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Synopsis·

In this paper there has been considered transient response of an asymmetric, one-story building structure due to an idealized ground motion. Torsional vibration of the structure has been of interest, but the analysis was not so simple because the motion is governed by a set of simultaneous, nonlinear differerential equations.

Approximate solutions of the differential equations of motion have been obtained by means of electrical analogy. Maximum distortions and twist of the structure during vibration were discussed for the variation in the intensity and duration of the ground motion, and the possibility of unstable motion of the structure was examined, so as to draw some conclusions related to the earthquake-resistant design of building structures.

Introduction

Response of an asymmetrical structure due to an earthquake contains some components of torsional vibration. A translational ground motion induces rotation, about a vertical axis in the structure, coupled with horizontal translations. Most building structures are asymmetrical in the mass- and rigidity-distributions with respect to their principal axes.

A primary consideration on this phenomenon was made as early as 1934 by Professor Ryo Tanabashi, the senior author of the present paper, when he dealt with the free- and forced-vibrations of asymmetrical, multi-story frame structures[1] Professor Kiyoshi Muto et al. of Tokyo University, Tokyo, Japan, carried out similar theoretical and experimental analyses of the problem since 1941, just twenty years ago[2], while Professor Robert S. Ayre has made an extensive investigation in 1943 at Stanford University, California, to obtain experimental response of an asymmetric, one-story building model to an idealized, transient ground motion[3] A number of investigations have thereafter been carried out to clarify and estimate the importance of torsional vibration of structures, but most of them were treated by basing upon the theory of linear oscillations.

However, it is broadly recognized that, when a ductile framed structure is subjected to a sufficiently strong ground motion, its oscillations will produce strains that exceed the elastic limit[4] The usual procedures of earthquake resistant design are based on the supposition that very strong ground motion may cause overstressing and that elasto-plastic characteristic or ductility capacity is sufficiently effective to control the vibrations without being collapse of danger. Hence, the clarification of nonlinearity observed from the earthquake response of structures is of great significance.

The present paper reports an investigation of the effects of an idealized ground motion on a single story building structure with bilinear, elasto-plastic restoring force characteristics. The structure was assumed to consist of a single rigid mass, free to move in a horizontal plane, and supported on four columns. The rigidities of these columns were chosen unequal so as to make the structure symmetrical with respect to one axis but unsymmetrical in the perpendicular direction.

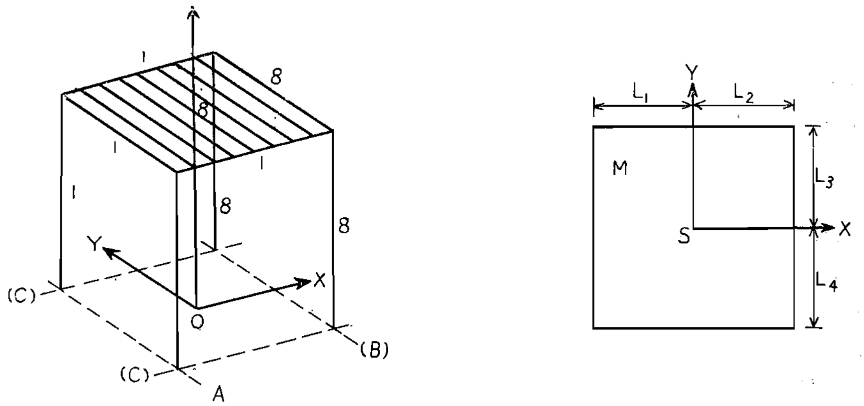
Transient responses of the structure to a highly idealized ground motion of versed-sine pulse type were studied by means of electrical analogy. The frequency or the duration of the ground motion was varied over a range wide enough to include the natural frequencies of the structure when it oscillates freely with very small amplitudes ; the magnitude of ground acceleration was also varied so that we can observe oscillatory behaviors of the structure which produce strains at a time that are either below or beyond the elastic limit. With increase in the magnitude of the ground acceleration, transition of the behaviors of the structure from the linear oscillation range into nonlinear oscillation range can be observed and this is the point of interest in the present analysis.

In this paper, the horizontal ground translation was confined only in one direction parallel to the pair of frames of different rigidities ; namely, large twisting of the structure can mostly be expected for this direction of ground motion. This was also for the purpose to examine the possibility of unstability in the response of the structure due to sufficiently strong ground excitations, as was discussed two years ago[5]

Assumptions

The Mechanical System

The mechanical system considered here is a single-story framed structure as shown in Fig. 1. This is the one that the authors had previously made some numerical analyses and whose dynamic characteristics, when it oscillates freely at a very small amplitude, were therefore already known[6].



NUMERALS SHOW THE RATIO OF ELASTIC BENDING RIGIDITIES

(a)

RECTANGULAR ROOF-FLOOR

(b)

Fig. 1. Asymmetric, single-story frame structure analyzed.

Dead and live loads of the structure are assumed to be uniformly distributed on the rectangular roof-floor, so that the center of mass is supposed to be located at the center of the floor. Two of the columns of the structure have the elastic bending rigidity which is 8 (eight) times as large as those of the remainders. Therefore, the center of rigidity of the structure is located somewhere apart from the center of mass. If we take the X-axis of a rectangular coordinate system that passes through the centers of mass and rigidity, it can be said that the structure has a certain static eccentricity existing in the X-direction.

Professors Bustamante and Rosenblueth of the University of Mexico stated in their paper[7] that dynamic studies of single-story buildings[8] indicate clearly that differences between dynamic and static eccentricity in ideal elastic buildings may be important. This would mean that the eccentricity is influenced by the frequency of ground motion even in the ideal elastic buildings, and further that in the case of an elasto-plastic building the time-effects of ground motion on the difference between dynamic and static eccentricities will be much more apparent. As a matter of fact, when a ductile, asymmetric structure undergoes a large deformation, it would be anticipated that the eccentricity may change not only its magnitude but also the direction. Therefore, in this investigation, we shall not be concerned with the eccentricity but shall find the response of the structure directly from the nonlinear equations of motion.

The mechanical system may be regarded to consist of three kinds of frames; (**a**) flexible, (**b**) rigid, and (**c**) medium rigidity. The initial tangent moduli of the restoring force-distortion curves for these frames have been calculated and they were found to be k, $8k$, and $2.5k$, respectively.

Since most ductile framed structures show an elasto-plastic characteristic in the restoring force-distortion relationship, and since many experimental investigations on steel and reinforced-concrete structures have furnished us some knowledge on the general features of this relationship, which can be approximated to a bilinear type, let us assume here that each frame has an idealized, bilinear hysteretic restoration characteristic as shown in Fig. 2. Namely, the restoring force of each frame is proportional to the lateral deflection in the plane up to the yield point and then increases or decreases linearly with another slope. When the velocity changes sign the restoring force decreases along a line parallel to the inital linear portion. The restoring force-distortion diagram is thus composed of linear segments. Such restoration is seen for any spring element in which the elastic limit of its constituent material is exceeded so that plastic deformations occur with accompanying internal irrecoverable work of hysteresis.

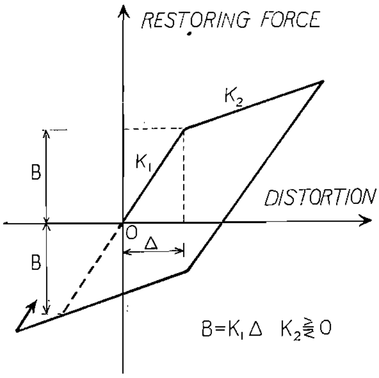


Fig. 2. Bi-linear restoring force-distortion relationship for each frame of the structure.

Originally, the initial tangent, k_1, of the restoring force-distortion diagram for a frame of the structure is associated with the moments of inertia of the cross-sections of its columns and beam as well as Young's modulus of elasticity for the material, while the elastic limit is given by the section moduli of the members and the yield-point stress of the material. In other words, the elastic limit and the slope k_1 of the restoring force are to be determined from the different quantities so that they are independent from each other. Further, the value of slope, k_2, in Fig. 2 is concerned with the plastic characteristics of the material.

Therefore, each frame of the structure can practically have a wide variety in the restoration characteristics which will have to be expressed in terms of the slopes, k_1 and k_2, and the elastic limit of restoring force or the corresponding yield displacement, Δ.

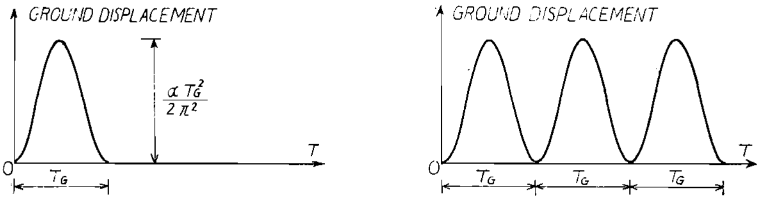
When the frames are subjected to a large dynamic load, the internal irrecoverable work of hysteresis or the loss of the kinetic energy in the structure will attenuate the vibration of the structure as if there exists an equivalent source of viscous damping in the system. The external viscous damping may also be of interest, but less emphasis will be laid in this paper on such damping other than the one due to hysteresis.

Ground Motions

Since there are not enough reliable strong-motion seismograph records available to employ them in analyzing their effects on structures, a comprehensive research on the structures subjected to all possible patterns of earthquake motions will not be feasible although it is greatly desirable and indispensable. Of course, many investigations were carried out so far on the earthquake responses of structures by basing upon a small number of past earthquake records such as the El Centro acceleration record of May 18, 1940, but it may be regarded that the results of the analyses are true and valid only for those specific earthquakes, namely, that our knowledge on earthquakes is still meager to draw some generalized criteria of earthquake-resistant design, and there is need for much additional data of actual earthquakes.

Usually, an earthquake motion consists of minor tremors, of relatively short duration, impulses followed by free vibrations as well as of forced vibration with all their transients superimposed. There are no regularities nor common patterns observed from the ground motion records. However, if we confine our attention to any ground displacement-time pattern, recorded at a reasonably long distance from the epicenter, it may be well noted that large-amplitude, transient ground vibration of a small number of cycles is predominant and it seems to play an important role to cause much damages or collapse of structures.

The simplest type of ground motion resembling the significant part of an actual earthquake motion will be with one degree of freedom and will be of the one that the ground translates to a distance and comes back within a finite period of time as shown in Fig. 3. Hence, in this paper,



(a) (b)

Fig. 3. Idealized ground motions.

let us assume an earthquake motion of a versed-sine pulse and/or its repetition of a small number of cycles, with the maximum acceleration α and the period or the duration T_G. Consequently, the amplitude, or the maximum ground displacement, of the versed-sine ground pulse will be $\alpha T_G^2/2\pi^2$.

We are chiefly concerned with the effects of variation in the ground acceleration and the duration of the ground motion, relative to the natural periods of oscillation of the structure. The free vibration characteristics of one- and two-story unsymmetrical structures have been discussed previously[1] It has been shown that in general an elastic, single-story structure has three natural modes of vibration ; but the present system in the elastic range has two such modes that rotation about a vertical axis is coupled with a horizontal translation, in addition to a natural mode which is entirely translational because the structure is symmetrical with respect to one of the principal axes.

