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A STUDY ON RUNOFF PATTERN AND ITS CHARACTERISTICS

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1. Introduction

Since L. K. Sherman's study of unit-graph method in 1932, many studies on the flood runoff caused by storm rainfall have been carried out by a large number of researchers. They are classified into two groups : one is the theoretical derivation based on the hypothesis of linearity as used in the unit-graph method ; the other is the empirical science based on the actual hydrologic observation. However, the dynamic mechanism in the runoff process has not yet been completely disclosed and, therefore, satisfactory results have not been obtained.

The runoff phenomena are essentially the stochastic process. In order to understand them, it is necessary to find out a macroscopic law on hydrologic transformation from rainfall to discharge and to study the statistical characteristics contained in the law. This paper describes, as the first step of the study, the quasi-deterministic analysis of the transformation system and the macroscopic law in the system.

Physically speaking, the transformation system in the runoff phenomena contains several different sequences such as the averaging of rainfall over time and space, the infiltation of rain-water to the deep soil stratum, the rain-water flow near basin surface, and so on. In order to construct a universally valid theory of runoff, we assume the space invariant of rainfall. Furthermore, we focus our attention on the behaviour of rain-water flow near the basin surface, because it is of most importance in flood runoff. In these respects, the various hydraulic mechanisms of rain-water flow and their relationships are described. As a result, the fundamental runoff patterns as unification of the interior mechanism are derived and these characteristics are discussed from a universal point of view.

2. Flow Behaviour of Rain-Water on Mountain Slope

The mountain slopes in a river basin are primarily classified into the following two types by the surface configuration ; the first is the type of slope covered with porous surface stratum that has high permeability such as in a well vegetated or well forested basin ; the second is the type having no porous surface stratum, as in a non-vegetated basin. The flow behaviour of rain-water, and especially the runoff mechanism of interflow, on these two types of slopes will not be similar to each other hydraulically. This is the resson why the runoff hydrographs from the two types show different shapes under the same conditions of rainfall. The theoretical and experimental results obtained by us and others with respect to these flow behaviours of rain-water are stated briefly in this chapter.

(1) Water flow in surface stratum

Fig. 1 shows the schematic profile of a mountain slope covered by a surface stratum in which the water that has penetrated flows downstream. In the figure, H represents the water depth, D the thickness of surface stratum, x the distance from the upstream end of the slope, L the length of the slope, θ the inclination angle of the slope, r the intensity of rainfall, and i the maximum infiltration rate into the sub-stratum lying under the surface stratum.

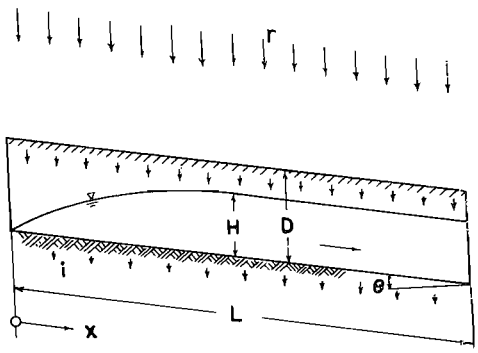

Fig. 1. Schematic representation of the lateral water flow in surface stratum.

Considering that the rate of rain-water that penetrates into the surface stratum is usually more than several hundred mm per hour, all the rain-water falling on the surface will penetrate during the initial period of rainfall. Therefore, the rain-water supplied substantially to the surface stratum is equal to the excess rainfall defined by $r_e = r - i$.

By using the equations of continuity and dynamics of flow, in which the law of Darcy is applied, the fundamental equation of water flow in the surface stratum can be written as follows.

$$kH\frac{\partial^2 H}{\partial x^2}+k\left(\frac{\partial H}{\partial x}-\sin\theta\right)\frac{\partial H}{\partial x}-\gamma\frac{\partial H}{\partial t}=-r_e \tag{1}$$

in which k is the coeffcient of permeability and γ the non-capillary porosity. This is a non-linear partial differential equation of advective diffusivity. Since it can be assumed in an actual river basin that $\partial H/\partial x/\sin\theta \ll 1$, Eq. (1) will approximately become a quasi-linear equation. Moreover, the diffusion term can be ignored in the equation, because it can be shown from the approximate solution of 2nd order that the order of diffusion term is less than that of advection term[1]. Under these assumptions, the approximate solution of Eq. (1) is given as follows.

