

Hydraulic Model Experiment Involving Tidal Motion

Part IV. Tidal Mixing

1. Introduction

In the coastal sea water, various materials are contained, which do not appear at all or scarcely in the open ocean. For example, there are the suspension load and the nutrient carried there by the river flow, a part of which is the diet of the marine organisms, and becomes the constitution or the source of the vitality, and influences the basic productivity in the coastal area. On the other hand, there are the noxious chemical materials involved by the industrial waste, which influence them in the wrong sense. The most important current as the transporter of these materials is the tidal current, and its effect is macroscopically regarded as a sort of diffusion.

Recently, the study on the tidal mixing rapidly developed in oceanography and from the results of the observations it becomes clear that the diffusivity changes in a very wide range due to the scale of the motion. Since the similitude of hydraulic models was ascertained in the above experiments this important subject was examined in the same model.

2. Procedure and results of the experiment

The flow pattern in each phase and the loci of the floats were obtained throughout the period by photographing the floats continuously from the top of the tower in both models used for the experiments of the bay model and the inlet model. In the inlet model, the observation was done in Miho Bay by photographing from the top of tower, 10 m in height.

The results are shown in Fig. 65~68. Fig. 65 and 66 show the loci in Hiroshima Bay when $x_r/h_r=2$ and the former is obtained in the model without the training dyke, corresponding with Fig. 14 and 15, and the latter is that with the dyke, corresponding with Fig. 16 and 17. Comparing with each other, it is found that after the training dyke was constructed the locus becomes considerably small in the innerpart of the bay. Fig. 67 and 68 show the loci in Miho Bay in the model with the dredged channel, and the

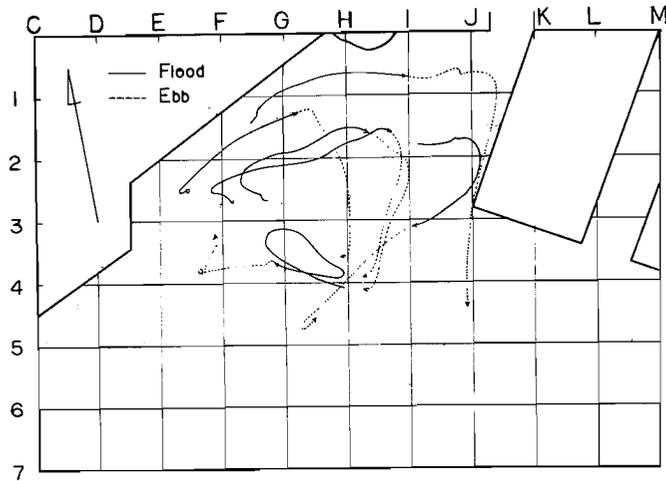


Fig. 65. Loci of the semidiurnal tide in Hiroshima Bay, $x_r/h_r=2$. Corresponding to Fig. 14 and 15.

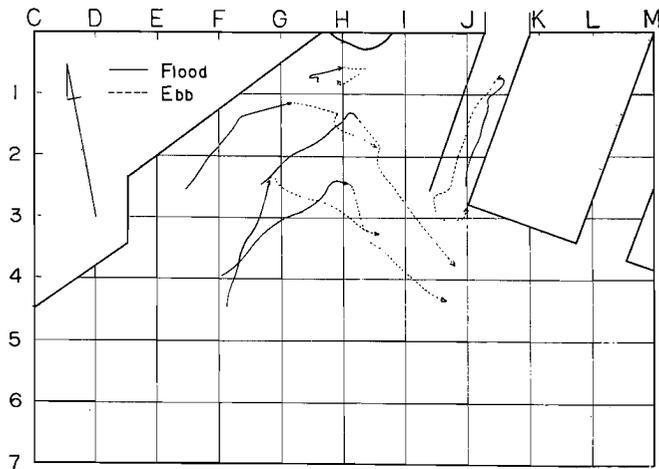


Fig. 66. Effect of the training dyke on the loci of the semidiurnal tide in Hiroshima Bay, $x_r/h_r=2$. Corresponding to Fig. 16 and 17.

former was obtained when the channel was open to Nakaumi, and the latter when the channel was closed at "b" in Fig. 27. From these observation it is found that when the channel was closed, the loci near the tip of the training dyke became considerably small. In these figures the full lines show

