment of inertia of bridge pier about origin of relative coordinates	(ton·m·sec ²)
$k(y)$ = Linearity difference of restoring force characteristic for oscillating system	(m)
l = Distance between center of gravity and origin of coordinates for pier-and-foundation system	(m)
M = Mass of bridge pier	(ton·sec ² /m)
p = Circular natural frequency of motion for oscillating system	(rad/sec)
P = Potentiometer	
$P(\theta)$ = Restoring force of pier-and-foundation system	(ton)
$P(\theta_c)$ = Restoring force of system at yield point	(ton)
r = Resistance in backlash element circuit	(ohm)
S = Summing amplifier	
t = Time; independent variable	(sec)
T = Duration of ground motion	(sec)
T_0 = Natural period of vibration of oscillating system in small-amplitude vibration	(sec)
ΔW = Total loss of vibration energy in oscillating system	(ton·m)
X_1 = Input of electrical circuit	(volt)
X_0 = Output of electrical circuit	(volt)
y = Relative displacement of system at ground surface	(cm)
\dot{y} = Relative velocity of system at ground surface	(cm/sec)
y_0 = Ground displacement	(cm)
Y_M = Maximum-amplitude or distortion of system	(cm)
$\alpha = M\ddot{y}_0/P(\theta_c)$, ratio of maximum inertia force to yield-point restoring force of oscillating system	(none)
β = Slope of $P(\theta)$ - θ curve for prototype of bridge pier	(rad)
θ = Angular displacement of pier-and-foundation system	(rad)
θ_c = Angular displacement of system at yield-point restoring force	(rad)

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