On the characteristics :

$$x-x'=f\cdot(t-t') \tag{2}$$

the following relation must be satisfied.

$$\gamma H=R_e(t)-R_e(t')+\gamma H(x',t') \tag{3}$$

in which x' and t' are the values of coordinates representing the initial point of characteristics on the $x-t$ plane, $H(x',t')$ the water depth at point (x',t'), $f=(k/\gamma)\sin\theta$, and $R_e(t)=\int_0^t r_e(t)dt$. The non-capillary porosity γ seems to be independent of the intensity of rainfall and the time, as shown in Table 1 representing a part of the results obtained from the laboratory tests, made at Kyoto University, on runoff by the use of artificial rainfall and characterized by the soil structure[2].

Table 1. Results of laboratory test on runoff.

Run	Length of slope (m)	Material of surface stratum	Depth of surface stratum (cm)	Void ratio (%)	Initial water content (%)	Rainfall intensity (mm/hr)	γ	Roughness coefficient n ($m^{-1/3}$.sec)
A	7.50	glass wool	1.00	—	—	362	—	0.3 -0.4
B	6.00	sand 0.62 mm	12.0	48.2	7.6	157	0.266	—
	6.00	〃	〃	〃	8.6	112	0.249	0.02-0.03
C	6.00	sand 0.36 mm	〃	47.8	14.7	163	0.178	0.01-0.02
	6.00	〃	〃	〃	13.8	122	0.187	—

(2) Effective rainfall and occurrence area of overland flow

In this paper, the effective rainfall is defined as the rainfall which supplies effectively the overland flow, and the occurrence area of overland flow

as the area in which the overland flow occurs on the surface stratum. These are two elemental factors in the runoff process, and they must be evaluated by considering both the conditions of the soil of a slope, and the conditions of the rainfall.

For a slope with no surface stratum, the intensity of effective rainfall r_f is equal to that of excess rainfall r_e, and the occurrence area of overland flow F is the whole drainage area A. For a slope with surface stratum, the runoff phenomenon becomes considerably complicated, and two different phases of runoff will occur, depending on whether or not the water surface in the surface stratum reaches its limit. This limit is given by the following relation, from Eqs. (2) and (3).

$$R_e(t)-R_e\left(t-\frac{L}{f}\right)=\gamma D \tag{4}$$

in which $H(x', t')$ in Eq. (3) is assumed to be equal to zero. This simple relation is a very important one, as will be explained in the next chapter, and may be called the transition condition of the system change.

If the value of the terms on the left hand side of Eq. (4) is always less than γD, the overland flow may occur only in temporary streams or gullies distributed on a slope. The effective rainfall which supplies the overland flow is equal to the sum of the excess rainfall which directly supplies the streams, and the seepage rate to rain-water from the saturated zone of the surface stratum into the overland flow. The average intensity of effective rainfall r_f per unit area, for practical purposes of runoff analysis, is given as:

$$r_f=\frac{1}{A}(A_e r_h+A_0 r_e) \tag{5}$$

in which $A_e=2NL_dH$, and N is the number of temporary streams per unit area, L_d the average length of the streams, r_h the seepage rate of rain-water to the streams, and A_0 the area of the streams in the plan. Defining the effective rainfall as that given by Eq. (5), it must be noted that the occurrence area of overland flow is replaced by the whole drainage area, and that the average intensity of effective rainfall r_f varies from point to point, corresponding to the water depth H in the surface stratum. The runoff in this regime is called the sub-surface runoff.

If the rainfall intensity becomes greater and the condition, $H \leqq D$, is not maintained, the overland flow occurs also on the surface stratum. The

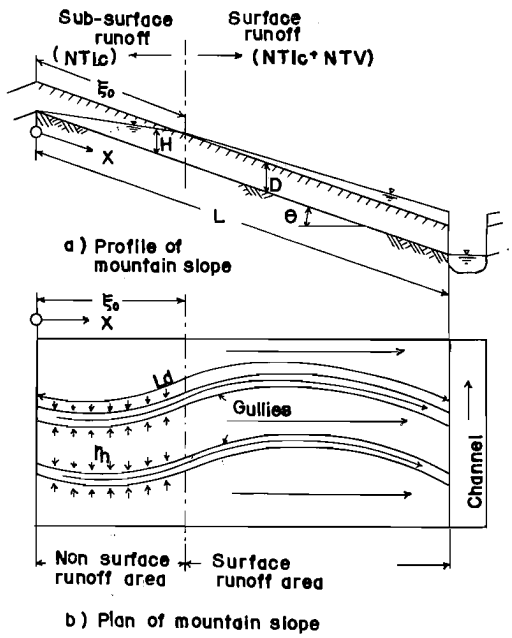

Fig. 2. Schematic representation showing the occurrence of surface runoff.