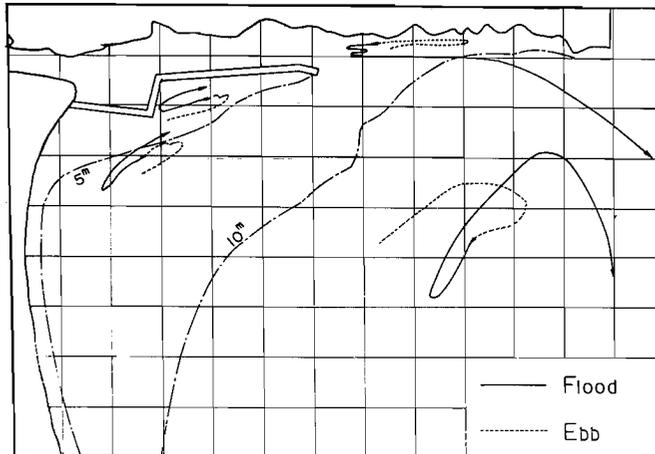


Fig. 67. Loci of the semidiurnal tide in Miho Bay. Model, Sakai Channel has been dredged and Nakaumi has the area of the present state.

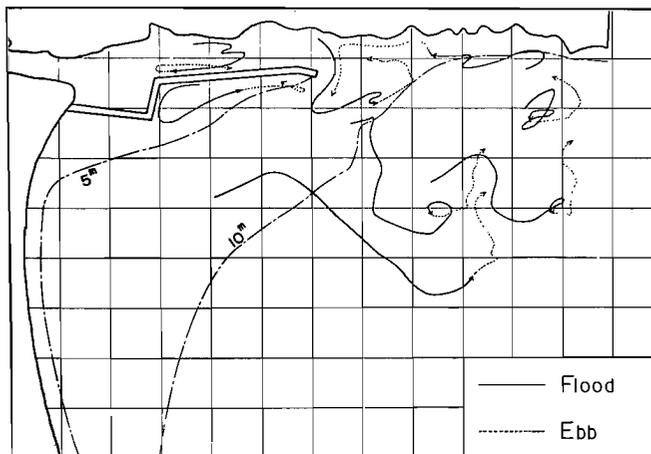


Fig. 68. Loci of the semidiurnal tide in Miho Bay. Model, Sakai Channel has been dredged and Nakaumi has the area as large as under planning.

the loci during the flood tide, that is, from low water to succeeding high water, and the broken lines show the loci during the ebb tide, that is, from high water to succeeding low water, and each locus is shown during one period.

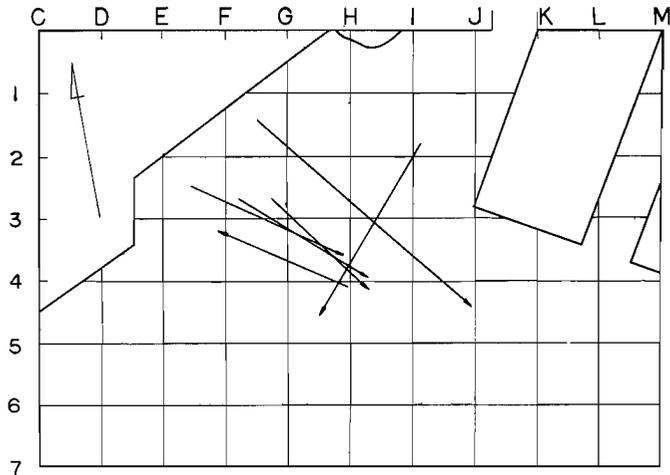


Fig. 69. Tidal Residues in Hiroshima Bay.
Corresponding to Fig. 65.

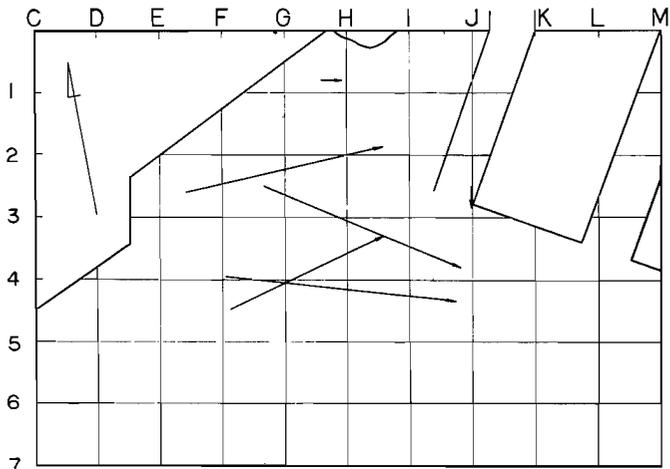


Fig. 70. Effect of the training dyke on the tidal residue.
Corresponding to Fig. 66.