Fundamental Equations of Motion of an Asymmetric, One-story Structure

Let us consider a simplified structure as illustrated in Fig. 1. The distortion at each frame of the structure is further assumed to contribute a restoration in the horizontal direction in the plane of the frame. In other words, the frame has a negligible rigidity for the distortion out of its plane.

The relative motion of the structure, when it is subjected to a horizontal ground translation, will be expressed by the following equations with respect to the rectangular, moving coordinates, with Z-axis originally passing through the center of mass of the floor.

$$\begin{cases} M\dfrac{d^2Y}{dT^2}+R_a(Y-L_1\Theta)+R_b(Y-L_2\Theta)=-M\dfrac{d^2Y_G}{dT^2}=-M\alpha_Y f_Y(T) & (1) \\ M\dfrac{d^2X}{dT^2}+R_c(X+L_3\Theta)+R_c(X+L_4\Theta)=-M\dfrac{d^2X_G}{dT^2}=-M\alpha_X f_X(T) & (2) \\ I\dfrac{d^2\Theta}{dT^2}-L_1R_a(Y-L_1\Theta)-L_2R_b(Y-L_2\Theta) & \\ \qquad +L_3R_c(X+L_3\Theta)+L_4R_c(X+L_4\Theta)=0 & (3) \end{cases}$$

In Eqs. (1), (2) and (3), constants M and I represent the mass and the moment of inertia of the structure, respectively, while L_i stand for the distance between a frame and the center of mass, G. The bilinear, hysteretic restoration R's of the frames can be expressed by geometry in terms of the dependent variables, X, Y and Θ, denoting the translations and rotation of the center of mass of the structure, relative to the ground motion, and as functions of time, T. Rotation, Θ, takes positive sign when it is measured clockwise.

Simply for the sake of convenience, the equations of motion may be represented in a non-dimensional form, by introducing such reference dimensions as k_0 (kg/cm), L_0 (cm), and λ_0 (rad.). Both independent and dependent variables in Eqs. (1), (2) and (3) are therefore made dimensionless as follows:

$$\begin{cases} \tau=\sqrt{\dfrac{k_0}{M}}T. \qquad \dfrac{d}{dT}=\sqrt{\dfrac{k_0}{M}}\dfrac{d}{d\tau}, & (4) \\ \xi=\dfrac{X}{L_0}, \qquad \eta=\dfrac{Y}{L_0}. \qquad \text{and} \qquad \theta=\dfrac{\Theta}{\lambda_0} & (5) \end{cases}$$

The canonical form of the equations of motion then becomes

$$\begin{cases} \dfrac{d^2\eta}{d\tau^2}+\kappa_a\varphi_a(\eta-\lambda_0 l_1\theta)+\kappa_b\varphi_b(\eta-\lambda_0 l_2\theta)=-\alpha_\eta f_\eta(\tau) & (6) \\ \dfrac{d^2\xi}{d\tau^2}+\kappa_c\varphi_c(\xi+\lambda_0 l_3\theta)+\kappa_c\varphi_c(\xi+\lambda_0 l_4\theta)=-\alpha_\xi f_\xi(\tau) & (7) \end{cases}$$

$$\left\{\begin{aligned}&\mu\lambda_0\frac{d^2\theta}{d\tau^2}-l_1\kappa_a\varphi_a(\eta-\lambda_0 l_1\theta)-l_2\kappa_b\varphi_b(\eta-\lambda_0 l_2\theta)\\&\qquad+l_3\kappa_c\varphi_c(\xi+\lambda_0 l_3\theta)+l_4\kappa_c\varphi_c(\xi+\lambda_0 l_4\theta)=0\end{aligned}\right. \tag{8}$$

where $l_i=L_i/L_0$, $\mu=I/ML_0^2$, and κ_i is the ratio of the initial slope of the restoring force curve for each frame to the reference spring constant, k_0.

Since the product of a constant and a function, $\kappa_i\varphi_i$, as a whole corresponds to the restoring force function R_i divided by the reference length, L_0, attention should be called to the fact that the initial slope of the non-dimensional restoring force curve φ_i should always be drawn equal to unity in the diagrams, and that the non-dimensional yield displacement should be taken equal to Δ_i/L_0.

The maximum accelerations and the complementary functions representing the pattern of ground excitation are also written in the dimensionless form as

$$\left\{\begin{aligned}&\alpha_\xi=\frac{\alpha_X M}{k_0L_0}, \qquad f_\xi(\tau)=f_X\left(\sqrt{\frac{M}{k_0}}\,\tau\right) && (9)\\&\alpha_\eta=\frac{\alpha_Y M}{k_0L_0}, \qquad f_\eta(\tau)=f_Y\left(\sqrt{\frac{M}{k_0}}\,\tau\right) && (10)\end{aligned}\right.$$

Eqs. (6), (7) and (8) are now called the fundamental equations of motion of the asymmetrical, single-story frame structure subjected to horizontal translations of the ground.

Electrical Analogy

Approximate solutions of the simultaneous, nonlinear differential equations of motion of the structure subjected to an idealized seismic motion will be obtained by using an electronic analog computer of slow-speed, indirect type. Simulation of the mechanical equations is made by assembling a circuit of integrators and summing amplifiers in such a manner that the voltage relations are analogous to the relations of some particular variables in the mechanical system.

The advantage of an analog computer is that the necessary linear as well as nonlinear components are already organized in a fashion which enables any particular circuit to be readily assembled. The bilinear restoring force characteristic of each frame of the structure can be simulated by means of utilizing a backlash element whose electric composition is shown

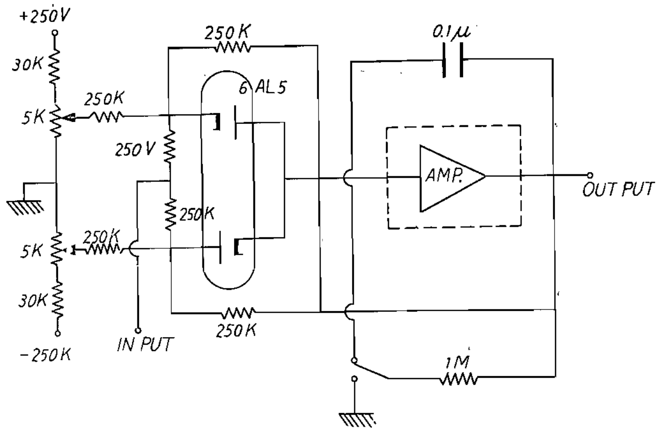


Fig. 4. Electric circuit of a backlash element.

in Fig. 4[9][10]

All linear components of the computer, located at the Department of Architecture, Kyoto University, are accurate to within 0.2% and the overall linearity of the amplifiers is reliable with a possible error less than 0.5% in the slow-speed range of operation from 0.1 to 1.0 cycle/sec. The accuracy of the nonlinear elements such as the backlash is less excellent, but it is estimated to fall within a 2.0% error. However, in simulating the mechanical equations, errors are accumulated from individual components, so that it is expected that the analog responses will check favorably with the rigorous responses, differences being on the order of ten or fifteen percent.

Scaling of Variables for the Electric Analogy

Every component of the electronic analog computer has, however, a certain limit for the feeding in-put voltage, beyond which the linearity of the component will not be guaranteed or the out-put voltage may fall in the "saturated" condition. To simulate variables of the mechanical system into the ones of the electrical circuit within the most suitable and reliable, linear voltage ranges of every component, scaling of the magnitude of the variables will therefore be desirable.

In order that the equations of motion can be expressed in terms of voltage relations, let us once again introduce a couple of parameters and transform the variables as follows :

$$\left\{\begin{array}{ll} t=p\tau=p\sqrt{\dfrac{k_0}{M}}T, \quad \dfrac{d}{dt}=\dfrac{1}{p}\dfrac{d}{d\tau}=\dfrac{1}{p}\sqrt{\dfrac{M}{k_0}}\dfrac{d}{dT} & (11) \\ x=q\xi=\dfrac{qX}{L_0}, \quad y=q\eta=\dfrac{qY}{L_0}. \quad \text{and} \quad w=q\theta=\dfrac{q\Theta}{\lambda_0} & (12) \end{array}\right.$$

The constant, p, will be chosen to afford a proper speed of computation, while the magnification factor, q, with the dimension of voltage, will be selected for the permissible voltage ranges of the components.

The equations of motion in the voltage relationships are now :

$$\left\{\begin{array}{ll} \dfrac{d^2y}{dt^2}+\dfrac{\kappa_a}{p^2}r_a(y-\lambda_0 l_1 w)+\dfrac{\kappa_b}{p^2}r_b(y-\lambda_0 l_2 w)=-\alpha_y f_y(t) & (13) \\ \dfrac{d^2x}{dt^2}+\dfrac{\kappa_c}{p^2}r_c(x+\lambda_0 l_3 w)+\dfrac{\kappa_c}{p^2}r_c(x+\lambda_0 l_4 w)=-\alpha_x f_x(t) & (14) \\ \mu\lambda_0 q\dfrac{d^2w}{dt^2}-l_1\dfrac{\kappa_a}{p^2}r_a(y-\lambda_0 l_1 w)-l_2\dfrac{\kappa_b}{p^2}r_b(y-\lambda_0 l_2 w) & \\ \qquad +l_3\dfrac{\kappa_c}{p^2}r_c(x+\lambda_0 l_3 w)+l_4\dfrac{\kappa_c}{p^2}r_c(x+\lambda_0 l_4 w)=0 & (15) \end{array}\right.$$

in which

$$\left\{\begin{array}{ll} \alpha_x=\dfrac{q\alpha_\xi}{p^2}=\dfrac{q\alpha_X M}{p^2 k_0 L_0}. \quad f_x(t)=f_\xi\left(\dfrac{t}{p}\right)=f_X\left(\dfrac{1}{p}\sqrt{\dfrac{M}{k_0}}\,t\right) & (16) \\ \alpha_y=\dfrac{q\alpha_\eta}{p^2}=\dfrac{q\alpha_Y M}{p^2 k_0 L_0}, \quad f_y(t)=f_\xi\left(\dfrac{t}{p}\right)=f_Y\left(\dfrac{1}{p}\sqrt{\dfrac{M}{k_0}}\,t\right) & (17) \end{array}\right.$$

It should be noted that the restoring force functions with respect to these coordinate systems are associated with each other in the relationship, that is,

$$\frac{q}{k_0 L_0}R_a(Y-L_1\Theta)=q\kappa_a\varphi_a(\eta-\lambda_0 l_1\theta)=\kappa_a r_a(y-\lambda_0 l_1 w) \qquad (18)$$

and so forth. The yield displacement of each frame, with the magnitude of Δ, will be Δ/L_0 in the dimensionless coordinates, and will now be equal to $q\Delta/L_0$ in the electrical analogy. The secondary slope, k_2, of the restoring force curve should also be scaled in proportion to the initial slope, k_1.