schematic flow pattern is shown in Fig. 2. The occurrence area of overland flow on the surface stratum varies with time, corresponding to the variation of the intensity of excess rainfall. The distance ξ_0 from the upstream end of a slope to the initial point of occurrence area is, using Eqs. (2) and (3),

$$R_e(t)-R_e\left(t-\frac{\xi_0}{f}\right)=rD, \quad \xi_0>L \tag{6}$$

The surface stratum under the occurrence area of overland flow must be fully saturated and, therefore, $r=r_e$ within this region. Accordingly, by the use of the condition of continuity, the occurrence area of overland flow becomes

$$F=(1-L_r)A, \quad L_r=\frac{\xi_0}{L} \tag{7}$$

(3) Overland flow

Since the overland flow on the surface stratum is assumed to be approximately uniform, the equations of motion and continuity become, respectively,

$$h=Kq^p \tag{8}$$

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=r_f \tag{9}$$

in which h is the water depth of overland flow, q the discharge rate per unit width, and

$$K=\left(\frac{n}{\sqrt{\sin\theta}}\right)^p \tag{10}$$

In Eq. (10), n is the Manning roughness coefficient and p the numerical constant, being equal to 0.6. The applicability of the Manning formula to

overland flow has been ascertained by laboratory tests carried out at Kyoto University and other place, but the value of n varies widely with the surface condition of the slope. According to the experimental results, the value of n for overland flow on a bare slope or a temporary stream distributed on the surface stratum is 0.01~0.03 ($m^{-\frac{1}{3}}$, sec) and the value on a vegetated surface is 0.3~0.4 ($m^{-\frac{1}{3}}$, sec). This difference of n, due to the surface condition, is remarkable[2].

Applying the theory of characteristics to Eqs. (8) and (9), the solution is as follows[3]. On the characteristics :

$$x-\xi=\frac{1}{pK}\int_{\tau}^{t}ds\left[\int_{\tau}^{s}\frac{r_f}{K}dz+\{q(\xi,\tau)^p\}\right]^{\frac{1}{p}-1} \tag{11}$$

the following relation must be satisfied.

$$q=\int_{\tau}^{t}\frac{r_f}{K}dt+\{q(\xi,\tau)^p\}\Big]^{1/p} \quad \text{or} \quad q=\int_{\xi}^{x}r_f dx+q(\xi,\tau) \tag{12}$$

in which $q(\xi,\tau)$ is the discharge rate at the point (ξ,τ), which represents the starting point of characteristics under consideration on the $x-t$ plane. Putting $\xi=\xi_0$, Eqs. (11) and (12) give the desired solution for overland flow on the surface stratum mentioned earlier.

3. Ruuoff Pattern and its Characteristics

By the hydraulic mechanism of rain-water flow on a mountain slope explained above, the runoff process of storm flood in a river basin can be classified and analyzed in a quasi-determinisitc way.

(1) Runoff Process and transformation system

The storm-runoff process is stochastic and the transformation from rainfall to discharge at a gauging station constitutes an ensemble of stochastic transformations. This ensemble has a non-deterministic property which involves a certain regularity or law as the ensemble average over time and space.

Thus, the time change of direct runoff, dQ_e/dt, at a gauging station in any river basin may be represented by the following non-linear stochastic functional form.

$$\frac{dQ_e{}^p}{dt}=g\left[\left(r_f(t)-\frac{d\tau}{dt}r_f(\tau)\right)/K,\ \int_0^t P(Q_e,E,t\,;t')E(t')dt'\right] \tag{13}$$

The first term in the functional form can be derived from Eq. (12),

and expresses the influence of effective rainfall r_f, within a certain finite time interval $(t-\tau)$, on the time change of the discharge rate at the present state. The finite time-interval varies with time because of the non-linearity of overland flow and the variation property of occurrence area of direct runoff. This time change of the time-interval plays an important role in making clear the fundamental characteristics of runoff process, as will be explained later. The following relation is obtained from Eqs. (11) and (12).

$$\frac{d\tau}{dt}=1-\frac{d}{dt}\left\{K(L-\xi_0(\tau))^p\left(\int_\tau^t r_f(t')dt'/(t-\tau)\right)^{p-1}\right\} \tag{14}$$

Let us call this, $d\tau/dt$, "lag change".