The locus generally showed an arc of a circle or ellipse, and the float did not return to initial position after one cycle, that is, the residue remains. This is named as the "tidal residue". The tidal residues in the preceding cases are shown in Fig. 69~72, which correspond with Fig. 65~68 respectively.

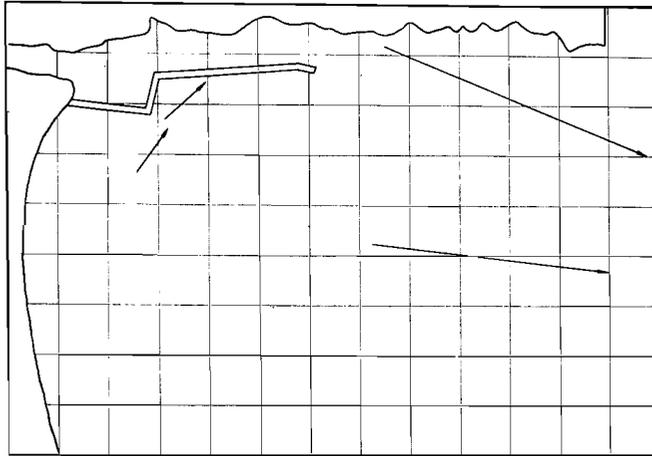


Fig. 71. Tidal residues in Miho Bay. Corresponding to Fig. 67.

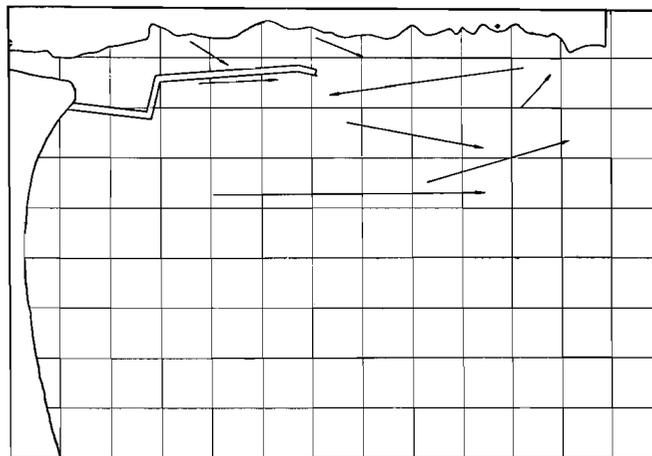


Fig. 72. Tidal residues in Miho Bay. Corresponding to Fig. 68.

3. Consideration

What controls the horizontal diffusion is the disturbance of the horizontal current, and the tidal mixing is a kind of horizontal diffusion. Although the flows in these models were laminar, such a disturbance is evidently observed in the flow patterns and the loci. In these experiments, although the similitude was assured only on the maximum velocities by the comparison with the prototype, it is certain that the similitude holds approximately

for the entire flow pattern, therefore for the loci. Although in the prototype, of course, there are the disturbances of various scales, and the mixing and the diffusion depend on their scale, it is considered that, in these model experiments, the similitude holds good for the disturbance of order of the length of the tidal excursion in the same degree as the tidal current.

Generally, expressing the conservative concentration in the sea by θ , the diffusion of θ is governed by the equation :

$$\frac{\partial \theta}{\partial t} + \sum U_i \frac{\partial \theta}{\partial x_i} = \sum \frac{\partial}{\partial x_i} \left(k_i \frac{\partial \theta}{\partial x_i} \right), \quad i = 1, 2, 3 \quad (26)$$

where k_i is the diffusivity.

Considering the shallow sea, regarding θ as the average value from the sea surface to the bottom, and neglecting the vertical component of the velocity, it is written that $i=1, 2$, in the equation (26). The condition of the similitude for this equation is written as follows,

$$\frac{\theta_r}{t_r} = U_r \frac{\theta_r}{x_r} = V_r \frac{\theta_r}{y_r} = k_{xr} \frac{\theta_r}{x_r^2} = k_{yr} \frac{\theta_r}{y_r^2} \quad (27)$$

Then

$$U_r = \frac{x_r}{t_r}, \quad V_r = \frac{y_r}{t_r}, \quad k_{xr} = \frac{x_r^2}{t_r}, \quad k_{yr} = \frac{y_r^2}{t_r} \quad (28)$$

In these equations since the model is not distorted horizontally, $x_r = y_r$, and considering those in the broad area, it is assumed that $k_x = k_y$.

