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THE EFFECT OF SURFACE TEMPERATURE ON THE CRUSTAL DEFORMATIONS

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Chap. 6. Introduction

The local aspect of crustal movement can be perceived through a record at an observatory, but the record is, in its crude state, a great accumulation of various effects of phenomena. These effects are mixed to various extents according to each of circumstances, so confusedly rather than complicatedly, that one might even suspect that the record is meaningless at all. If we can give an appropriate estimation to each effect of the "external" factors (the word "external" indicates "out of the earth's surface"; meteorological, astronomical and so on), and take it off from the record, then we shall obtain the strains resulting only from mechanical anomalies in the earth's crust, and then, unusual crustal movements accompanied with big earthquakes will appear quite exactly in the record.

This "purely mechanical" crustal strain was discussed in the preceding part. For the practical use of observational data, however, we should study at first the effects due to these external factors. As M. Takada[1] stated in his study of the observation at the Ide Observatory, the secular change in crustal strains will be made clearer and the prediction of earthquake would become more probable if we could investigate the intrinsic nature of annual variations and deduce their behaviors in advance. (The annual variations, or, the periodic variations throughout a year are evidently due to "external factors", to which we shall refer later, while the secular change corresponds to the mechanical anomaly in the crust.)

A great many external causes are considered contributing to the crustal deformations, but several investigators found experimentally what kinds of the factors are contributing particularly to the records of their own observatories and how they can be estimated and eliminated, through their long observations and accurate analyses of the data. A usual treatment of the elimi-

nation of effects of these external factors is to take an overrunning mean of the strains observed ; to take the mean value of records during a constant period (13 months, for example) and to regard it as the value at the central time of the period. This will make the secular change conspicuous, but not clear the present state of the change till half the period later (half a year later, as to the above example). I. Ozawa[2] proposed a width of fluctuations of strain variations. So long as the datum lies within this width, it will not signify any unusual movement due to an internal mechanical disturbance but only the movements caused by meteorological factors, while it denotes a mechanical anomaly (perhaps accompanied by a strong earthquake) when it runs off from the width. In fact, this deviation occurs prior to some big earthquakes in his record at the Osakayama Observatory (see Fig. 1. The hatched part in (a) denotes the width. Arrows point out the occurrences of a few big earthquakes ; Fukui earthquake on 28, June, 1948, Yoshino earthquake on 18, July, 1952 and the earthquake in the middle part of Kyoto Prefecture on 24, Nov., 1953, respectively.). The width $M \pm E$ is defined by the average M of values on the same date through several years and the half length $E = c\varepsilon$ of width, where $c = 2.33$ and ε is the (mathematical) standard deviation of M. V. V Popov and M. K. Chernyavkina[3] took it for granted that the record at their Yalta station would consist of a linear component bt (secular change) and a periodic term $\sum_k a_k \cos(k\omega t + \varphi_k)$, and having obtained the constant b for every

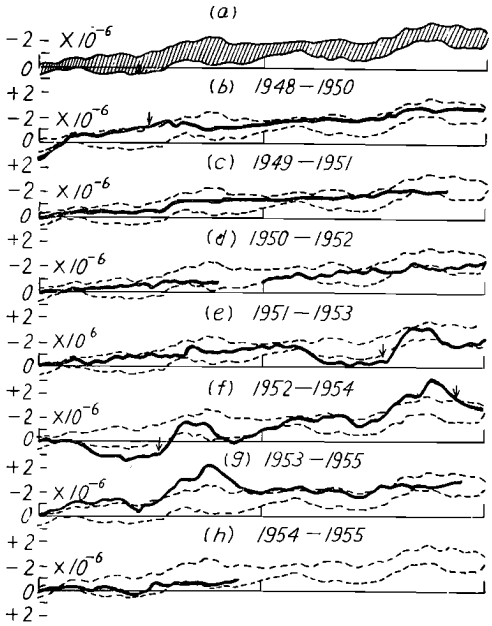

Fig. 1. The record of strain variation at the Osakayama Observatory from 1948 to 1955 and the width of fluctuation proposed by Ozawa. (After Ozawa. The fat curves denote the actual variation and both boundaries of width are indicated by dotted lines.)

day, they subjected to harmonic analysis the average hourly data per decade, which still remained in the record after exclusion of the term *bt* and which were regarded by them as purely periodic components. Though they failed to show the annual variation, they concluded the presence of daily periodic displacements with harmonics of the first and second orders. Besides those two components they took into consideration, we should think moreover of the effects due to the sudden or irregular changes of external circumstances. Rainfall, for example, must be one of the most remarkable, especially in our country where the circumstances of weather are much more changeable than in many other countries and states.

However, the crustal deformations brought by the sudden and irregular changes of external circumstances will almost vanish by the above treatments. These must be eliminated, indeed, for the survey of general aspect. The effects of sudden changes are quite hard to estimate, and so, impossible to treat theoretically. Hence the most interesting of the external factors from a theoretical standpoint should be those giving periodic movements on a constant cycle.

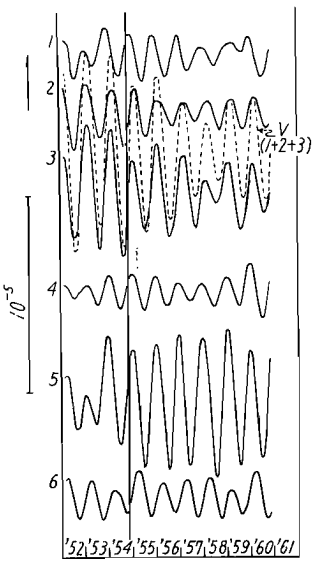

Fig. 2. Annual variations at Ide. (After Takada. The continued lines, numbered 1 to 6, denote the variations of different six components of the ground-strain, while the dotted curve represents the volume dilatation.)