The second term may be regarded as the departure from the time change of discharge rate determined by the first term, which has a certain statistical distribution. The characteristics of this distribution may depend on the entire history of the runoff, and the statistical variables may have some interrelation with each other. Under these considerations, the second term representing the statistical departure from the first term is expressed formally in Eq. (13), in which the statistical vector E, changing with time, has a certain probability P influenced by the state in the past, the statistical vector and the present time. Fig. 3 will illustrate the meaning of the previous statement.

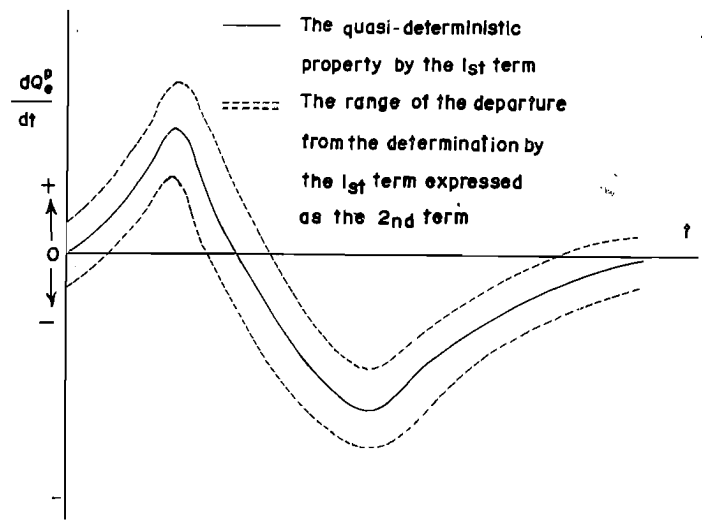

Fig. 3. Schematic illustration representing deterministic and statistical effects to the time change of discharge rate at a gauging station.

Although the transformation system from rainfall to discharge is stochastic, the first step in determining the system structure is to discover the equivalent systems in respect to dynamics, and to make clear the mechanism and the characteristics of these systems by the deterministic way which is suggested implicitly in the first term in Eq. (13). The statistical evaluation of the departure from the determination by the first term is the second step, which is meaningless without the performance of the first step.

Furthermore, it may be possible make smaller the effect of the second term in Eq. (13), which has unknown characteristics, by more precise investigation of the first term because the statistical approach represented by the second term seems to result from physical uncertainty. Thus, in subsequent sections, the approach to runoff analysis is quasi-deterministic, and L and K in Eqs. (4) and (10) are defined as the representative values of a basin.

(2) **Equivalent transformation system**

From the mathematical point of view, the system of transformation from a physical state or quantity, such as input r_e, to another state or quantity, such as output, Q_e, may be generally repesented by the following differential-difference equation of the functional form

$$\sum_{i=0}^{N} a_i \frac{d^i Q_e}{dt^i} = g\left[\sum_{i=0}^{N-1} b_i \frac{d^i r_e}{dt^i},\ \sum_{i=0}^{N-1}\sum_{j=0}^{N} C_i \frac{d^i r_e(t-t_j)}{dt^i},\ t\right] \quad (15)$$
$$;\ 0 \leqq t_0 < t_1 \cdots\cdots < t_j < t_{j+1} < \cdots\cdots < t_n$$

in which N and n are integers, a_i, b_i and c_i the coefficients, $t-t_j$ the time-interval.

The transformation system is characterized by coefficients, a_i, b_i and c_i, and the difference between time-intervals, $t_{j+1}-t_j$, in Eq. (15). Thus, four equivalent transformation systems, in the sense of dynamics, are classified as follows.

(i) Linear time-invariant system (LTI).
a_i, b_i, c_i and $(t_{j+1}-t_j)=\text{const.}$

(ii) Linear time-variant system (LTV).
a_i, b_i, c_i or $(t_{j+1}-t_j)=f_n(t)$.

(iii) Non-linear time-invariant system (NTI).
a_i, b_i, c_i or $(t_{j+1}-t_j)=f_n(Q_e,\ dQ_e/dt)$.

(iv) Non-linear time-variant system (NTV).
a_i, b_i, c_i or $(t_{j+1}-t_j)=f_n(Q_e,\ dQ_e/dt,\ t)$.

These classifications may be suitable to a quasi-deterministic system such as runoff phenomena, and can be understood physically depending upon whether the character of a system (a) varies with time (LTV and NTV) or not (LTI and NTI), and whether (b) is influenced by the input, such as the intensity of rainfall (NTI and NTV) or not (LTI and LTV).

Since the characters of the four transformation systems classified above are essentially different from each other, the determination of the interior dynamic structure in the runoff process must be based upon the clear classification of the equivalent transformation systems of runoff, determined by the basin characteristics and the rain-water flow behaviour.