Therefore they become,

$$U_r = V_r = \frac{x_r}{t_r} = \frac{y_r}{t_r} \quad (29)$$

$$k_r = k_{xr} = k_{yr} = \frac{x_r^2}{t_r} \quad (30)$$

As already stated, the exchange of sea water by the tidal current and the accompanying distribution of the conservative concentration is regarded as a kind of horizontal diffusion of large scale. As to the horizontal diffusivity, various values have been proposed by many workers. Among these, for the tidal mixing, it is asserted that the diffusivity in the prototype is expressed by $A_p \cdot U_p \cdot L_p$ on the basis of the idea of the mixing length, in which A_p is a constant, U_p is the mean velocity, and L_p is the length of the tidal excursion. Assuming that the diffusion in the model belongs to the same regime as in the prototype in such a scale as is now under consideration, the diffusivity in the model is to be expressed by $A_m U_m L_m$, and the

ratios by

$$A_r U_r L_r = A_r \frac{x_r^2}{t_r} \quad (31)$$

Comparing this with the equation (30), if $A_r=1$ or $A_p=A_m$, it is considered that the diffusion phenomena in the model are similar to the prototype. Since this value is nondimensional, if the current is similar it is probable that $A_p=A_m$ as will be stated later.

As to the length of the tidal excursion L in the model, assuming that the water particles move sinusoidally, it may be expressed as follows, with the use of the maximum velocity.

$$L = \int_0^{T/2} U dt = \int_0^{T/2} U_{max} \sin \frac{2\pi}{T} t dt = \frac{U_{max}}{\pi} T \quad (32)$$

in which T is the period. The relation between theoretical L calculated by the equation (32), and observed L in the experiment of the bay model, is shown in Fig. 73. Although in this figure it seems that the observed L is slightly larger, it may be considered that this equation is valid because of the slightness of the difference. Therefore the diffusivity k is proportional to the square of the mean velocity.

In the figures of the velocity distribution after the training dyke was constructed in the bay model (Fig. 16 and 17), the velocities in the area of the western side of the dyke is decreased to less than 1/2 as compared with that in the absence of the dyke (Fig. 14 and 15), especially the reduction is greater near the dyke. As stated above, the diffusivity is proportional to the square of the mean velocity, so that it is decreased to less than 1/4, and it is inferred that the degree of the diffusion is largely decreased.

In discussing the diffusion by the tidal current, although the length of the tidal excursion L is regarded as one of the measures, the length of the

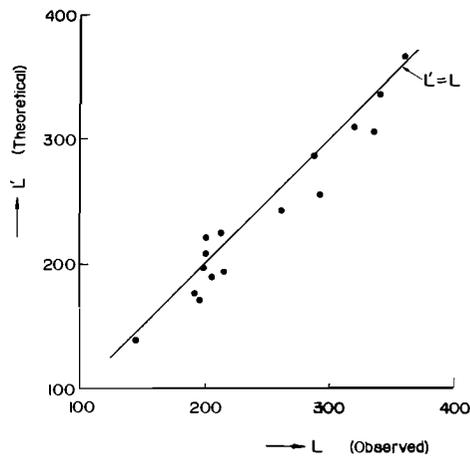


Fig. 73. The lengths of tidal excursions (cm), observed and theoretical, in the model of Hiroshima Bay.

tidal residue R is also an important quantity, which must not be overlooked. Indeed, what can be observed directly is a great advantage of the model experiment. Although the tidal residue appears when a constant flow exists, if the direction is at random it may be regarded as a measure of the diffusion. The relation between R and L is shown in Fig. 74. From this figure it is found out that R is proportional to L . Since the dimension of the diffusivity is $[L]^2[T]^{-1}$, when R is used as the measure of the diffusion and the period of the semidiurnal tide as the time factor, R^2/T is considered to express the diffusivity. This will be called the tidal diffusivity (k_R) tentatively. This is equivalent to regarding R as the mixing length and R/T as the turbulent velocity in the mixing length theory for the diffusion. The relation between the tidal diffusivity and the length of the tidal excursion is shown in Fig. 75. From this figure it is found that the tidal diffusivity is proportional to the square of L , that is

$$k_R = \frac{R^2}{T} = \beta L^2 \quad (33)$$

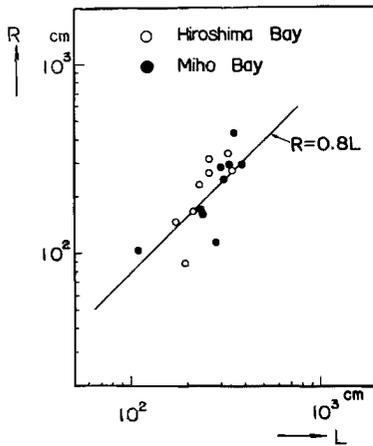


Fig. 74. Relation between the length of the tidal excursion and the tidal residue.