Most records modified in the above manner, still have components of periodic movement on a day or a year cycle. (Popov and Chernyavkina are not so cautious of the component of secular change itself, as is obvious from the fact that they take a linear term *bt* as its representative.) For example, the result obtained by Takada[4] is shown in Fig. 2.

There exist many causes to give the periodicity to the crustal deformations, but we confine ourselves to the study of annual periodicity, for we seek the long secular change and try to obtain from it the mechanical anomalies in the crust. The revolution of the earth as a planet will make nothing of such local movements, and we might consider the temperature variation as the most essential one. Rainfall, together with atmospheric pressure, will also offer

the periodic effect, though not so precisely as temperature will. At the present step of observations, the above three factors are generally to be regarded as the principal ones acting constantly to give the nonsecular change as a whole in the crustal strains.

In this part, we shall estimate only the effects of temperature theoretically. Naturally, other factors also should be studied at the same time, and so, in this sense, the following study will not allow us to obtain any closed conclusion. K. Hosoyama[5] states from his observations that the principal causes giving annual periodic variation of strains are atmospheric pressure varying throughout a year as well as temperature variation on the earth's surface.

Most observatories in our country are situated scores of meters below the earth's surface, and at the same depths the temperature scarcely varies but keeps constant throughout a year. (The room temperatures in observatories have, of course, greater variations than theoretical results, for they are connected with the surface by some tunnels or caves. For example, see Ozawa's[6] report, and yet the amplitude in his report does not exceed 0.4 to 0.5°C throughout the year.) In other words, the temperature variation attenuates with depth very sharply. T. Matsuzawa[7] once treated this problem and disscussed the stress at the interior of crust when temperature varies sinusoidally and propagates as a plane wave along the earth's surface. It is his conclusion that it yields considerable stresses in the crust of a depth comparable with the wave length of temperature variation, which is reproved still valid in most cases we take up here with more physically interesting cautions. S. Homma[8] studied an extension of Matuzawa's calculation to the case of neighborhood of ocean where the ground temperature is almost constant. He still investigated the effects of the shape of each observatory that generally forms a tunnel or a gallery.

As stated later, the attenuations of temperature and of strain are, together with the local reliefs and irregularities of the earth's surface, the most remarkable points in our study, which will leave many to be discussed in future. Recently Popov[9] took up the same problem, arriving at the similar conclusion with Matuzawa.

On the other hand, the theory of thermoelastical dynamics with irreversible process was constructed by M. A. Biot[10] Using Biot's theory we discuss the effects of temperature variation upon an internal point of the crust under

the condition that a sinusoidal variation of temperature propagates along the earth's surface. In the foregoing researches the problem was mostly simplified by neglecting the inertia terms in the equations of motion and the term representing heat expansion in the equation of heat conduction, although the neglect of the latter term might be a little absurd. Evidently the study performed here also can not help being accompanied with some simplifications of the phenomena. It will be very difficult to answer the whole question, when we take in minds those complicated records, but we shall find through our investigation the governing role of temperature variation in crustal deformations.

Chap. 7. The Effects of Surface Temperature to the Periodic Movement of the Earth's Crust

1. The Establishment of Problem and the Boundary Conditions

At first we construct two equations according to Biot; one (a vector equation) describes the motion of particle, caused by the variation of thermal condition in an elastic solid with the Lamé constants λ and μ, the density ρ and the coefficient of thermal expansion α, while the other (a scalar equation) expresses the variation of temperature causing this motion. Let the strain components and the temperature at an element of the medium be e_{ij} $(i, j=1, 2, 3)$ and $T+\theta$, where θ is an increment of temperature above a reference absolute temperature T for the state of zero stress and strain. The equations of state are written in the form

$$\sigma_{xx}=2\mu e_{xx}+\lambda e-\beta\theta \qquad \sigma_{xy}=\mu e_{xy}$$
$$\sigma_{yy}=2\mu e_{yy}+\lambda e-\beta\theta \qquad \sigma_{yz}=\mu e_{yz}$$
$$\sigma_{zz}=2\mu e_{zz}+\lambda e-\beta\theta \qquad \sigma_{zx}=\mu e_{zx},$$

β denotes the product of the bulk modulus and of the coefficient α, and $e=e_{xx}+e_{yy}+e_{zz}$ is the voluminal dilatation.

The three equations of motion are derived from the condition of equilibrium, the derivation of which will not need to be repeated here, and we have

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}+\rho\left(X-\frac{\partial^2 u_x}{\partial t^2}\right)=0$$
$$\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}+\rho\left(Y-\frac{\partial^2 u_y}{\partial t^2}\right)=0$$

$$\frac{\partial \sigma_{zx}}{\partial x}+\frac{\partial \sigma_{zy}}{\partial y}+\frac{\partial \sigma_{zz}}{\partial z}+\rho\left(Z-\frac{\partial^2 u_z}{\partial t^2}\right)=0.$$

We shall confine ourselves to the case that body forces shall not exist. Substituting the equations of state into the above as to the stress components, we find

$$\mu\nabla^2 u+(\lambda+\mu)\,\mathrm{grad}\, e-\beta\,\mathrm{grad}\,\theta=\rho\frac{\partial^2 u}{\partial t^2}$$

On the other hand, the first law of thermodynamics can be expressed as

$$dh=dU-\sum_{\mu,\nu}\sigma_{\mu\nu}de_{\mu\nu},$$

where U is the internal energy of an element of unit size, and h the heat absorbed by it. Dividing this by $T_1=T+\theta$, we obtain the entropy

$$ds=\frac{dh}{T_1}=\frac{dU}{T_1}-\frac{1}{T_1}\sum_{\mu,\nu}\sigma_{\mu\nu}de_{\mu\nu},$$

or, rewriting the differential dU as a linear combination of dT_1 and de_{ij},

$$ds=\frac{1}{T_1}\frac{\partial U}{\partial T_1}dT_1+\sum_{\mu,\nu}\left(\frac{\partial U}{\partial e_{\mu\nu}}-\sigma_{\mu\nu}\right)de_{\mu\nu}.$$