The runoff process in an actual basin varies with the conditions of both the surface soil and the rainfall, as stated in the previous chapter. In a river basin with no surface stratum, no sub-surface flow occurs, and the occurrence area of runoff phenomena does not vary with time. Therefore, the transformation system in such a basin is time invariant. On the other hand, in a river basin with surface stratum, two different conditions may arise, depending upon whether the occurrence area of surface runoff is limited in the temporary streams, or whether it is not, and the transition condition is given by Eq. (4). Furthermore, the effective rainfalls for the two conditions are different from each other in their evaluation. In addition, since the numerical constant p in Eqs. (10), (11) and (12) is equal to 0.6 for overland flow, each dynamic transformation system of runoff is non-linear.

Table 2. Classification of equivalent transformation system in runoff process.

Condition of surface stratum			Transformation system
$D=0$	$\xi_0=0,$	$r_f=r_e$	NTI
$D>0$	$\xi_0=0,$	$r_f \doteqdot \overline{r_h}=A_e r_h A^{-1}$	NTI_c
	$\xi_0=f_n(t, r),$	$r_f=r_e+\overline{r_a}$	NTV

Table 2 shows the result of the above discussion on the equivalent transformation system of runoff, representing the relation between the surface stratum condition and the transformation system. In this table, $\overline{r_h}$ represents the average seepage rate r_h over the area, $\overline{r_a}$ is $(\overline{r_h})_{max}$, and NTI_c system represents the region of subsurface runoff, in which subscript c shows that

the effective rainfall r_T varies with time and space.

(3) Runoff pattern

By reason of the existence of three equivalent transformation systems, as shown in Table 2, three fundamental patterns in runoff process result, as tabulated in Table 3.

Table 3. Fundamental runoff patterns derived from transformation system.

Transformation system				Runoff pattern
NTI	$\xi_0(\tau)=0,$	$F=A$	$Q_e=Q_s$	NTI
NTI_c	$\xi_0(\tau)=0,$	$F=A$	$Q_e=Q_i$	NTI_c
$NTI_c \rightarrow (NTI_c+NTV) \rightarrow NTI_c \cdots\cdots \rightarrow NTI_c$	$\xi_0(\tau)=0,$ $\xi_0(\tau)=f_n(t, r),$	$F=A,$ $F=(1-L_r(\tau))A,$	$Q_e=Q_i$ (for NTI_c) $Q_e=Q_s$ (for NTV)	$NTI_c \sim NTV$

Subscripts e, i and s of Q represent the values of direct runoff, subsurface runoff and surface runoff, respectively. When NTV transformation system arises it is always attended with NTI_c system as shown as (NTI_c+NTV) in Table 3.

Fig. 4 shows runoff patterns and transformation system schematically. The actual runoff process is always in the area enveloped by the solid line, in any basin and in any condition of rainfall. The area enveloped by the dotted line, LTI, is the system in which the method of runoff analysis based on linear time-invariant assumption such as unit-graph method holds good.

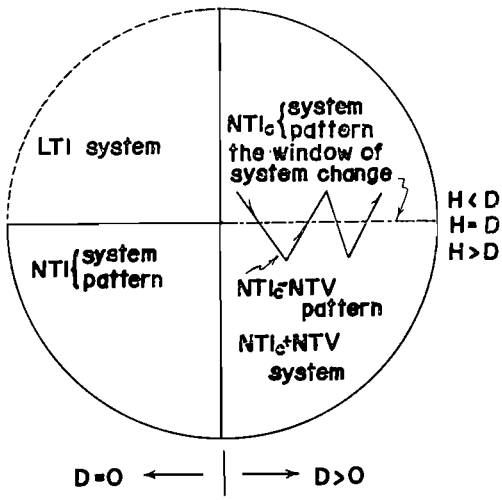

Fig. 4. Diagram representing the relation of equivalent transformation systems and runoff patterns.

NTI pattern occurs in a basin with no surface stratum ($D=0$) such as a barren basin, in which the dynamic property is relatively simple. How-

ever, a basin with no surface stratum is scarcely ever found among the usual mountain basins.

Mountains are generally well vegetated, and have a surface stratum ($D>0$). The runoff process in such basins is very complicated, and the system analysis plays a particularly vital role. If the transition condition given by Eq. (4) is not satisfied, the system and the runoff pattern are both NTI_c. If the transition condition is satisfied, however, the runoff pattern is $NTI_c \sim NTI$, the transformation systems transit as $NTI_c \rightarrow (NIT_c + NTV) \rightarrow NTI_c \rightarrow (NTI_c + NTV) \cdots \rightarrow NTI_c$, in which arrows represent the system transition. These are shown in Fig. 4.