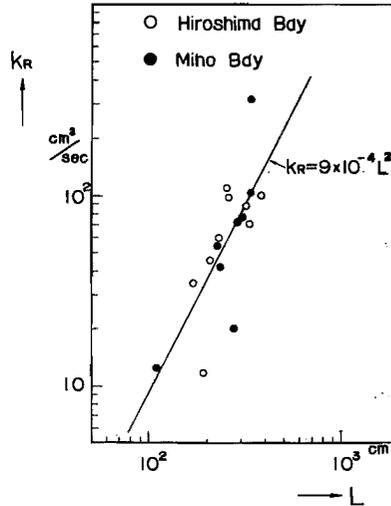


Fig. 75. Tidal diffusivity.

On the other hand as stated above the relation between R and L is as follows ;

$$R = aL \quad (34)$$

so that

$$a^2 = \beta T \quad (35)$$

The length of the tidal excursion is

$$L = \frac{U}{2} T, \quad (36)$$

in which U is the mean velocity. Then the diffusivity becomes

$$k = AUL = \frac{2A}{T} L^2 \quad (37)$$

If k is equal to k_R , then ;

$$k = k_R = \frac{R^2}{T}$$

Therefore from equations (33), (35), and (37) the following relation is obtained

$$A = \frac{\beta T}{2} = \frac{\alpha^2}{2} \quad (38)$$

This equation provides the meaning to A . From Fig. 74 we obtain that $\alpha = 0.8$, therefore $A = 0.32$ by the equation (38). When $R = 0$, A is naturally zero.

For the value of A in the prototype, S. Hayami, Y. Fukuo, and D. Yoda (1956) obtained $A_p = 0.13$ by the study on the tidal current in Akashi Channel. This value of A_p is nearly equal to the average value of the former two, $A_m = 0.32$ and $A_m = 0$. It is suggested by this fact that $A_p = A_m$, that is, the tidal diffusion in the model is similar to the prototype.

From the above inference, the order of the tidal diffusivity in the prototype as Hiroshima Bay and Miho Bay may be estimated as $5 \times 10^4 \sim 5 \times 10^6$ (c.g.s.) by the equation (30).

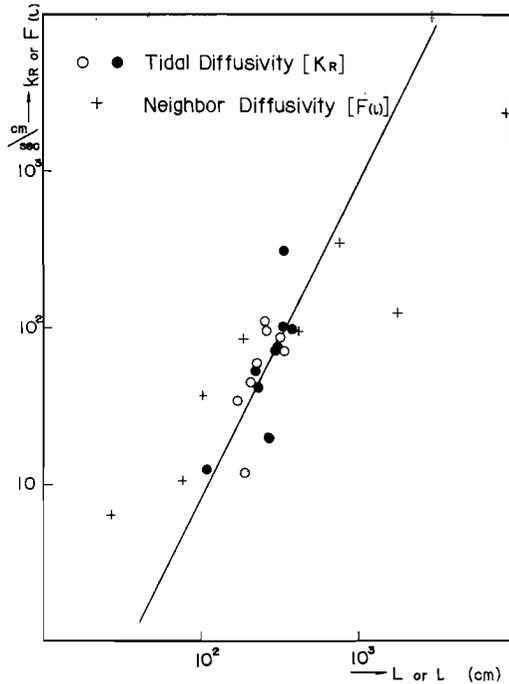


Fig. 76. Tidal diffusivity and neighbor diffusivity.

As one of treatments of the diffusion in the ocean there is the idea of the neighbor diffusivity proposed by L. F. Richardson and H. Stommel, in which they showed that the "4/3" law holds between the neighbor separation l and the neighbor diffusivity $F(l)$. This law was found from the observation in the atmosphere, and is said that this can be roughly applied to the ocean. This diffusivity in the prototype is compared with the tidal diffusivity in Fig. 76. Although the treatments in these two diffusivities are different from each other, it is interesting to note that if l is regarded to be equivalent to L , the value of both diffusivities are approximately the same.

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