The block diagram of assembling necessary electronic elements to obtain approximate solutions of Eqs. (13), (14) and (15) is shown in Fig. 5. At the top of the figure there is shown a circuit to produce a sinusoidal forcing function, while the bottom of the figure is the circuit simulating the simultaneous differential equations of motion of the structure. Capital

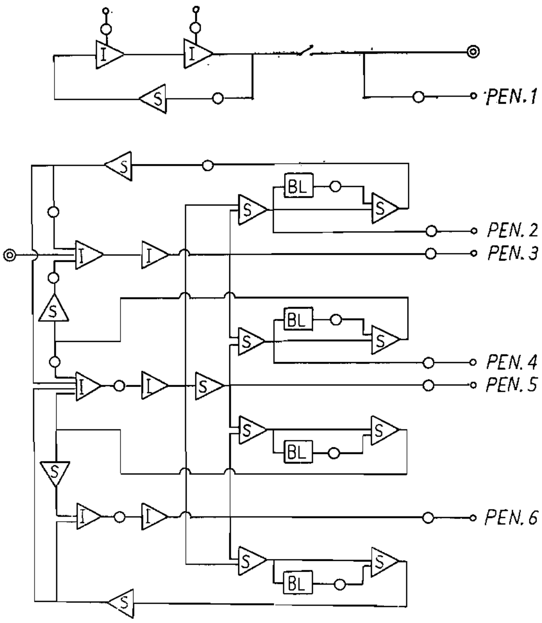


Fig. 5. Block diagrams simulating the equations of motion of the structure and the forcing functions. I: Integrator; S: Summing amplifier; BL: Backlash element; and circles in the diagrams denote potentiometers.

letters, I and S, in the triangles represent the integrators and summing amplifiers, and small circles stand for the potentiometers by which the coefficients in the equations can be adjusted.

The results of the computation, the displacements or velocities of the frames, are recorded on a direct-writing oscillograph, whose elements are illustrated in Fig. 5 by Pens No. 1 to No. 6.

Analytical Results

The maximum dynamic response of each frame of the structure during the periods of forced- and free-vibration eras was of primary concern in this investigation. Therefore, all results are presented by plotting dimensionless stбry displacement ratios against either the intensity of the ground motion or the fundamental elastic period of vibration of the structure.

For this particular structure, the X-component of the ground motion may be of less interest, since the excitation of the X-component alone will cause only translations of the floor. A large torsion of the structure is expected mainly due to the Y-component of the ground motion. Therefore, the analysis has been carried out by assuming that the ground acceleration in the X-direction is always equal to zero.

Intensity of the idealized ground motion of a versed-sine pulse type,

$$Y_G = \frac{a_Y T_G^2}{4\pi^2}\left(1 - \cos\frac{2\pi T}{T_G}\right), \tag{19}$$

against the maximum of the elastic strength of the structure has been expressed in terms of a parameter,

$$C = a_Y M/(B_a + B_b). \tag{20}$$

Namely, the meaning of this parameter will be comprehensible in such a fashion that a static force, of the magnitude corresponding to $C > 1$, will cause plastic deformations in both of the frames in parallel with the Y-axis, and vice versa, if the reaction of each frame is proportional to the elastic rigidity.

Most of the analytical results have been related to idealized structures consisting of the frames of equal span length, whose secondary slopes k_2 of the restoring force curves were all made equal to zero. Dimensionless quantities in Eqs. (13), (14) and (15) were chosen in the electric analogy analysis as follows:

$$l_1 = -l_2 = l_3 = -l_4 = \frac{1}{2}, \qquad \lambda_0 = 1, \qquad \text{and} \qquad \mu = \frac{I}{ML_0^2} = \frac{1}{6} \tag{21}$$

Also, concerning to the variation in the plastic restoring force of each frame, a comparison has been attempted on the dynamic responses obtained for two types of idealized structures. For idealized structure ***A***, all yield displacements were taken to be equal so that the yield point restoring force of each frame is associated directly with the elastic rigidity, while for idealized structure ***B*** the yield-point restoring forces were assumed to be proportional to the square root of the elastic rigidity of each frame. The assumptions are purely hypothetical, but it was stated earlier that such mechanical characteristics of structures can be designed practically.

The periods of linear vibration of the structures considered were given in terms of the initial slope, k_1, of frame (a) and the mass of the structures, M, as follows[1] :

$T_X = 2\pi/\sqrt{5.00\,k/M}$ ······vibration in the plane parallel to the X-axis,

${}_1T_{Y\Theta} = 2\pi/\sqrt{4.55\,k/M}$ ······fundamental mode of coupled vibration, and

${}_2T_{Y\Theta} = 2\pi/\sqrt{25.47\,k/M}$······higher mode of coupled vibration.

In addition, it may be of interest to refer the periods of vibration of the structures with a pair of either flexible or rigid frames being installed in parallel with the Y-axis. The periods of linear vibration for the flexible and rigid structures are :

$T_f = 2\pi/\sqrt{2.00\,k/M}$ ······for the flexible structure, and

$T_r = 2\pi/\sqrt{16.00\,k/M}$······for the rigid structure.

All basic results of the investigation are shown in Figs. 6–17. In these figures, the dimensionless intensity parameter C ranges from 0.1 to 1.25, the parameter standing for the duration of the idealized ground motion, $T_G/{}_1T_{Y\Theta}$, ranges from 0.435 to 3.461, and zero viscous damping is generally considered.

Discussion of Results

A thorough observation of the dynamic response curves of the idealized structures has clarified that the maximum story-displacement or the maximum amplitude of oscillation of the structures has almost always occurred during the forced vibration eras, and in many cases of a large value of intensity parameter C, the maxima have been found within the first cycle of the ground motion. A pretty large amount of vibration energy has been found to dissipate from the systems due to the elasto-plastic characteristics of the restoring forces, and the amplitudes of the structures were appreciably reduced in the residual, free vibration eras.

Figs. 6(**a**), (**b**), (**c**) and (**d**) were illustrated first to compare the plots under the action of a three-cycle ground displacement with those for only one-cycle of the idealized ground motion as correspondingly shown in Figs. 8(**a**), (**b**), (**c**) and (**d**). Coincidence of the plots for the two cases, in the magnitudes and the general features, would confirm us that for the bi-

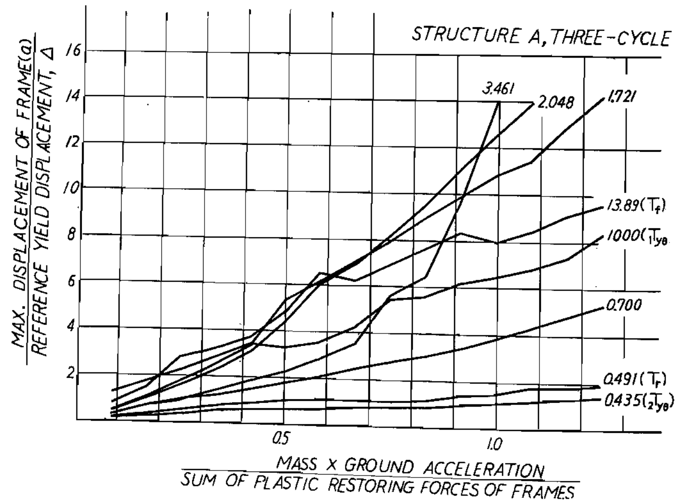


Fig. 6(a).

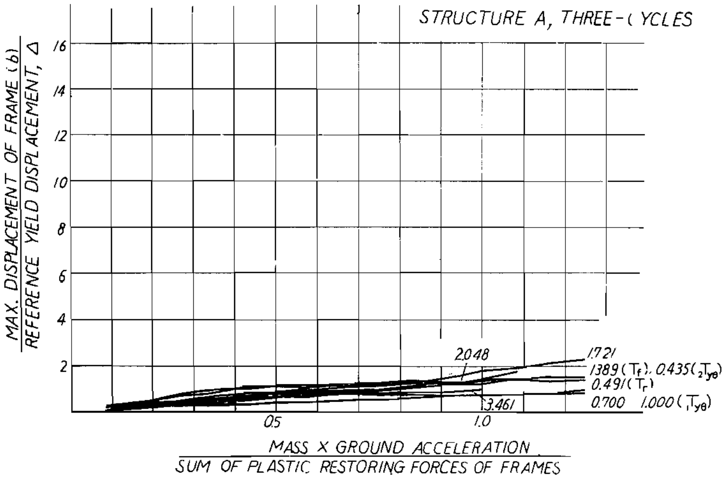

STRUCTURE A, THREE-(YCLES
MAX. DISPLACEMENT OF FRAME (b)
REFERENCE YIELD DISPLACEMENT, Δ
16
14
12
10
8
6
4
2
2.048
1.721
1.389 (Tf), 0.435 (2Tye)
0.491 (Tr)
3.461
0.700 1.000 (1Tye)
0.5
1.0
MASS X GROUND ACCELERATION
SUM OF PLASTIC RESTORING FORCES OF FRAMES

Fig. 6(b).

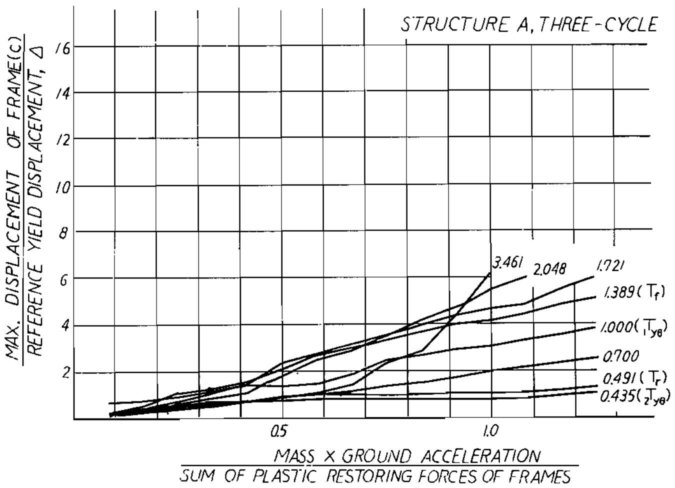

STRUCTURE A, THREE-CYCLE
MAX. DISPLACEMENT OF FRAME (c)
REFERENCE YIELD DISPLACEMENT, Δ
16
14
12
10
8
6
4
2
3.461
2.048
1.721
1.389 (Tf)
1.000 (1Tye)
0.700
0.491 (Tr)
0.435 (2Tye)
0.5
1.0
MASS X GROUND ACCELERATION
SUM OF PLASTIC RESTORING FORCES OF FRAMES

Fig. 6(c).