Thus, the broken line in Fig. 4 may be called "the window of system transition". The number, the time positions, and the magnitudes of the window gratly influence the configuration of runoff, particularly during a flood, and are determined by using the transition condition Eq. (4).

(4) **Lag as transformation operator**

The runoff pattern depends on the character of transformation system, in which the most fundamental factor is the lag. Here, the lag must be defined as the transformation operator in physical significance, which connects rainfall and discharge.

Under this consideration, the time-interval $(t-\tau)$ of the propagation of disturbance on the characterestics represents the lag determined by the interior structure of each pattern for overland flow defined by Eqs. (11) and (12), because the discharge rate at any time and any point on $x \sim t$ characteristic plane can be essentially calculated by using the effective rainfall r_f during the time-interval $(t-\tau)$. That is, the discharge rate of direct runoff Q_e can be expressed from Eq. (13) as follows, putting $T_c = t-\tau$,

$$Q_e = (F/T_c)\int_{T_c} r_f dt \tag{16}$$

Therefore, the lag T_c becomes the transformation operator.

Next we will discuss the lag in some detail. The elapsed time at a gauging station of discharge may on the whole be classified into two domains, depending upon whether the disturbances created at the upstream end of a representative slope reach the station or do not. The former may be defined as disturbance domain (D.D.) and the latter as non-disturbance domain (U.D.). In addition, (U.D.) and (D.D.) are each divided into two domains depending upon whether the effective rainfall is supplied entirely or partly

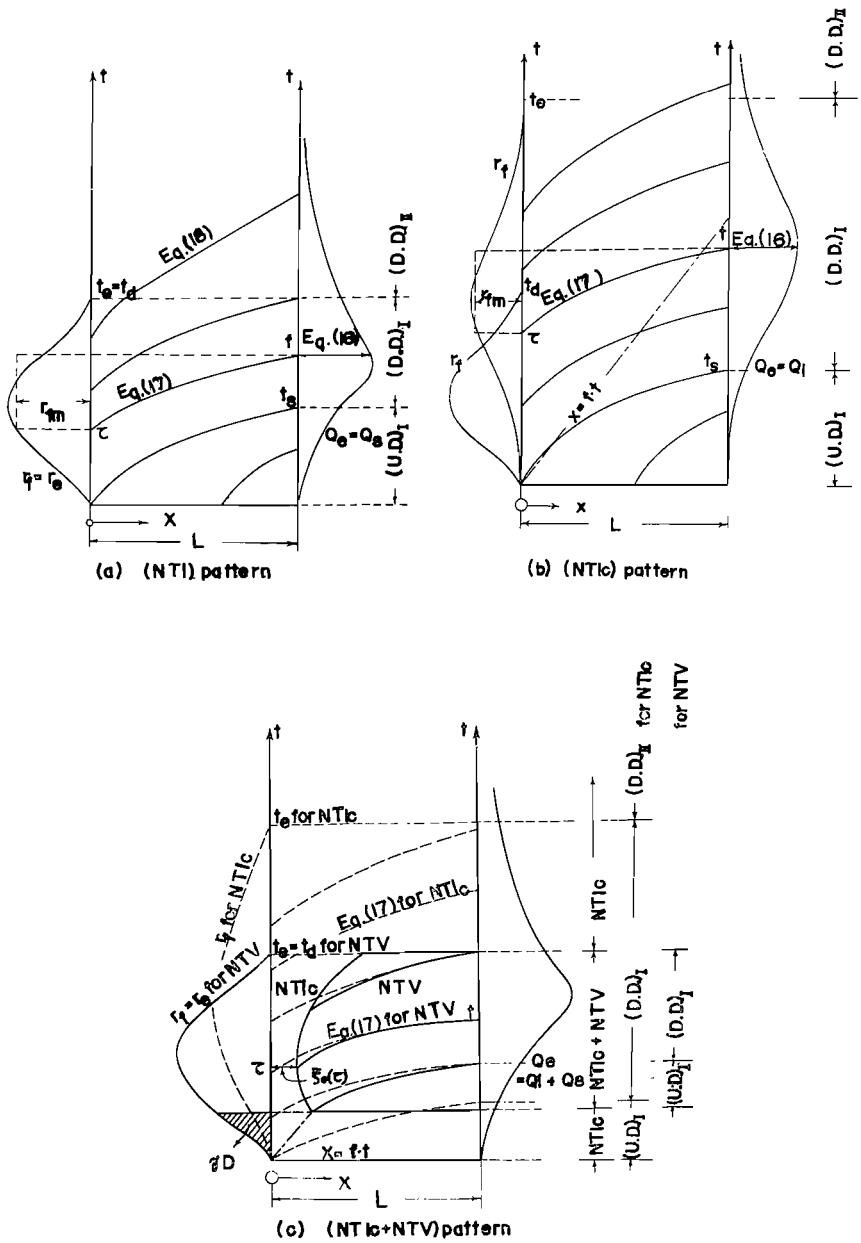

Fig. 5. Schematic representation showing the characteristics of transformation system in each pattern.