From the second law of thermodynamics it is required that ds is an exact differential in T_1 and e_{ij}, that is,

$$\frac{\partial \sigma_{\mu\nu}}{\partial e_{ij}}=\frac{\partial \sigma_{ij}}{\partial e_{\mu\nu}}\quad\text{and}\quad\frac{\partial U}{\partial e_{\mu\nu}}=\sigma_{\mu\nu}-T_1\frac{\partial \sigma_{\mu\nu}}{\partial T_1}.$$

Differentiating the preceding equations of state by T_1 (i.e. by θ), and substituting the results intn the above in the case of $\mu=\nu$, we have

$$\frac{\partial U}{\partial e_{\mu\nu}}-\sigma_{\mu\nu}=\beta T_1,$$

and of $\mu\neq\nu$,

$$\frac{\partial U}{\partial e_{\mu\nu}}-\sigma_{\mu\nu}=0.$$

Hence, the entropy can be also expressed as follows

$$ds=c'\frac{dT_1}{T_1}+\beta de,$$

where c' denotes the specific heat of the unit volume in the absence of deformation. Integrating this we obtain

$$s=c'\log\left(1+\frac{\theta}{T}\right)+\beta e.$$

θ being sufficiently small compared with T, this is nearly equivalent to the relation

$$s=\frac{c'\theta}{T}+\beta e \quad \text{or} \quad h=c'\theta+T_{\beta}e.$$

Still, by the law of heat conduction we have a well-known formula

$$\frac{\partial h}{\partial t}=k\nabla^2\theta,$$

where k is the coefficient of heat conduction. Substitution of h just represented as a linear expression of θ and e, into the equation of heat conduction yields

$$k\nabla^2\theta=c'\frac{\partial\theta}{\partial t}+T\beta\frac{\partial e}{\partial t}.$$

by which and the preceding vector equation of motion we can determine the motion of a particle as well as its thermal field in the elastic solid. Rewriting c' by $c\rho$, where c denotes the specific heat per unit mass, and k by a^2, the temperature conductivity, we can also express the scalar equation as follows,

$$a^2\nabla^2\vartheta=\frac{\partial\vartheta}{\partial t}+\frac{\beta T}{c\rho}\frac{\partial}{\partial t}\operatorname{div} u.$$

(Similar investigation was performed by M. Päsler[11] earlier than Biot. But it seems that Biot has the advantage of more systematic construction in the theory.)

Suppose a temperature variation exist on the surface of a semi-infinite elastic solid as the following form

$$\vartheta_0\cos(\omega t-kx).$$

ϑ_0 is a half difference between the maximal and minimal values of temperature throughout a year. It must be noted that the value of ϑ_0 is unknown so far as the observatory has no peculiar apparatus measuring the surface temperature in its neighborhood. Generally speaking, ϑ_0 must be larger than half the greatest variation in atmospheric temperature of the district. ω is equal to $2\pi/T$ and T denotes a year. k, the wave number, never originates only in the earth's revolution but it should be rather considered as a parameter resulting from the above supposition with respect to the temperature variation. As is shown later, k has tolerably large value (which means the wave length l is tolerably small), and so, l is to be regarded as a parameter

expressing the local irregularities or reliefs and can not be given the numerical value from the beginning of study. A similar variable appears in the V. V. Popov's work, who stated that the value 1000 m as l would be valid, though his statement of the reason is not so clear. However, since we demand the solution under the above condition of ϑ, the strains obtained will have the same wave number with ϑ, for which reason we shall be able to decide the value of k from the actually observed values of strains. In other words, the parameter k can be significant as the wave number of the strains, and it seems unreasonable to estimate k only from the fact that it denotes the wave number in the temperature variation, disregarding other effects of k. k would be rather considered as a quantity compensating the losses caused by such a treatment of the problem under those simple postulates submitted to the temperature and the shape of the earth's surface. We take the x and z axes, along and perpendicularly to the surface, respectively. Positive directions of both axes coincide with that of the propagation of ϑ and downward from the boundary. Thus settled, the problem will be worked in the xz plane (a two-dimensional problem). The potentials φ and ψ are defined as in the usual theory of elastic waves, that is,

$$u=\frac{\partial\varphi}{\partial x}-\frac{\partial\psi}{\partial z} \quad w=\frac{\partial\varphi}{\partial z}+\frac{\partial\psi}{\partial x}$$

Formally these correspond to the P and S potentials, though they will not necessarily possess the usual physical significances. In fact, substituting these into the Biot's formulae and separating them as to φ and ψ, then we have the following three equations,

$$\rho\frac{\partial^2\varphi}{\partial t^2}=(\lambda+2\mu)\nabla^2\varphi-\beta\vartheta, \quad \rho\frac{\partial^2\psi}{\partial t^2}=\nabla^2\psi, \quad a^2\nabla^2\vartheta=\frac{\partial\vartheta}{\partial t}+\frac{\beta T}{c\rho}\frac{\partial}{\partial t}\nabla^2\varphi.$$

Therefore, the problem will come to the acquirement of three functions φ, ψ and ϑ satisfying the above equations. Expressing the stress components in X_x, X_y,$\cdots\cdots Z_z$ following Love, we write down the boundary conditions as follows,

$$\vartheta(x,0,t)=\vartheta_0 e^{i(\omega t-kx)}, \quad (Z_x)_{z=0}=0 \quad \text{and} \quad (Z_z)_{z=0}=0,$$

however, if we donote by p_{xx}, p_{xy},$\cdots\cdots p_{zz}$ the formal components of stresses defined mathematically by the actual strains through the well-known stress-strain relation in usual elastic theory, the stress parts in the boundary conditions can be rewritten in the form

$$(p_{zx})_{z=0}=0 \quad \text{and} \quad (p_{zz})_{z=0}=\beta\vartheta_0 e^{i(\omega t-kx)}$$