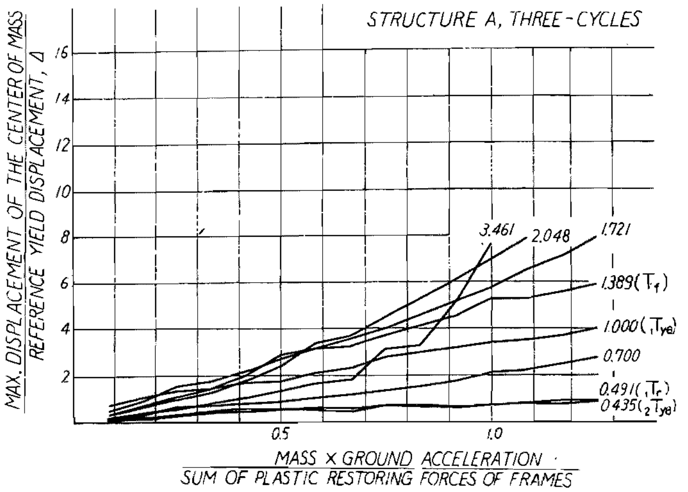


Fig. 6(d).

Fig. 6. Maximum displacements of structure *A* subjected to a three cycle, versed-sine ground displacement.

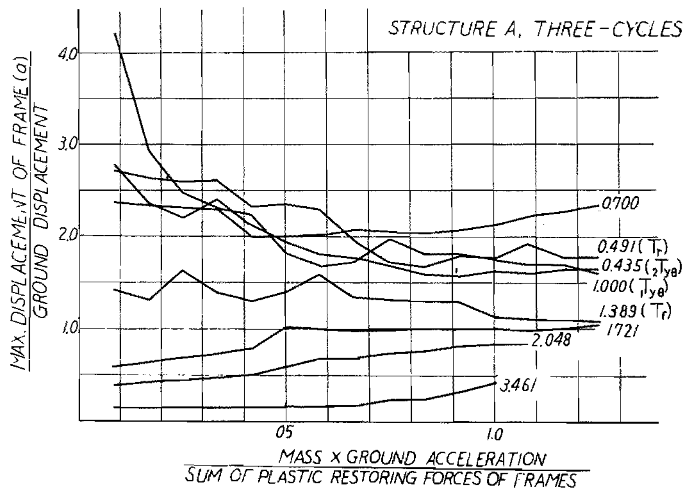


Fig. 7. Ratios of the maximum displacements to the ground displacement.

linear vibration systems the effects of the number of cycles of the transient excitation on the maximum dynamic response of the system may be of less significance. Also, a comparison of Fig. 7 with Fig. 9(**a**) will suggest that the growth of the maximum displacement of frame (**a**) due to a repetitive action of the idealized ground displacement will be apparent only for a small value of C, namely, for almost elastic linear vibrations. Consequently, more attention will hereafter be paid to the maximum dynamic responses of the structures due to one cycle of the ground displacement.

Maximum Displacements

Elasto-plastic responses plotted in Figs. 8(**a**), (**b**), (**c**) and (**d**) will indicate the maximum displacements of each frame of idealized structure ***A*** and of the center of mass of the structure from the initial equilibrium positions. The abscissas of the figures are presented in terms of the intensity parameter C of the ground motion, while the ordinates are standing for the ratios of the elasto-plastic displacements to the reference elastic-limit displacement of the frame, which is sometimes called the "ductility ratio" or the "ductility factor". Most of the maximum displacements are found to be in excess of the elastic limit.

For a small value of the period of ground displacement, T_G, it will be seen that the maximum displacement of frame (**a**) varies approximately as the first power of the intensity parameter. However, for a larger value of T_G, the maximum displacement varies as a higher power of C. Since the maximum displacement of frame (**b**) is far small in comparison with those of frame (**a**), it will be noted that the displacement of the center of mass is about one half of the displacement of frame (**a**). The angle of rotation of the structure can be estimated from the maximum displacement of frame (**c**), which in this case is developed entirely due to the torsion of the structure.

In Figs. 9(**a**), (**b**), (**c**) and (**d**), the maximum displacements are plotted in terms of the ratios to the amplitude of the ground motion. It may be of interest to note that the maximum displacement ratios are less than unity for any value of C if the parameter $T_G/{}_1T_{Y\theta}$ is larger than a certain value ; i.e., $T_G/{}_1T_{Y\theta}\doteq 1.72$ for frame (**a**), 0.70 for frame (**b**), and 1.00 both for frame (**c**) and the reconciled maximum displacement of the center of mass of the structure. That is to say, the relative story displacements of

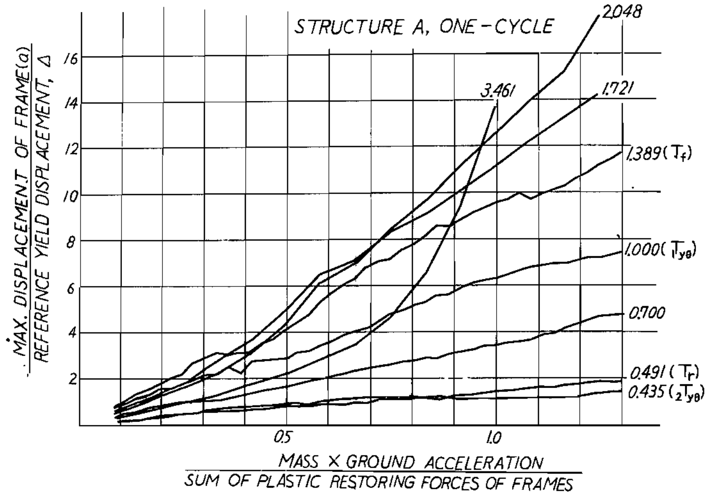

STRUCTURE A, ONE-CYCLE
MAX. DISPLACEMENT OF FRAME(a) / REFERENCE YIELD DISPLACEMENT, Δ
MASS X GROUND ACCELERATION / SUM OF PLASTIC RESTORING FORCES OF FRAMES
2.048
3.461
1.721
1.389 (Tf)
1.000 (1Ty8)
0.700
0.491 (Tr)
0.435 (2Ty8)

Fig. 8(a).

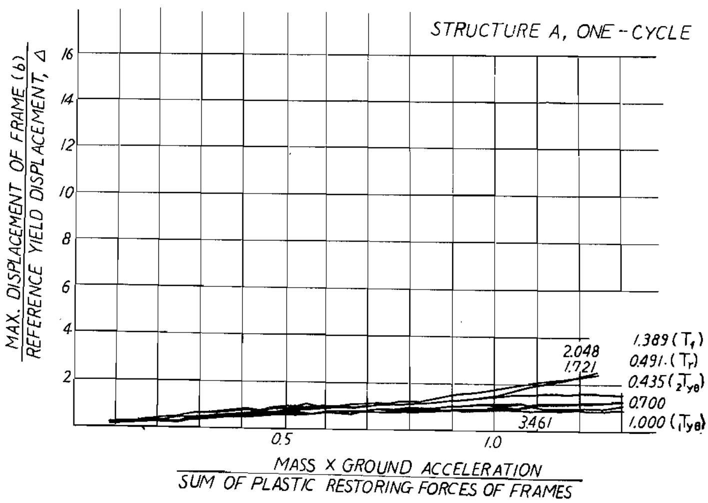

STRUCTURE A, ONE-CYCLE
MAX. DISPLACEMENT OF FRAME (b) / REFERENCE YIELD DISPLACEMENT, Δ
MASS X GROUND ACCELERATION / SUM OF PLASTIC RESTORING FORCES OF FRAMES
2.048
1.721
3.461
1.389 (Tf)
0.491 (Tr)
0.435 (2Ty8)
0.700
1.000 (1Ty8)

Fig. 8(b).

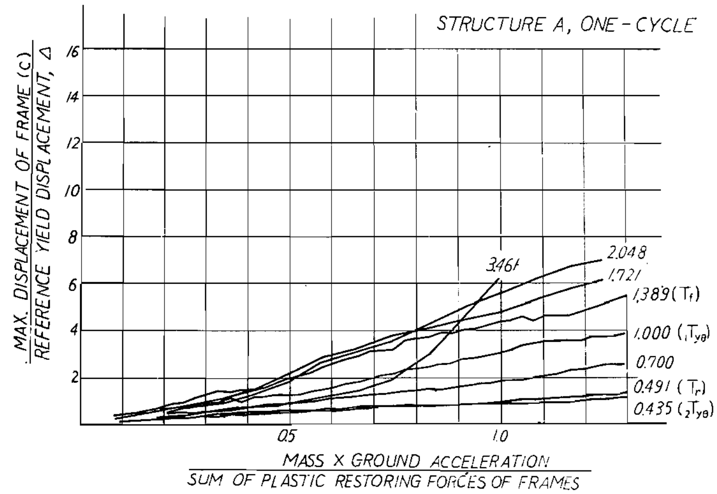


Fig. 8(c).

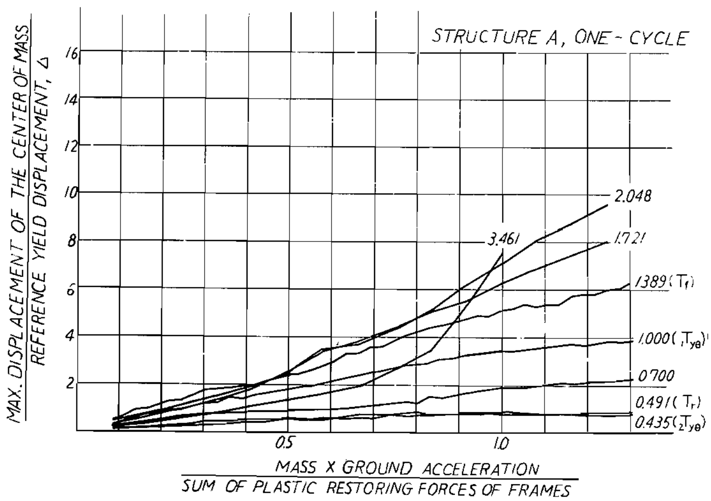


Fig. 8(d).

Fig. 8. Maximum displacements of structure A subjected to one-cycle, versed-sine ground displacement.

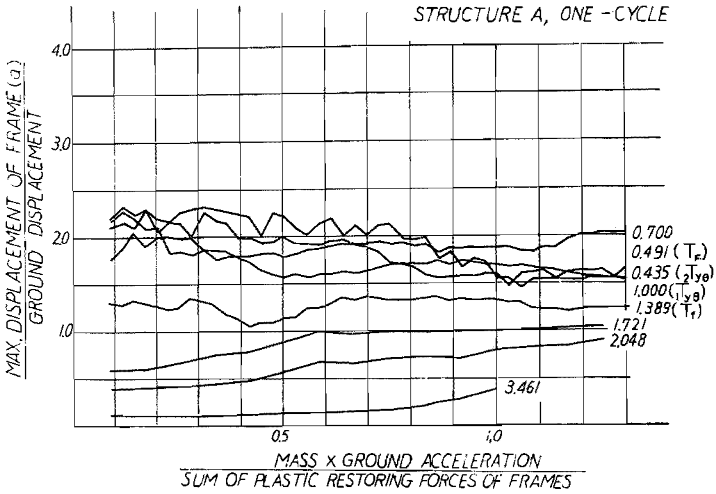

STRUCTURE A, ONE -CYCLE
MAX. DISPLACEMENT OF FRAME (a)
GROUND DISPLACEMENT
4.0
3.0
2.0
1.0
0.700
0.491 (T_F)
0.435 (₂T_yB)
1.000 (₁T_yB)
1.389 (T_f)
1.721
2.048
3.461
0.5
1.0
MASS x GROUND ACCELERATION
SUM OF PLASTIC RESTORING FORCES OF FRAMES

Fig. 9(a).

