# Hydraulic Model Experiment Involving Tidal Motion

## Part II. Similitude

### 1. Introduction

The most important problem in the hydrauric model experiment is the dynamical similitude between the prototype and the model. Since it is theoretically impossible to simulate the prototype and the model in every detail, that is, the law of dynamical similitude is different in accordance with every phenomenon, the term "similitude" is used in the sense that some particular relationship holds between the prototype and the model. Only in such a case, the hydraulic model experiment has meaning. In the present study restricted within the oscillations of long period such as tide, the law of dynamical similitude is discussed.

### 2. Theoretical consideration for dynamical similitude

As the tidal current is dominant in horizontal direction the vertical velocity may be neglected. Hence, if the mean velocities in horizontal directions are shown by $U$ and $V$, the equations of $U$ and $V$ are given by

$$\frac{\partial U}{\partial t}+sU\frac{\partial U}{\partial x}+sV\frac{\partial U}{\partial y}=-\frac{C}{2h}U^2-g\frac{\partial \zeta}{\partial x} \tag{1}$$

$$\frac{\partial U}{\partial t}+sU\frac{\partial V}{\partial x}+sV\frac{\partial V}{\partial y}=-\frac{C}{2h}V^2-g\frac{\partial \zeta}{\partial x} \tag{2}$$

where $x$ and $y$ are coordinates for space and $t$ for time, $\zeta$ is the displacement of the water surface in the vertical direction, $g$ is the accerelation of gravity, $h$ is the water depth, $C$ is the drag coefficient, and $s$ is the coefficient nearly equal to unity, decided by the vertical distribution of the velocity. In shallow water where the tidal current is dominant, the force due to density distribution can be neglected because it will be smaller than any other term of equations (1) and (2). Coriolis' force and wind stress are also neglected.

For the establishment of the dynamical similitude, it is necessary for the ratio of each term of equations in the prototype and the model to be the

same respectively. If the magnitudes for the prototype and the model are indicated by subscripts $p$ and $m$ respectively, and their ratio by $r$, we obtain

$$\frac{U_r}{t_r}=s_r\frac{U_r^2}{x_r}=s_r\frac{U_rV_r}{y_r}=C_r\frac{U_r^2}{h_r}=\frac{\zeta_r}{x_r} \tag{3}$$

$$\frac{V_r}{t_r}=s_r\frac{U_rV_r}{x_r}=s_r\frac{V_r^2}{y_r}=C_r\frac{V_r^2}{h_r}=\frac{\zeta_r}{y_r} \tag{4}$$

Assuming $s_r=1$, these equations are written as follows,

$$U_r=\frac{x_r}{t_r}=\zeta_r^{1/2} \tag{5}$$

$$V_r=\frac{y_r}{t_r}=\zeta_r^{1/2} \tag{6}$$

$$C_r=\frac{h_r}{x_r}=\frac{h_r}{y_r} \tag{7}$$

These equations require $x_r=y_r$, and $U_r=V_r$. In general $\zeta_r=h_r$, so that the conditions of the dynamical similitude are obtained as follows,

$$t_r=\frac{x_r}{h_r^{1/2}} \tag{8}$$

$$C_r=\frac{h_r}{x_r} \tag{9}$$

Equation (8) provides the relation between the time and space scales. It is nothing but the Froude number condition. Equation (9) provides the drag coefficient condition. When the drag coefficient of the model differs from the prototype, this equation requires that the vertical scale differ from the horizontal one, that is, the model must be distorted. The equation of continuity is not influenced because of the uniformity of density.

One of the most difficult problems in the hydraulic model experiment is on the frictional coefficient, because in most cases the coefficient of the prototype is uncertain ; and the difficulty lies not only in giving the model the frictional coefficient which satisfies the conditions of the dynamical similitude, but also because the coefficient itself changes depending on the Reynolds number.

Now when both the frictional coefficient of the prototype $C_p$ and that of the model $C_m$ are known, the ratio of the horizontal scale $x_r$ to the vertical scale $h_r$ is determined by the equation (9) and if one of these is decided, for example, the horizontal scale $x_r$ is decided, the others, $h_r$ and $t_r$, are determined by both equations (8) and (9).

Generally the frictional coefficient is the function of the Reynolds number. In the prototype, since the scale of the motion is very large and the flow belongs to complete turbulent regime in almost all cases, it is considered that $C_p$ must be constant. If the regime of flow in the model is the same as the prototype, the condition of similitude would be rather simple, but such cases can be scarcely expected. The real flow in the model belongs to the laminar or transient regime, so that the coefficient usually changes in a wide range. For this reason it is impossible to hold the similitude in the strict sense, so that we must be satisfied by making the similitude hold for mean values.

For this purpose, it may be convenient to introduce a constant frictional coefficient ($C_m$) at a characteristic Reynolds number. Although the Reynolds numbers in the laminar and transient regime are different from each other, the point of view is fundamentally the same.

The problem that must be decided by the experiment is to determine the best value of this characteristic Reynolds number, and this is one of the important subjects in this study.