Hence, the problem seems quite resemblant to that of the ordinary elasticity without energy dissipation under the existence of the normal pressure $\beta\vartheta_0 e^{i(\omega t-kx)}$ acting on the boundary plane. As is well-known, it is an elementary problem to get a solution of plane waves in an elastic half space with such boundary conditions. (See Ewing, Jardetzky and Press[12].) The former boundary condititions are rewritten in the following form, by use of φ and ψ,

$$Z_x=\mu e_{xz}=\mu\left(2\frac{\partial^2\varphi}{\partial x\partial z}+\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\right)=0 \quad (z=0)$$

$$Z_z=2\mu e_{zz}+\lambda(e_{xx}+e_{zz})-\beta\vartheta$$

$$=\lambda\nabla^2\varphi+2\mu\left(\frac{\partial^2\varphi}{\partial z^2}+\frac{\partial^2\psi}{\partial x\partial z}\right)=\beta\vartheta(x,0,t)=0 \quad (z=0).$$

2. The Exact Solution of Strains and Temperature Variation

Under the above boundary conditions the forms of φ, ψ and ϑ can not help being restricted as functions of x and t; they should have the form such as $f(z)e^{i(\omega t-kx)}$ in order to satisfy the boundary conditions. Writing φ, ψ and ϑ as $f_p(z)e^{i(\omega t-kx)}$, $f_s(z)e^{i(\omega t-kx)}$ and $g(z)e^{i(\omega t-kx)}$, successively, and substituting these into the equations, we have

$$f_p''(z)=(k^2-k_p^2)f_p(z)+\frac{\beta}{\lambda+2\mu}g(z)$$

$$f_s''(z)=(k^2-k_s^2)f_s(z)$$

$$g''(z)=\left\{k^2+i\frac{\omega}{a^2}\left(1+\frac{\beta^2T}{c\rho(\lambda+2\mu)}\right)\right\}g(z)-i\frac{\omega}{a^2}\frac{\beta T}{c\rho}k_p^2f_p(z),$$

where k_p and k_s denote ω/v_p and ω/v_s, and v_p, v_s are the propagation velocities in the elastic solid of the longitudinal and shear waves. These equations of f_p, f_s and g are ordinary differential equations of the variable z, in which $f_s(z)$ is determined independently of two other unknowns while the simultaneous relation holds between $f_p(z)$ and $g(z)$. Writing these three functions as follows,

$$f_p(z)=Ace^{-\nu z}+A_pc_pe^{-\nu_p z}, \quad f_s(z)=c_se^{-\nu_s z} \quad \text{and}$$

$$g(z)=ce^{-\nu z}+c_pe^{-\nu_p z},$$

we can obtain ν and ν_p by solving the quadric equation of the unknown ν^2

$$(\nu^2-k^2)^2+\left\{k_p^2-i\frac{\omega}{a^2}\left(1+\frac{\beta^2 T}{c\rho(\lambda+2\mu)}\right)\right\}(\nu^2-k^2)-i\frac{\omega}{a^2}k_p^2=0,$$

which is derived from substitution of the supposed forms of $f_p(z)$ and $g(z)$ into the equations. As to A and A_p, we seek them in substitution of the above ν into the form

$$(\nu^2-k^2+k_p^2)A=\frac{\beta}{\lambda+2\mu}$$

ν_s is immediately determined by the fact that $f_s(z)$ is to be a solution of the above equation and we find

$$\nu_s=\sqrt{k^2-k_s^2},$$

which is exactly equal to the value of S potential in the theory of elastic waves under the application of normal pressure cited in the preceding passage. On the other hand, c, c_p and c_s are integral constants depending on the boundary conditions, which are rewritten in the following form by using these constants,

$$c+c_p=\vartheta_0,\quad 2ik(\nu Ac+\nu_p A_p c_p)-(2k^2-k_s^2)c_s=0 \quad \text{and}$$
$$(2k^2-k_s^2)(Ac+A_p c_p)+2ik\nu_s c_s=0,$$

from which we can represent c, c_p and c_s as the functions of ν, ν_p, A and A_p as follows

$$c=\frac{A_pR(\nu_p)}{A_pR(\nu_p)-AR(\nu)}\vartheta_0,\quad c_p=-\frac{AR(\nu)}{A_pR(\nu_p)-AR(\nu)}\vartheta_0 \quad \text{and}$$
$$c_s=\frac{2ik(2k^2-k_s^2)(\nu-\nu_p)AA_p}{A_pR(\nu_p)-AR(\nu)}\vartheta_0.$$

Here we denote the Rayleigh function by $R(x)=(2k^2-k_s^2)^2-4k^2\nu_s x$; in the usual problem of elasticity each potential has $R(\nu_p)$ as denominator of its amplitude, while the denominators of the above c, c_p and c_s turn to $A_pR(\nu_p)-AR(\nu)$.