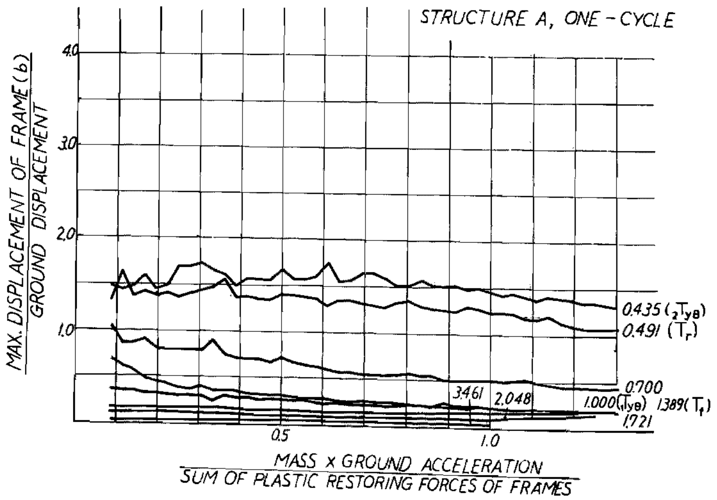

STRUCTURE A, ONE -CYCLE
MAX. DISPLACEMENT OF FRAME (b)
GROUND DISPLACEMENT
4.0
3.0
2.0
1.0
0.435 (₂T_yB)
0.491 (T_F)
0.700
3.461
2.048
1.000 (₁T_yB)
1.389 (T_f)
1.721
0.5
1.0
MASS x GROUND ACCELERATION
SUM OF PLASTIC RESTORING FORCES OF FRAMES

Fig. 9(b).

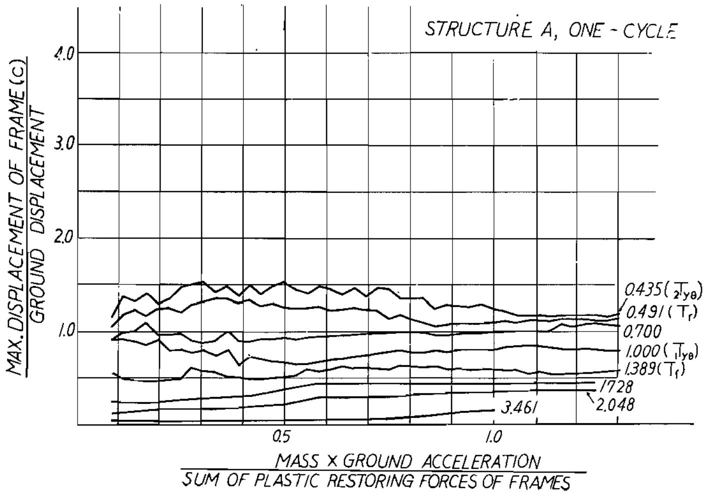


Fig. 9(c).

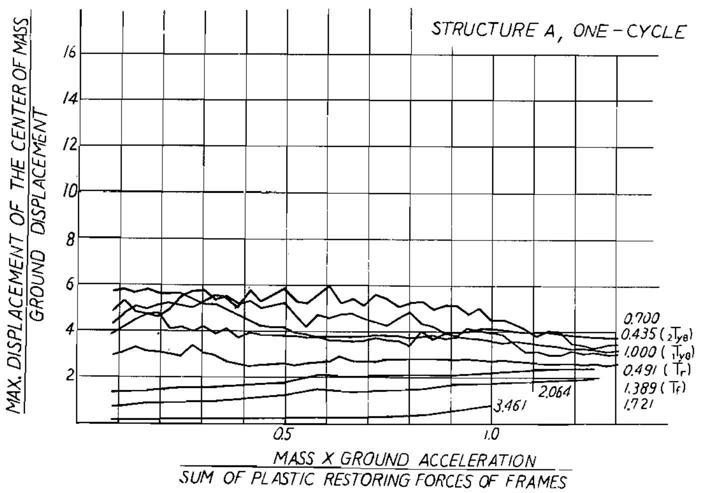


Fig. 9(d).

Fig. 9. Ratios of the maximum displacements to the ground displacement.

the frames can be expected to have smaller absolute values than that of the ground motion if the duration of the ground motion is long enough.

Another principal feature of the figures will be summarized in such a way that all the responses plotted do not depend very much upon the intensity parameter C but upon the period of ground motion. And, this fact may strongly suggest us that for the nonlinear, torsional vibration of one-story structures it will not be impossible to make a rough estimation of the maximum displacements of the structures by a reconciliation of the amplitude and the corresponding period of a ground motion if these values will be known.

Instead of the maximum displacements of the structure, the maximum total amplitudes, or the magnitudes of the maximum swing in every response-time curve of the frames, are plotted in Figs. 10 and 11. These quantities were interesting because we have taken into consideration the fact that the equilibrium position of each frame is shifted by the intensive ground motions to result the "permanent set" being increased or decreased. The maximum total amplitudes will in many cases be able to represent more clearly the maximum plastic distorions of the frames.

In Fig. 10, the principal features of the plots are somewhat similar to those mentioned already for the plots shown in Fig. 8. Namely, the exceedingly large distortions take place in frame (**a**) when the structure is subject-

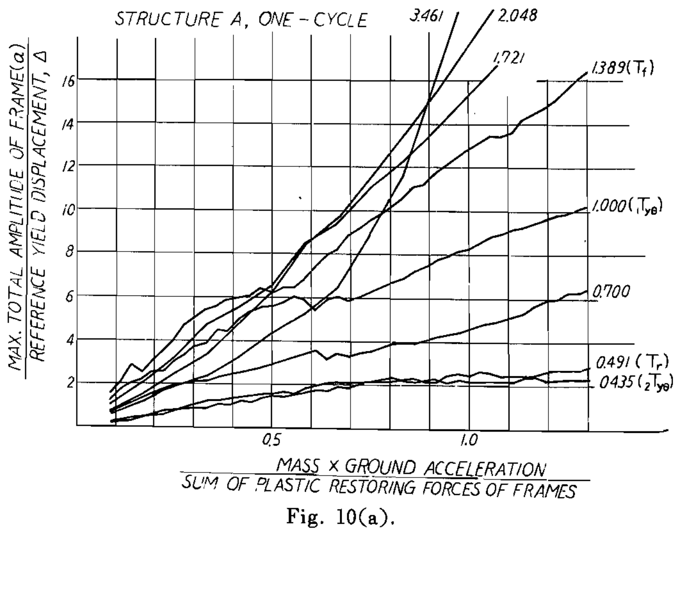


Fig. 10(a).

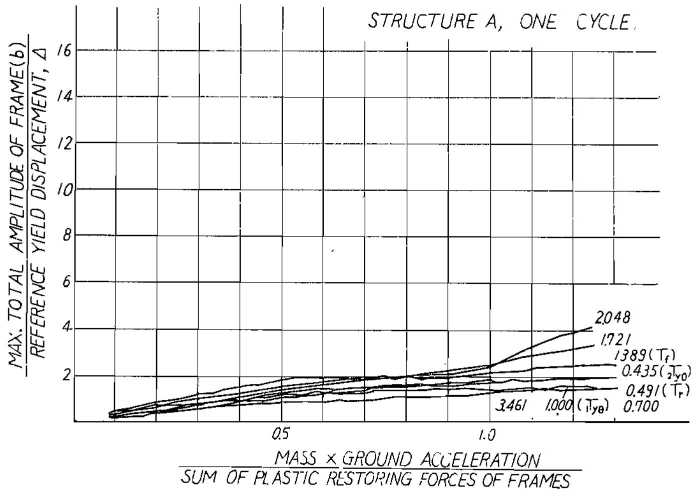


Fig. 10(b).

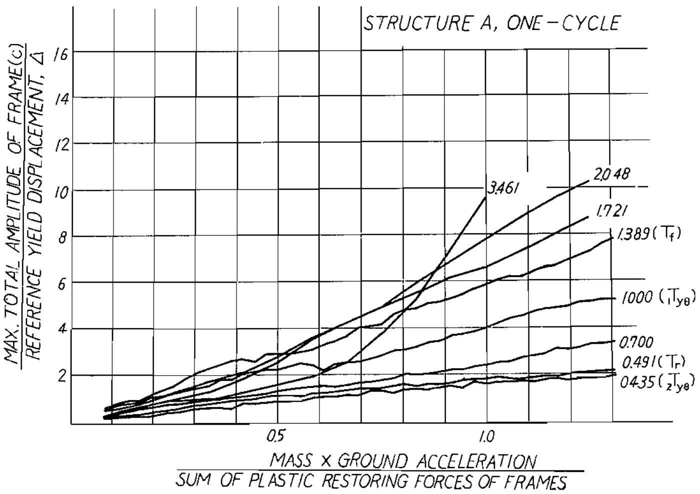


Fig. 10(c).

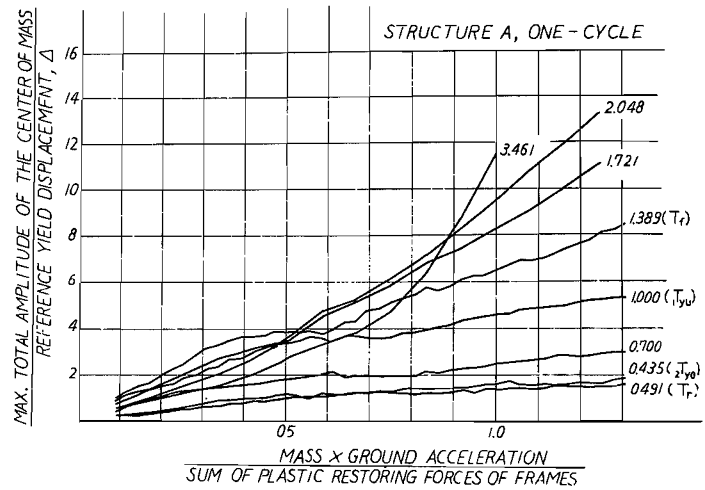


Fig. 10(d).

Fig. 10. Maximum total amplitudes of structure *A* subjected to one-cycle, versed-sine ground displacement.