to the characteristics representing the rain-water disturbance, adding the subscript I (entirly) and II (partly).

Denoting the time which expresses the boundary between (U.D.) and (D.D.) by t_s, and the time which represents the cessation of effective rainfall by t_e, the relation between t and τ corresponding to it becomes as follows[3], from Eqs. (11) and (12).

For $t \leqq t_s((\text{U.D.})_I$ or $(\text{U.D.})_{II})$; $\tau=0$

For $t_s \leqq t \leqq t_e((\text{D.D.})_I)$: $$t-\tau=K\cdot\{L-\xi_0(\tau)\}^p/r_{tm}{}^{1-p} \tag{17}$$

For $t_e < t((\text{D.D.})_{II})$: $$(t_e-\tau)^{1/p-1}\{(t-t_e)+p\cdot(t_e-\tau)\} = pK^{1/p}\{L-\xi_0(\tau)/r_{fm}{}^{1/p-1} \tag{18}$$

in which $r_{tm}=\int_\tau^t r_f dt/(t-\tau)$, $r_{fm}=\int_\tau^t r_f dt/(t_e-\tau)$.

In these equations the origin of time must be taken at the initial time of each transformation system. Fig. 5 shows the transformation patterns.

(5) Representative values of basin characteristics

A question will arise concerning the characteristics of representative values of a basin, L and K.

These values, L and K, are the criteria of a basin representing the length and the effect of roughness and inclination angle in a basin, re-

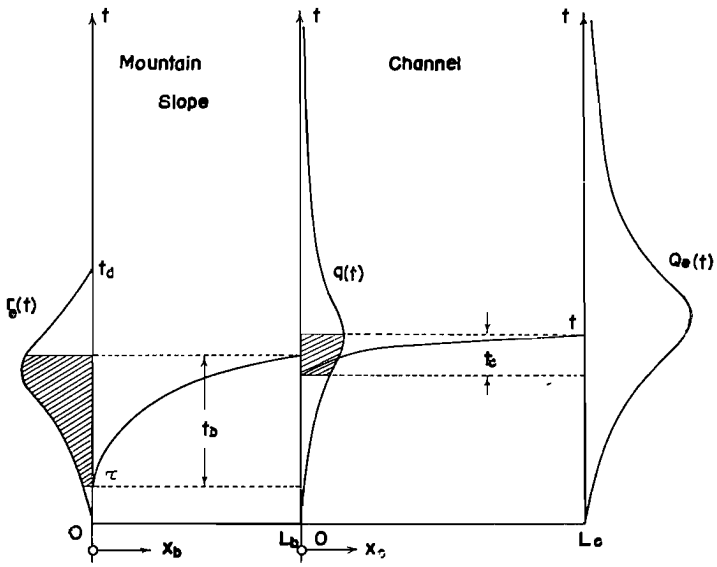

Fig. 6. Schematic representation showing propagation states of rainfall distarbance on mountain slope and channel, for NTI system.

spectively. Each value is to be divided into two parts, representing those of the mountain slope and those of the channel. This division of L may be independent of K, and vice versa, because of the physical significance. So, the subscript b and c will be used for the values of mountain slope and channel, respectively.

L_b, K_b and L_c, K_c determine not only the lags in the mnuntain slope and the channel but also the discharge rate at a gauging station as expressed in Eq. (16). Therefore, the effects of mountain slope and channel on runoff relation can be evaluated from a comparison of each lag.