Using the above results we can thus derive the exact expressions of general solution to this problem, still leaving unknown the values of ν, ν_p, A and A_p,

$$\vartheta=\frac{\vartheta_0 e^{i(\omega t-kx)}}{A_pR(\nu_p)-AR(\nu)}\{A_pR(\nu_p)e^{-\nu z}-AR(\nu)e^{-\nu_p z}\}$$

$$\frac{\partial u}{\partial x}=\frac{\vartheta_0 AA_p k^2 e^{i(\omega t-kx)}}{A_pR(\nu_p)-AR(\nu)}\{-R(\nu_p)e^{-\nu z}$$
$$+R(\nu)e^{-\nu_p z}+2(2k^2-k_s^2)\nu_s(\nu-\nu_p)e^{-\nu_s z}\}$$

$$\frac{\partial w}{\partial x}=\frac{\vartheta_0 A A_p e^{i(\omega t-kx)}}{A_p R(\nu_p)-AR(\nu)}\{\nu R(\nu_p)e^{-\nu z}$$
$$-\nu_p{}^2 R(\nu)e^{-\nu_p z}-2k^2(2k^2-k_s{}^2)\nu_s(\nu-\nu_p)e^{-\nu_s z}\}$$

$$\frac{\partial u}{\partial z}=\frac{i\vartheta_0 A A_p k e^{i(\omega t-kx)}}{A_p R(\nu_p)-AR(\nu)}\{\nu R(\nu_p)e^{-\nu z}$$
$$-\nu_p R(\nu)e^{-\nu_p z}-2(2k^2-k_s{}^2)\nu_s{}^2(\nu-\nu_p)e^{-\nu_s z}\}$$

$$\frac{\partial w}{\partial x}=\frac{i\vartheta_0 A A_p k e^{i(\omega t-kx)}}{A_p R(\nu_p)-AR(\nu)}\{\nu R(\nu_p)e^{-\nu z}$$
$$-\nu_p R(\nu)e^{-\nu_p z}-2k^2(2k^2-k_s{}^2)(\nu-\nu_p)e^{-\nu_s z}\}.$$

$\partial u/\partial x$ and $\partial w/\partial z$ are the dilatations of horizontal and vertical components, respectively, which will be actually observed by the extensometers in each observatory. The inclination to the horizontal plane corresponds to $\partial w/\partial x$ in our solution, which is measured by the tiltmeters on the actual observation.

3. Approximations with respect to the infinitesimal δ.

For an approximate evaluation of the constants ν, ν_p, A and A_p, a consideration will be taken about the parameter $\delta=\frac{\beta^2 T}{c\rho(\lambda+2\mu)}$ which makes our study conspicuous from among the foregoing investigations. In spite of its tolerably large magnitude δ has been treated as good as neglected in the former studies since the term of heat expansion has never been taken into account in the equation of heat conduction. And yet, to discuss such small strains as caused by the temperature variation on the earth's surface, we must pay attention about each infinitesimal parameter of the earth's crust, comparing it with the parameters denoting the external effects. In fact, when the wave length of temperature variation is so short enough that the following inequality may be applied

$$k>\sqrt{\frac{\omega}{a^2}}\,\delta,$$

the chief role in estimating each formula is mainly occupied by k, and so, the neglect of high powers of δ would make nothing of the correctness of the conclusions obtained. Hence, only the first approximation of δ will be concerned here and higher than the second power of δ is all taken off out of the formulae. It must be also noted that δ presents in the formulae

always as the product with ω/a^2. The above condition confining k will be discussed in the next chapter. Similarly, higher than the second power of $\varepsilon=\frac{k^2}{\omega/a^2}$ will be also neglected for the convenience of simplification ; k must be small enough to satisfy the relation

$$k<\sqrt{\frac{\omega}{a^2}}$$

We shall show the validity of this excessive restriction of k also in the next chapter. Still, two infinitesimals $\varepsilon_p=\frac{k_p^2}{\omega/a^2}$ and $\varepsilon_s=\frac{k_s^2}{\omega/a^2}$ much smaller than ε should be eliminated, and, finally, the ratios $\varepsilon_p/\varepsilon$ and $\varepsilon_s/\varepsilon$ are taken into consideration only the first powers of them.

Under the restrictions just mentioned ν, ν_p, A and A_p can be obtained approximately as follows,

$$\nu=\nu_r+i\nu_i,\quad \nu_p=\sqrt{k^2-k_p^2},$$

$$A=\frac{\beta}{(\lambda+2\mu)(\omega/a^2)^2}\left\{k_p^2-i\frac{\omega}{a^2}\left(1-\frac{\beta^2T}{c\rho(\lambda+2\mu)}\right)\right\},\quad A_p=\infty$$

$$\nu_r=\sqrt{\frac{\omega}{2a^2}}\left(1+\frac{\beta^2T}{2c\rho(\lambda+2\mu)}\right)+\frac{k^2}{\sqrt{8\omega/a^2}},$$

$$\nu_i=\sqrt{\frac{\omega}{2a^2}}\left(1+\frac{\beta^2T}{2c\rho(\lambda+2\mu)}\right)-\frac{k^2}{\sqrt{8\omega/a^2}}.$$

It is remarkable that A_p can be taken as infinity, which implies that the term with $R(\nu)$ vanishes and only $R(\nu_p)$ stays in each denominator of the solutions, just as in the solution of the elasticity theory. By putting $A_p=\infty$, the expression is awfully simplified and written as follows,

$$\vartheta=\vartheta_0e^{-\nu_rz}e^{i(\omega t-kx-\nu_iz)}$$

$$\frac{\partial u}{\partial x}=-\vartheta_0Ak^2e^{-\nu_rz}e^{i(\omega t-kx-\nu_iz)}+\vartheta_0Ae^{i(\omega t-kx)}$$

$$\times\left\{\frac{k^2R(\nu)}{R(\nu_p)}e^{-\sqrt{k^2-k_p^2}z}+\frac{2k^2(2k^2-k_s^2)\nu_s(\nu-\nu_p)}{R(\nu_p)}e^{-\sqrt{k^2-k_s^2}z}\right\}$$

$$\frac{\partial\omega}{\partial z}=\vartheta_0A\nu^2e^{-\nu_rz}e^{i(\omega t-kx-\nu_iz)}-\vartheta_0Ae^{i(\omega t-kx)}$$