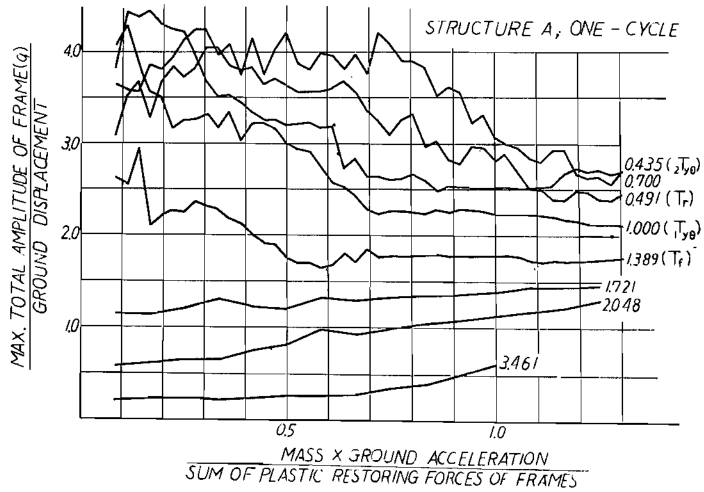


Fig. 11(a).

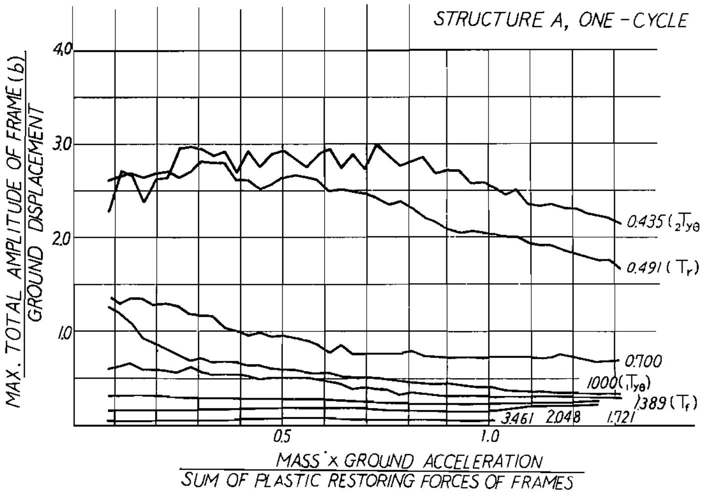


Fig. 11(b).

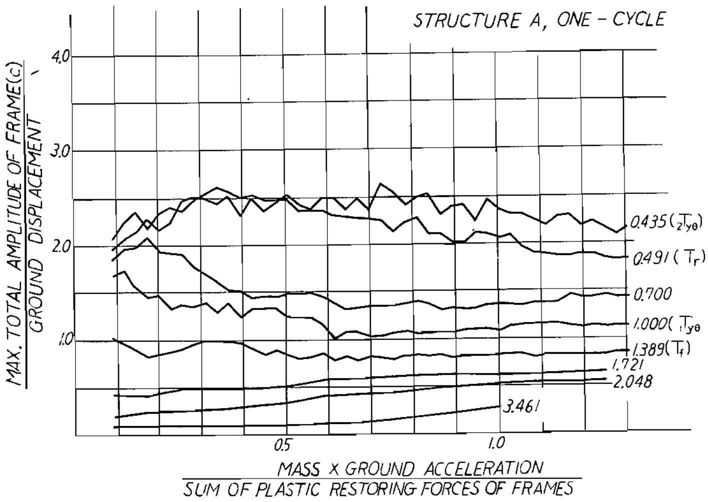


Fig. 11(c).

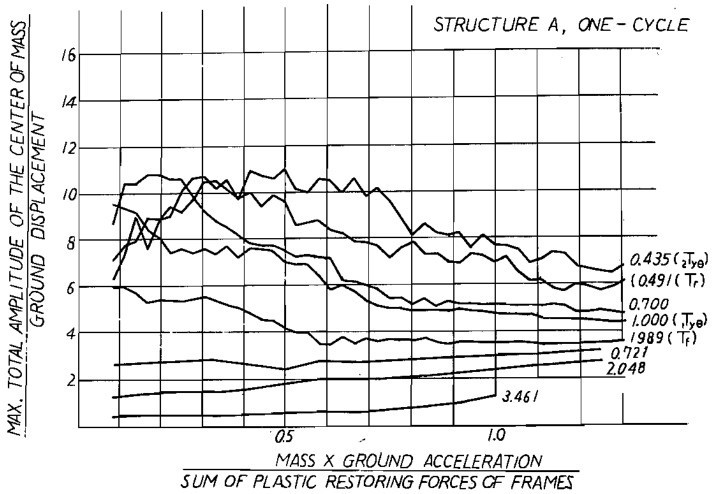


Fig. 11(d).

Fig. 11. Rations of the maximum total amplitudes to the ground displacement.

ed to a one-cycle ground displacement with a large value of C and with a duration longer than the fundamental period of vibration of the structure.

Figs. 11(**a**), (**b**), (**c**) and (**d**) indicates the ratios of the maximum total amplitudes to the ground displacement. The ordinates in this representation are approximately twice as large as those corresponding to Figs. 9(**a**), (**b**), (**c**) and (**d**), except in the range of a large value of C. It will be easily found that the differences between the plots for various values of $T_G/_1T_{Y\Theta}$ become smaller as the intensity parameter C is increased.

Maximum Dynamic Response of Structure B

Maximum displacements of frames (**a**), (**b**) and (**c**) of idealized structure B are shown in Figs. 12(**a**), (**b**) and (**c**). The structure is supposed to have the identical dynamic characteristics as those of idealized structure A unless the oscillation will produce strains which exceed the elastic limit. Simply because of the assumption on a different type of the plastic restoring force as well as the yield displacement for each frame, the responses shown in Fig. 12 have presented an interesting feature that, between the ductility factors of frames (**a**) and (**b**), any striking disparity will not be found.

Figs. 13(**a**) and (**b**) are to be compared with Figs. 9(**a**) and (**b**), respectively. The resembrance or mutual agreement of the general features

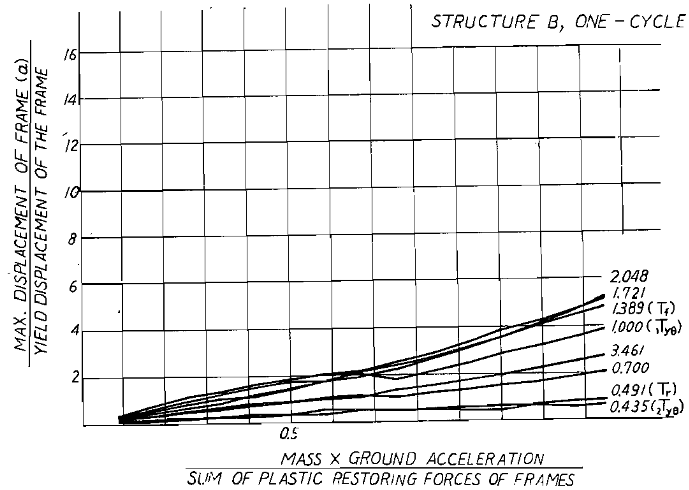


Fig. 12(a).

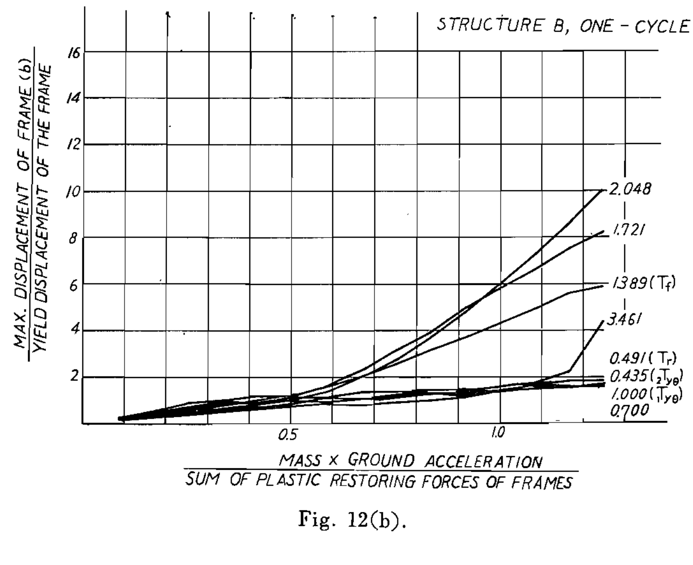


Fig. 12(b).

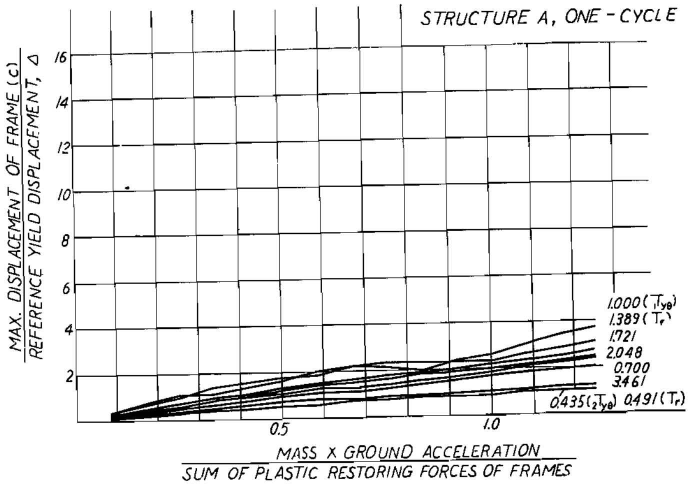


Fig. 12(c).

Fig. 12. Maximum displacements of structure ***B*** subjected to one-cycle, versed-sine ground displacement.

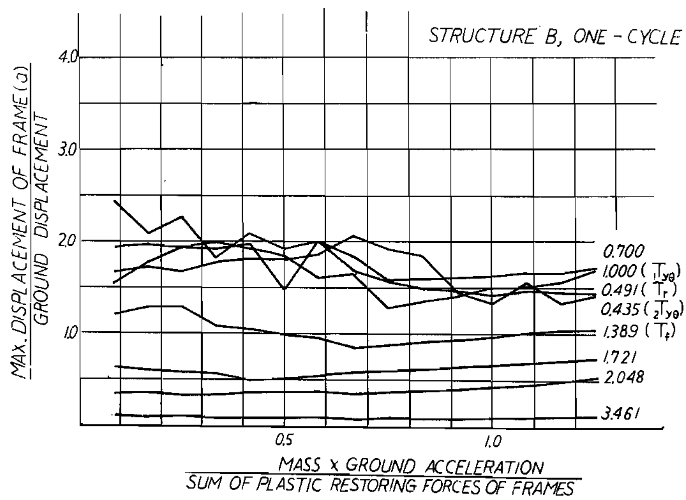


Fig. 13(a).

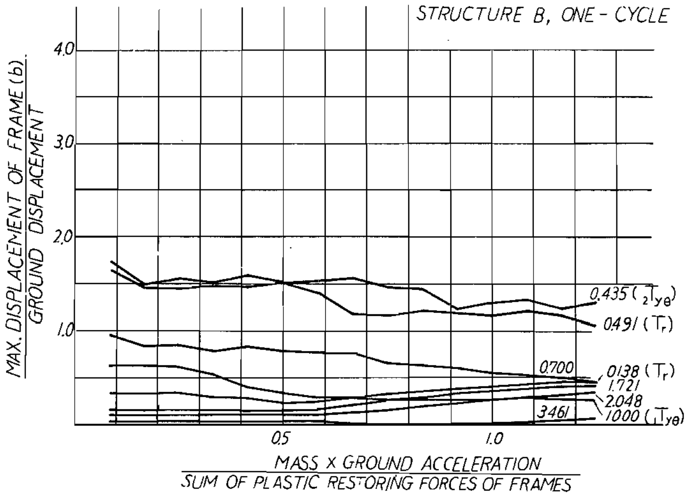


Fig. 13(b).

Fig. 13. Ratios of the maximum displacements to the ground displacement.

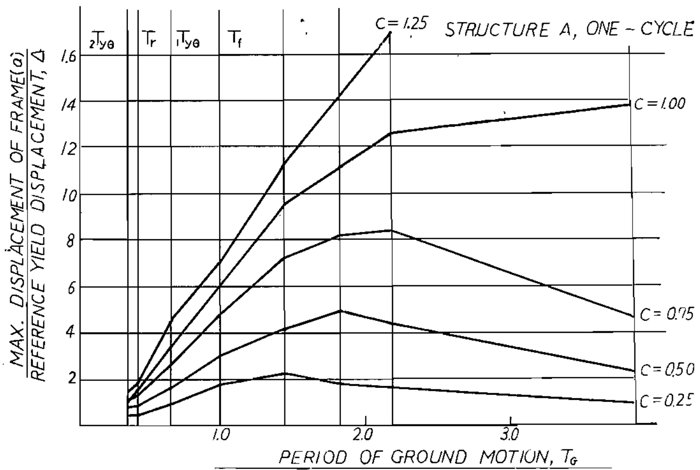


Fig. 14. Spectra of the maximum displacements of frame (**a**) in structure *A*, with C=Mass times ground acceleration/sum of the plastic restoring forces of frames (**a**) and (**b**) as a parameter : —— one-cycle ground displacement.

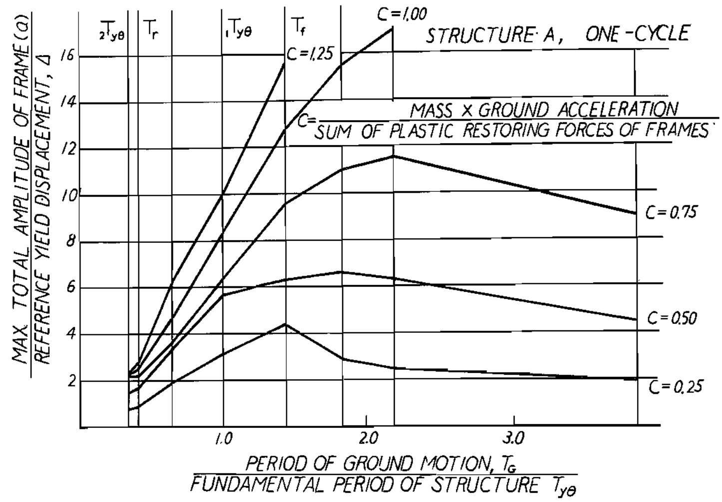


Fig. 15. Spectra of the maximum total amplitude of frame (**a**) in structure A with C as a parameter —— one-cycle ground displacement.

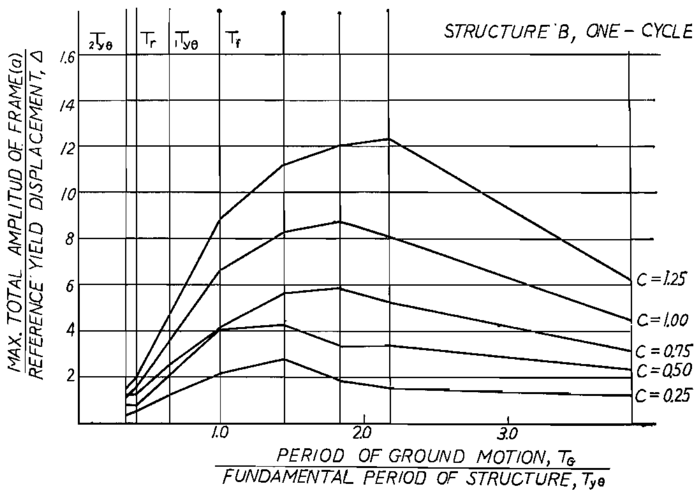


Fig. 16(a).

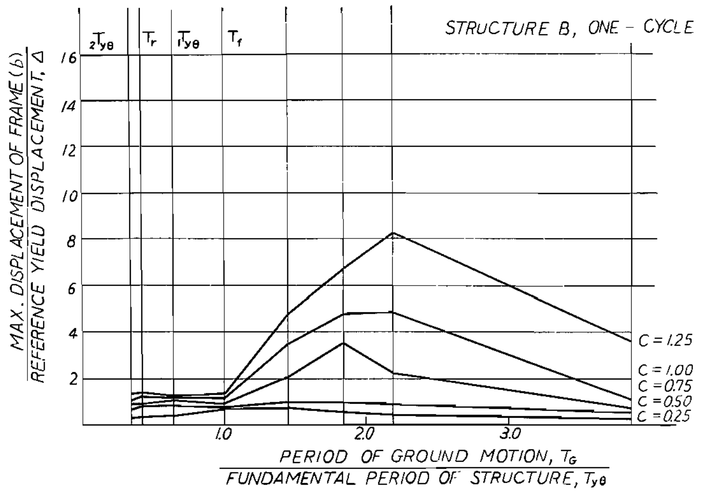


Fig. 16(b).

Fig. 16. Spectra of the maximum displacements of frames (a) and (b) in structure $\boldsymbol{B}$, with parameter C —— one-cycle ground displacement.

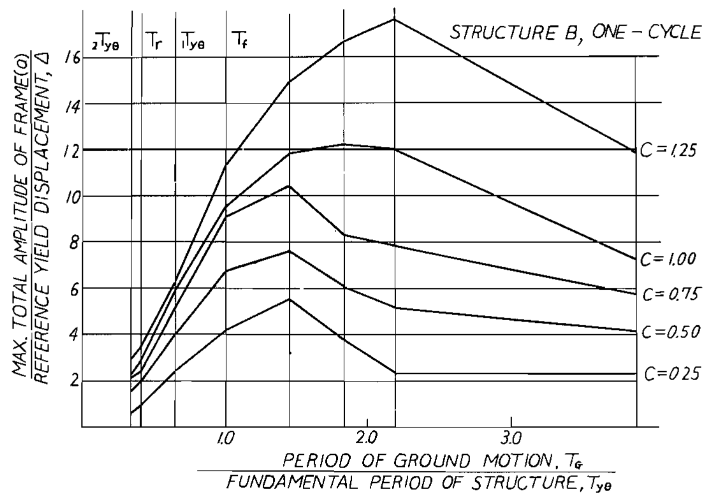


Fig. 17(a).

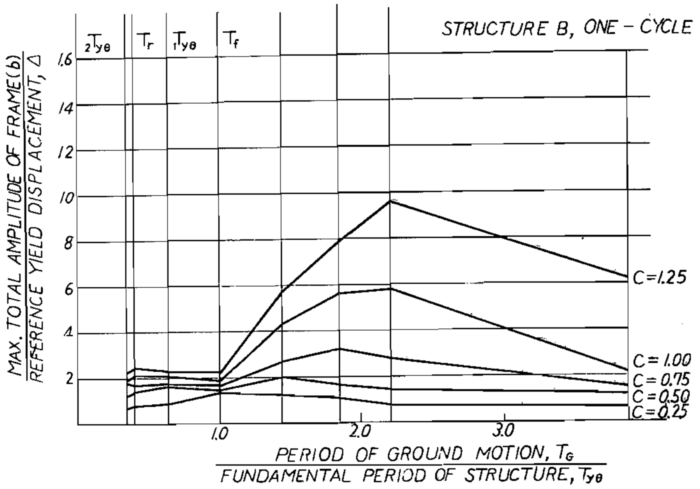


Fig. 17(b).

Fig. 17. Spectra of the maximum total amplitudes of frames (**a**) and (**b**) in structure B —— one cycle ground displacement.

of these figures would be enough to assure us of the facts having been discussed as for Fig. 9.

Elasto-plastic Response Spectra

In Figs. 14–17, the maximum dynamic responses are plotted against the dimensionless period of the ground motion. The effect of the period of the ground motion on the maximum displacement or the maximum total amplitude of each frame is now very appreciable. Most of the response spectra indicates to have an apparent peak located somewhere in the range of T_G greater than the fundamental period of vibation of the structure. With an increase in C, the peak shifts its location toward a larger value of $T_G/{}_1T_{Y\theta}$.

Conclusions

Based on the results of this investigation using the idealized ground motions, the following general conclusions are deduced :

1) For the simplified asymmetric structures, effect of the repetitive action of idealized ground motions on the maximum dynamic response of the structures may be of little significance.

2) The maximum responses of the structure will depend mostly on the

ground displacement if the probable range of the period of ground motion will be known.

3) A rough estimation of the maximum distortions of the structure will possibly be made by reconciliation, if dynamic characteristics of the ground motion as well as the structure will be given.

The possibility of unstable responses of the asymmetric structures has been examined. The results of analysis will be illustrated in the following Appendix.

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Appendix

On the Unstability in the Dynamic Response of Asymmetric Structure

In the foregoing analysis, little attention has been paid to the motion of the center of mass in the direction parallel to the X-axis. Under the assumptions of the analysis, one might have understood tacitly that the center of mass of the floor will move back and forth only in the direction parallel with the Y-axis and displacement X will not develop, provided that the X-component of the ground motion is zero. Now, the questions arise: "May this statement be true even for a large amplitude of rotation? Will the equilibrium of the structure at $X=0$ be stable when the restoring force of each frame exceeds the elastic limit?"

We proceed to study the problem on the stability of the motion of the structure at the center of mass under the condition that $d^2X_G/dT^2=0$. Instead of the bilinear restoring force, let us now consider a soft spring type restoration characteristic for the asymmetric, one-story structure. The soft spring type restoration will be in the form

$$R(u)=ku-\beta u^3+\gamma u^5+\cdots\cdots \qquad (k>0, \text{ and } \beta>0) \tag{A-1}$$

This assumption is usually justified for a certain type of structures, if the distortion u is not too large.

For a small value of u, the restoration may be replaced with the first two terms, so that the equation of motion of the structure in the X-direction can be written from Eq. (2) as

$$M\frac{d^2X}{dT^2}+\left(2k-6\beta\frac{L^2}{4}\Theta^2\right)X-2\beta X^3=-M\frac{d^2X_G}{dT^2} \tag{A-2}$$

When the ground acceleration, d^2X_G/dT^2, is always zero, it is easily noted that a solution of Eq. (A-2) will be $X=0$. However, the nonlinear differential equation will be with a variable coefficient since the rotation Θ

arises from the ground displacement in the Y-direction, so that the discussion on stability of the solution $X=0$ may be important.