$$\times\left\{\frac{\nu_p^2R(\nu)}{R(\nu_p)}e^{-\sqrt{k^2-k_p^2}z}+\frac{2k^2(2k^2-k_s^2)\nu_s(\nu-\nu_p)}{R(\nu_y)}e^{-\sqrt{k^2-k_s^2}z}\right\}$$

$$\frac{\partial u}{\partial z}=i\vartheta_0Ak\nu e^{-\nu_rz}e^{i(\omega t-kx-\nu_iz)}-\vartheta_0Ae^{i(\omega t-kx)}$$

$$\times\left\{\frac{k\nu_p R(\nu)}{R(\nu_p)}e^{-\sqrt{k^2-k_p^2}z}+\frac{2k(2k^2-k_s^2)\nu_s^2(\nu-\nu_p)}{R(\nu_p)}e^{-\sqrt{k^2-k_s^2}z}\right\}$$

$$\frac{\partial\omega}{\partial x}=i\vartheta_0 Ak\nu e^{-\nu_r z}e^{i(\omega t-kx-\nu_i z)}-i\vartheta_0 Ae^{i(\omega t-kx)}$$

$$\times\left\{\frac{k\nu_p R(\nu)}{R(\nu_p)}e^{-\sqrt{k^2-k_p^2}z}+\frac{2k^3(2k^2-k_s^2)(\nu-\nu_p)}{R(\nu_p)}e^{-\sqrt{k^2-k_s^2}z}\right\}.$$

Each expression representing strains consists of three kinds of terms, the attenuations of which with the depth are $e^{-\nu_r z}$, $e^{-\sqrt{k^2-k^{p2}}z}$ and $e^{-\sqrt{k^2-k_s^2}z}$ respectively. The first term will attenuate in the same way as the temperatur will, while two others are damping in the same manner as ordinary two elastic waves. Hence each of three terms may correspond to strains caused by its own particular process, which will be also discussed in the next chapter.

Taking int consideration the supposition for approximate procedure above mentioned, we shall be able to write the coefficients as follows,

$$-\vartheta_0 Ak^2\sim\frac{i\beta\vartheta_0}{\lambda+2\mu}\varepsilon,\quad \vartheta_0 A\nu^2\sim\frac{\beta\vartheta_0}{\lambda+2\mu}\{1-i(\varepsilon-\varepsilon_p)\},$$

$$ik\nu A\vartheta_0\sim\frac{\beta\vartheta_0}{\sqrt{2}(\lambda+2\mu)}\varepsilon^{1/2}\left\{\left(1-\frac{\delta}{2}(1+i)+\left(\frac{\varepsilon}{2}-\varepsilon_p\right)(1-i)\right\},\right.$$

$$\vartheta_0 Ak^2\frac{R(\nu)}{R(\nu_p)}\sim\frac{\sqrt{2}\beta\vartheta_0}{(\lambda+2\mu)(\varepsilon_p-\varepsilon_s)}\varepsilon^{3/2}(-1+i),$$

$$-\vartheta_0 A\nu_p^2\frac{R(\nu)}{R(\nu_p)}\sim-\frac{\sqrt{2}\beta\vartheta_0}{(\lambda+2\mu)(\varepsilon_p-\varepsilon_s)}\varepsilon^{1/2}(\varepsilon-\varepsilon_p)(-1+i),$$

$$-i\vartheta_0 Ak\nu_p\frac{R(\nu)}{R(\nu_p)}\sim\frac{\sqrt{2}\beta\vartheta_0}{(\lambda+2\mu)(\varepsilon_p-\varepsilon_s)}\varepsilon^{1/2}\left(\varepsilon-\frac{\varepsilon_p}{2}\right)(1+i),$$

$$\vartheta_0 A\frac{2k^2(2k^2-k_s^2)\nu_s(\nu-\nu_p)}{R(\nu_p)}\sim\frac{\sqrt{2}\beta\vartheta_0}{(\lambda+2\mu)(\varepsilon_p-\varepsilon_s)}\varepsilon^{1/2}\left(\varepsilon-\frac{\varepsilon_p}{2}\right)(1-i),$$

$$-i\vartheta_0 A\frac{2k(2k^2-k_s^2)\nu_s^2(\nu-\nu_p)}{R(\nu_p)}\sim-\frac{\sqrt{2}\beta\vartheta_0}{(\lambda+2\mu)(\varepsilon_p-\varepsilon_s)}\varepsilon^{1/2}(\varepsilon-\varepsilon_s)(1+i),$$

$$-i\vartheta_0 A\frac{2k^3(2k^2-k_s^2)(\nu-\nu_p)}{R(\nu_p)}\sim-\frac{\sqrt{2}\beta\vartheta_0}{(\lambda+2\mu)(\varepsilon_p-\varepsilon_s)}\varepsilon^{3/2}(1+i).$$

Especially the second and the third terms in each formula of the strain turn to

$$e^{-\nu_p z}\left(1+\frac{\varepsilon_p}{2\varepsilon}kz\right)e^{-kz},\quad e^{-\nu_s z}\left(1+\frac{\varepsilon_s}{2\varepsilon}kz\right)e^{-kz},$$

by which they can be unified. Finally, taking only the real part in each

solution we can obtain an approximate solution in the form

$$\vartheta=\vartheta_0 e^{-\nu_r z}\cos(\omega t-kx-\nu_i z)$$

$$\frac{\partial u}{\partial x}=-\frac{\beta\vartheta_0}{\lambda+2\mu}\Big\{\rho^2 e^{-\nu_r z}\cos\Big(\omega t-kx-\nu_i z-\frac{\pi}{2}\Big)^*$$

$$+\rho\Big(kx-\frac{\lambda+2\mu}{\lambda+\mu}\Big)e^{-kz}\cos\Big(\omega t-kx-\frac{\pi}{4}\Big)\Big\}$$

$$\frac{\partial w}{\partial z}=\frac{\beta\vartheta_0}{\lambda+2\mu}\Big\{e^{-\nu_r z}\cos(\omega t-kx-\nu_i z)^*+\rho^2 e^{-\nu_r z}\cos\Big(\omega t-kx-\nu_i z-\frac{\pi}{2}\Big)^*$$

$$+\rho\Big(kz-\frac{\lambda}{\lambda+\mu}\Big)e^{-kz}\cos\Big(\omega t-kx-\frac{\pi}{4}\Big)\Big\}$$

$$\frac{\partial u}{\partial z}=\frac{\beta\vartheta_0}{\lambda+2\mu}\Big\{\rho\Big(1-\frac{\beta^2 T}{2c\rho(\lambda+2\mu)}\Big)e^{-\nu_r z}\cos\Big(\omega t-kx-\nu_i z+\frac{\pi}{4}\Big)^*$$

$$+\frac{1}{2}p^3 e^{-\nu_r z}\cos\Big(\omega t-kx-\nu_i z-\frac{\pi}{4}\Big)^*$$

$$+p\Big(kz-\frac{2\lambda+3\mu}{\lambda+\mu}\Big)e^{-kz}\cos\Big(\omega t-kx+\frac{\pi}{4}\Big)\Big\}$$

$$\frac{\partial w}{\partial x}=\frac{\beta\vartheta_0}{\lambda+2\mu}\Big\{p\Big(1-\frac{\beta^2 T}{2c\rho(\lambda+2\mu)}\Big)e^{-\nu_r z}\cos\Big(\omega t-kx-\nu_i z+\frac{\pi}{4}\Big)^*$$

$$+\frac{1}{2}p^3 e^{-\nu_r z}\cos\Big(\omega t-kx-\nu_i z-\frac{\pi}{4}\Big)^*$$

$$+p\Big(kz+\frac{\mu}{\lambda+\mu}\Big)e^{-kz}\cos\Big(\omega t-kx+\frac{\pi}{4}\Big)\Big\},$$

where

$$p=\frac{k}{\sqrt{\omega/a^2}},\quad \nu_r=\sqrt{\frac{\omega}{2a^2}}\Big(1+\frac{\beta^2 T}{2c\rho(\lambda+2\mu)}\Big)+\frac{p}{2\sqrt{2}}$$

$$\nu_i=\sqrt{\frac{\omega}{2a^2}}\Big(1+\frac{\beta^2 T}{2c\rho(\lambda+2\mu)}\Big)-\frac{p}{2\sqrt{2}}$$

Asterisks show the heat terms of which we shall discuss in detail in the next chapter, together with other questions reserved here.

Chap. 8. Estimations of the Thermoelastic Strains from the Theoretical and the Observational Results

In the approximate solution we arrived at in the preceding chapter is only the first power considered as to the infinitesimal δ. In fact, among

some infinitesimals higher than the first or the second powers of which were neglected, δ, ε_D and ε_S are the constants depending only of the physical properties of the earth (and not of the given form of temperature variation). Even compared with δ^2, ε_D and ε_S have still much smaller magnitudes and naturally they ought to be taken off, while ε varies as a function of k and it may get tolerably large in certain cases. We restricted ε by a little complicated condition ; first, it must be such an infinitesimal as its second power may be negligible, so we have the relation about k

$$k \ll \sqrt{\frac{\omega}{a^2}}\,.$$

second, in order to ignore higher than the second power of δ another limitation to the value of k

$$k \gg \sqrt{\frac{\omega}{a^2}}\,\delta$$

must hold, which is equivalent to the condition of ε that $\varepsilon \gg \delta^2$ Hence, ε can not also be so small as it becomes comparable with δ^2. Only under these restrictions about ε the above solution can be justified and regarded as an extension of the antecedent results ; an extension with the closer consideration of the effects of heat process.

At first sight th above restrictions look quite inadequate to an investigation of the problem for their strong restraint on the range of value permissible to take as the wave length, l, of temperature variation. However, when we take the following values as the physical constants of the earth's crust,[13]

$$a^2 \sim 1\times10^{-2} \qquad c \sim 2\times10^{-1}\ (\mathrm{cal.g^{-1}.deg^{-1}.})$$
$$\alpha \sim 8\times10^{-6} \qquad \rho \sim 2.8$$
$$T \sim 273^\circ\mathrm{K} \qquad \sigma \sim 0.25,$$

the above restrictions imply, with respect to the wave length, that

$$10^3 < l < 10^7\ (\mathrm{cm}).$$

In our problem it will not need to take notice of the wave length smaller than over ten meters, and so, the existence of lower limit of l in the above inequality will be no obstacle to our investigation. Still, as stated later, the following fact makes the existence of upper limit also available that the actual value of wave length may be regarded as a few kilometers or so, which lies evidently within the above range of value. Therefore we shall be

sufficiently able to study the general aspect of this problem, using the approximate solution obtained in Chap. 7, though mathematically it brings only a very particular case where l is restricted within so limited values. Hence, in the case, for example, that l denotes the effect of movement of the earth as a planet, of course, we can not say anything from this approximate formula.

Each term in the expressions can be distinguished one another by its own physical significance. The expressions describing the variations of strains will be separated into two kinds of terms ; the asterisked in the preceding chapter have ν_r as their common coefficient of attenuation (with the depth) which coincides with that of the temperature variation, and the other terms, consisting of two terms originally (see the exact solution) and unified by the procedure of approximation, have the same types with P and S waves of the ordinary elasticity under the normal pressure

$$\beta\vartheta_0 e^{i(\omega t-kx)}$$

on the boundary plane. Since the temperature variation causes this pressure on the surface and the non-asterisked terms correspond to the solution caused by this mechanical pressure, they will represent the deformations resulting through mechnical process from the existence of the above normal stress on the surface. In other words, these terms correspond to the strain caused by the energy which has been already transformed from hear to elasticity on the boundary surface. These terms are to be called "elastic terms" in this paper. On the contrary, the asterisked terms will correspond to the strain caused by the energy which propagates to the interior point in question as heat and yields the elastic deformation just at this point. These terms will be named here "heat terms" Thus, in the expressions of strains, we find a reparation into two terms each of which is characterized by its own particular process of generation.