It is perhaps of interest to interpolate at this point the treatment of the stability problem on a phase-plane analysis, as one does in the theory of nonlinear oscillations. In accordance with our usual practice we replace d^2X/dT^2 in Eq. (A-2) by $V(dV/dX)$ to obtain the first order equation

$$\frac{dV}{dX}=-\frac{1}{M}\frac{a_1X+a_3X^3}{V} \tag{A-3}$$

with a_1 and a_3 being

$$\begin{cases} a_1=2k-\dfrac{3}{2}\beta L^2\Theta^2 & \text{(A-4)} \\ a_3=-2\beta & \text{(A-5)} \end{cases}$$

Consideration about the characteristic of Eq. (A-3) at the origin, $X=0$, will be analogous to examining the stability of the equation

$$\frac{dV}{dX}=-\frac{1}{M}\frac{a_1X}{V} \tag{A-6}$$

on a phase plane. The origin ($X=0$, $V=0$) in the phase plane corresponds to a singular point, and the sign of a_1 is now decisive for the characteristic of the singularity. Poincare-Liapunof's theory states that if the ceofficient a_1 is positive the equilibrium point (0, 0) in the phase plane is called "center", while it is a saddle point" if a_1 is negative[11]

Physically, it will be illustrated in such a fashion that, if $a_1>0$, a slight disturbance from the equilibrium position will result in a small oscillation of frames. However, if $a_1<0$, the motion of the center of mass of the structure departs widely from the equilibrium position upon the slightest disturbance.

From Eq. (A-4), we let

$$a_1=2k-\frac{3}{2}\beta L^2\Theta^2=0 \tag{A-7}$$

and this will now furnish us the critical value of Θ,

$$\Theta_{crit.}=\pm\sqrt{\frac{4k}{3\beta L^2}} \tag{A-8}$$

Hence, the above discussions will be stated once again as;

1) if $\Theta<\Theta_{crit.}$, oscillation of the structure in the X-drection will be stable, but

2) if $\Theta > \Theta_{crit.}$, the oscillation is unstable.

Further discussions of the unstability of the oscillation of the structure were made in the previous works of the authors[5),6)]. In this paper, it has been attempted to make sure that the results of the discussions will hold and whether such unstable phenomena will be observed at $X=0$ in the analog computer analysis. Therefore, a similar setting up of the problem has been made, and transient vibration of the asymmetric, single-story framed structure (Fig. 1) were studied.

In Fig. 18, dynamic response curves are shown for the structures with frames (**a**), (**b**) and (**c**), all of which have bilinear restoration characteristics. Each set of the response curves illustrates the changes in the quantities, x, $-\frac{1}{2}w$, $-\left(y-\frac{1}{2}w\right)$, y, $-\left(y+\frac{1}{2}w\right)$, and the forcing function

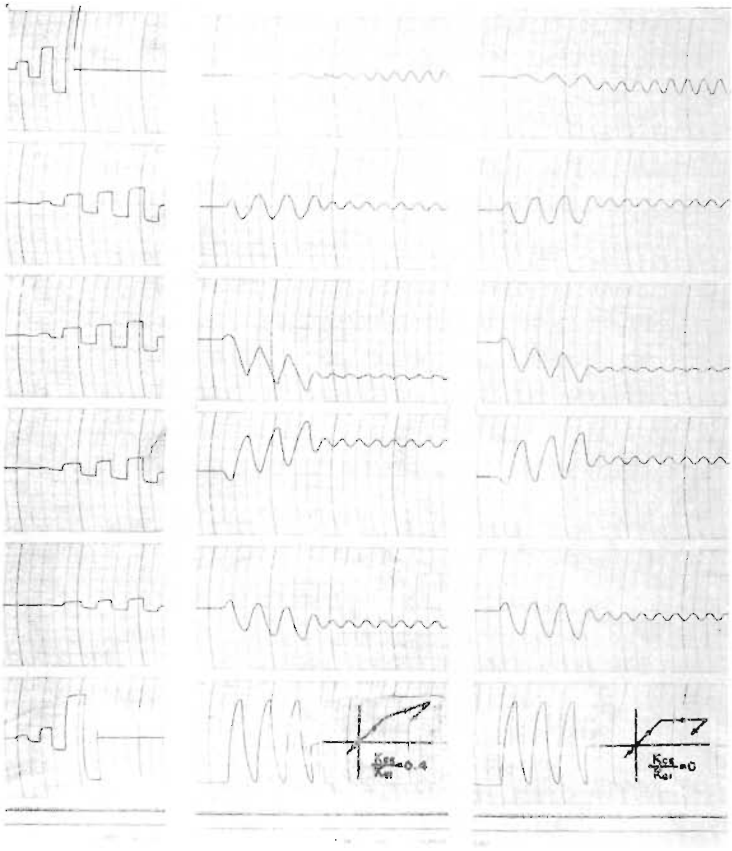

Fig. 18(a). Fig. 18(b). Fig. 18(c).

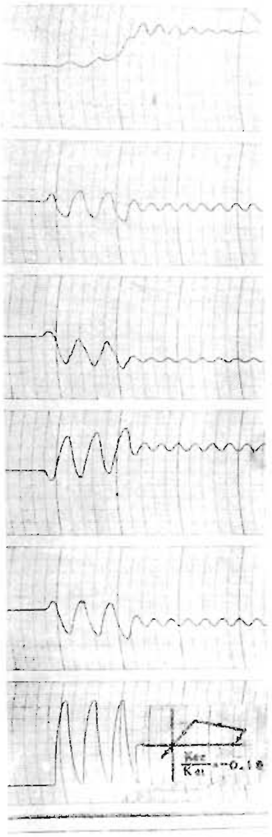

Fig. 18(d).

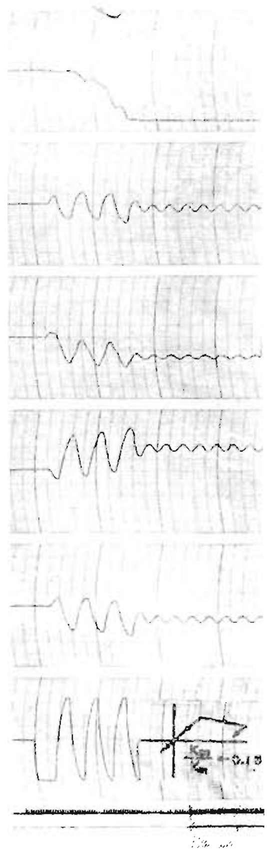

Fig. 18(e).

Fig. 18. Dynamic response curves of the structures with various restoring force characteristics of bilinear type, recorded with a direct-writing oscillograph.
(a) Calibrations,
(b) $k_2/k_1=0.4$, response curve for x is stable,
(c) $k_2=0$, response x is likely unstable,
(d) and (e) $k_2/k_1=-0.18$, response curve for x is apparently unstable, but other response curves are stable.

with respect to time. The voltages of these quantities should be checked upon the calibrations shown in Fig. 18(**a**), since the amplitudes of the curves were enlarged or reduced with different magnification factors of the direct-writing oscillograph in order that all the response curves would fall within a moderate amplitude range. It should also be noted that the amplitude of response x is enlarged about ten times as large as the other responses.

Three types of bilinear, hysteretic restoring forces were assumed here for frame (**c**) of the structure. Namely, the ratios of the secondary slope k_2 of the restoring force curve to the initial reference slope k_1 were chosen as: (i) $k_2/k_1=0.4$, (ii) $k_2/k_1=0$ (ideal plastic), and (iii) $k_2/k_1=-0.18$.

The response of x, the motion of the center of mass of the structure in the direction parallel to the X-axis, is now interesing. Fig. 18(**b**) shows that for the case of an increasing slope k_2 the response x is stable and that a slight disturbance has resulted in a small vibration about the equilibrium position. The response x in Fig. 18(**c**) now suggests a possibility of unstable response as the motion of the center of mass of the structure departs from the position of equilibrium after the action of the ground motion is terminated.

Finally, it should be noticed that two cases of computation as shown in Figs. 18(**d**) and (**e**) were made under the identical conditions of ground excitation and structure, respectively. However, the sign of the responses is opposite each other and the response curves have departed widely from the equilibrium position during the forced vibration eras.

The results of the electric analogy analysis may be one of the evidences for the possibility of unstable responses. It can be stated that an unstable response of an asymmetric structure will be largely possible if the restoring force curve for frames has a decreasing secondary slope k_2. Further quantitative discussions will, however, need more extensive data of analysis.

Nomenclature

		Unit:
a, b, c :	Suffices denoting frames, (a), (b) and (c)	
α :	Maximum ground acceleration	cm/sec^2
B_i :	Yield-point restoring force of each frame	kg.
β, γ :	Coefficients in the expression of the soft spring type restoring force of frame (c)	kg/cm^3, kg/cm^5
C :	$\alpha_Y M/(B_a+B_b)$, the ratio of the maximum inertia force to the sum of the yield-point restoring forces of frames in the Y-direction	none
Δ :	Yield displacement of frame	cm
$f(T)$:	Complementary function of the ground mo-	

		Unit :
	tion	none
I :	Moment of inertia of the structure	kg.cm.sec^2
k_i :	Elastic rigidity of frames	kg/cm
k_0 :	Reference spring constant	kg/cm
k_1 :	Initial slope of the restoring force-distortion diagram of frame	kg/cm
k_2 :	Secondary slope of the restoring force-distortion diagram of frame	kg/cm
$\kappa_i = k_i/k_0$:	Ratio of the initial slope of the restoring force curve for each frame to the reference spring constant	none
L_i :	Distance between a frame and the center of mass of the structure	cm
L_0 :	Reference length	cm
l_i :	Non-dimensional length between frame and the center of mass of the structure	none
λ_0 :	Reference dimension for rotation	rad.
M :	Mass of the structure	kg·sec^2/cm
μ :	I/ML_0^2	
p :	Reference frequency for the electric analogy	rad./sec
R_i :	Bilinear, hysteretic restoring force of each each frame	kg
φ_i :	Non-dimensional bilinear, hysteretic restoring force	none
r_i :	Bilinear, hysteretic restoring force for the electrical analogy	volt
Θ :	Rotation of the structure about a vertical axis	rad.
$\Theta_{crit.}$;	Critical value of Θ, as a criterion of unstability	rad.
θ :	Dimensionless rotation of the structure	none
T, τ, t :	Time ; independent variables	sec, none
T_G :	Duration of a cycle of ground pulse	sec
${}_1T_{Y\theta}$, ${}_2T_{Y\theta}$:	Natural periods of vibration of the structure	sec
T_f :	Natural period of vibration of the flexible	

		Unit:
	structure	sec
T_r :	Natural period of vibration of the rigid structure	sec
V :	dX/dT, velocity of the center of mass of the structure in the X-direction	cm/sec
w :	Rotation angle for electrical analogy	volt
X, Y, Z :	Rectangular coordinates	cm
x, y :	Rectangular coordinates for the electrical analogy	volt
ξ, η :	Non-dimensional, rectangular coordinates	none
X_G, Y_G :	X- and Y-components of the ground displacememt	cm

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