The heat terms, together with the temperature variation, attenuate as $e^{-\nu_r z}$, and the elastic terms as e^{-kz} The relative acuteness of both attenuations depends upon the relative magnitudes of ν_r and k. ν_r can be regarded approximately as

$$\nu_r \sim \sqrt{\frac{\omega}{2a^2}}+\frac{1}{2\sqrt{2}}\frac{k}{\sqrt{\frac{\omega}{a^2}}},$$

and taking account of the fact that $\sqrt{\frac{a^2}{\omega}}\sim 4.5\times 10^{-3}$ we find ν_r is evidently larger than any (positive) value of k. Namely, the heat term usually goes to vanish with depth more rapidly than the elastic term, which leads to an extension of the Matuzawa's statement from the more significant standpoint. (Matuzawa stated the difference on the attenuations between the temperature and the strain caused, while the statement mentioned here points out that (i) the strains caused by heat energy attenuate in the same way as the temperature and (ii) they (in which the temperature itself is comprehended) have the larger coefficient of attenuation than the strains caused by mechanical process.) ν_r will be substantially identified with $\frac{1}{2\sqrt{2}}\frac{h}{\sqrt{\omega/a^2}}$ and $\sqrt{\frac{\omega}{2a^2}}$, respectively according to $k\gtrless\frac{2\omega}{a^2}$, or $l\lesssim 1.6\times 10^5$ (cm.). As the actual l lies just near this boundary value, the above two terms of ν_r become almost equal each other, and so, both are useful as to the approximations of ν_r.

As the wave length is getting shorter, the attenuation of each term becomes more rapid. To discuss the case of the wave length shorter than over ten meters we must take the second or the higher than second powers of ε into the solution, for the upper restriction to k does not exist any longer. On the other hand, when l becomes long enough that the relation $k>\sqrt{\frac{\omega}{a^2}}\delta$ can not hold, the first approximation at to δ get really absurd. That is to say, we must pay attention about the effects of heat process more and more in proportion as the wave length becomes longer. However, if we return to the solution in its exact form (Section 2, Chap. 7) and investigate each term in the expressions of strain, we find that three kinds of terms constructing the exact solution have $\nu_r\sim\sqrt{\frac{\omega}{2a^2}}$, $\nu_p\sim\sqrt{\frac{\omega}{2a^2}}\frac{\delta}{2}(1+i)$ and $\nu_s\sim k$ as their coefficients of attenuation in the case that $k\ll\sqrt{\frac{\omega}{a^2}}\delta$. As to their physical meanings, the terms $e^{-\nu_r z}$ and $e^{-\nu_s z}$ correspond to heat and elastic terms, which is recognized by the same consideration as above, while the term $e^{-\nu_p z}$ is to be considered as a heat term rather than an elastic one since its phase as well as amplitude becomes resemblant to that of the temperature variation. This result coincides with the Biot's[14] statement that shear waves are generally unaffected by the thermal phenomenon, although in our approximate solution dilatational wave can be also separated into heat and elastic term. (This conciseness about dilatational wave perhaps results from the restriction imposed upon the wave length.) Even in such long waves as k is much smaller than, we admit also the prominent role of elastic

terms at a deep interior point, since the minimal value is taken by $\nu_s \sim k$, among the three coefficients of attenuation.

According to the observational results by I. Ozawa[15] at the Osakayama Observatory, the wave length of strain may be evaluated about 1.7 km, by use of a result from the amplitude ratio of dilatational components observed at two points 200 m distant each other, under the supposition that the dilatation component varies as a sinusoidal stationary wave on a year cycle. The amplitude of strain at the observatory being $2 \sim 3 \times 10^{-6}$, and the observatory being situated about a depth of 100 m, we can obtain ϑ_0 conversely, substituting the above values into l, $\frac{\partial w}{\partial z}$ and z. ϑ_0 is determined as 20~40°C. Thus, the surface temperature should vary to this extent of values if the annual change of crustal movement were regarded all as the result of temperature variation on the earth's surface.

We find this result to ϑ_0 may be valid, comparing it with the mean difference between the greatest and the least atmospheric temperatures throughout a year in Kyoto district. (The mean difference of atmospheric temperature is about 24°C in this district.) It must be noted as stated in Chap. 6, that the effects of rainfall and atmospheric pressure should be taken into account at the same time and yet, in our study those effects are taken off from the start. We shall be able to conjecture that this is the reason why ϑ_0 seems to take the larger value rather than that considered probable.

In regard to such a value of ϑ_0 the heat terms in the strains attenuate very rapidly and become unobservably small to 10^{-8} extent, already at a depth of a few meters, and hence, it is sure that the observational data will describe only the elastic terms of each strain.

At the beginning of the preceding chapter we noticed that l signifies the local irregularities or reliefs. Thus, 1.7 km as the value of l at the Osakayama Observatory must indicate some local irregularities in the neighborhood of this observatory ; it may display the existence of local reliefs, streams or bushes and, indeed, the variation of local temperature on the surface will be more changeable even in such a limited area as the atmospheric temperature can be regarded almost uniform on it. But unfortunately we have no basis to decide whether this value of l is appropriate or not to the neighborhood of the Osakayama Observatory. Therefore, as also pointed out in the same passage of the preceding chapter, it would be rather comprehensible that we think of l as the wave length of strain and that it happens to coincide with

the wave length of temperature for the simplification of form giving the variation of surface temperature.

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