For the sake of simplicity, we will here discuss the case of the time domain $(\mathrm{D.D.})_I$ in NTI system. It is to be noted that we compare the two lags, t_b and t_c, corresponding to the same part of rainfall as shown in Fig. 6. From Eq. (17) corresponding to $(\mathrm{D.D.})_I$ in NTI, t_b and t_c are expressed as follows,

$$t_b = K_b L_b^p / r_{tm}^{1-p} \tag{19}$$

$$t_c = K_c L_c^p / q_m^{1-p} \tag{20}$$

in which $K_c = \{n_c (\sin\theta_c)^{-1/2} K_1^{2/3}\}^p$, and $p = 3/(2z+3) \approx 0.6$, supplemented with the hydraulic radius R equal to $K_1 A_c^z$, the crossectional area A_c. And q_m which is the discharge rate per unit width of slope averaged over t_c must be represented as follows, for the reason mentioned before.

$$q_m = r_{tm} L_b \tag{21}$$

Consequently, the effect of mountain slope and channel on runoff relation is expressed in the form of the ratio of the two lags, using Eqs. (19), (20) and (21), that is,

$$S_t = t_c / t_b \approx K_c L_c^p / K_b L_b \tag{22}$$

Since S_t is influenced only by the basin characteristics and not by the rainfall condition, we can define S_t as an important non-dimensional value which represents the relation between the characteristics of a basin and the runoff.

Using Eq. (22), it will be seen that the value of S_t is considerably smaller than unity in a river basin less than several hundred km^2 in area, and that it is not necessary to take account of the effect of stream channels on runoff pattern. This is because of the fact that $p = 0.6$ and $K_b > K_c$, being derived from the experimental results on overland flow as stated in the previous chapter.

Thus, we may consider that the representative values of a basin L and K, are mainly influenced by the characteristics of the mountain slope. Assuming a quasi-uniform distribution of mountain slope characteristics, it may be possible to analyze the runoff process from the universal point of view by using the results in previous sections.

If the value of S_t is not less than unity, such as in a larger basin, we are forced to take account of the effect of stream channels even in the quasi-deterministic sense. A study in this field of hydrologic problem is now in progress.

(6) Characteristics of runoff pattern and its peak flow

It has been explained previously that the lag is the fundamental factor representing the character of transformation system. Accordingly the characters of individual runoff patterns can be visualized by the investigation of the characteristics of the lag. In this regard we will discuss briefly the characters of runoff patterns, especially the peak flow, and show some examples of application.

1) Non-linearity

First of all, the non-linearity of runoff phenomenon depends on the order of the time variation of T_c as transformation operator, and can be evaluated from the investigation of the structure of lag change $d\tau/dt$ defined by Eq. (14).

If lag change is always zero in the entire time domain at a gauging station, the propagation velocity of rain-water flow is constant and the transformation system is time invariant. In this case, a method of runoff analysis based on the assumption of linear time-invariant, such as unit-graph method, becomes an effective tool. The actual runoff phenomenon, however, is not linear, as stated in previous sections. Accordingly, evaluation of non-linearity is very important for the flood runoff, especially in a small basin, because of the weakness of the linearization by the effects of channel storage and other factors. The larger the time-variation of lag change becomes, the stronger the effect of non-linearity yields. As a result, the linear relationship between rainfall and discharge is unsatisfactory and the method of runoff analysis becomes very complicated.

The non-linearity of NTI pattern depends on only the non-linearity of overland flow. The effective rainfall r_f is equal to the excess rainfall r_e and there is substantially no effect on flattening it, so that the time varia-

tion of effective rainfall is larger than that of NTI_c. Moreover, since the roughness for overland flow is usually small in a basin having NTI system, the non-linear effect is very strong. This is the reason why the runoff is very intense in such a basin.

In a basin with surface stratum, NIT_c or $NTI_c \sim NTV$ pattern will appear depending on the rainfall condition.

As the rain-water falling on a ground is stored temporarily in the surface stratum for NTI_c, the range of time-variation of r_I is small, and for this reason the effect of non-linearity in appearance seems relatively weak.

The transformation systems for $NTI_c \sim NTV$ pattern pass through the windows which represent the system transition from NTI_c to ($NTI_c \sim NTV$), and vice versa. Hence the non-linearity of $NTI_c \sim NTV$ pattern depends on the nonlinear characters of overland flow and the system change. Since the index of the occurrence area of overland flow, $\xi_0(\tau)$, corresponding to (NTI_c +NTV) system acts to increase the non-linear character in runoff process, the configuration of runoff changes radically at the window and, therefore, the non-linear character of system change becomes of most importance in $NTI_c \sim NTV$ pattern.

As mentioned above, the non-linear structure of runoff phenomenon is very complicated even in the case of $S_t \ll 1$, owing to the deviations of rainfall and basin characteristics. By the system approach stated in previous sections, however, it may be possible to disclose the structure and to supply the basic knowledge by which we may discuss the applicability of many methods for runoff analysis hitherto presented, and construct a new method from the stand point of dynamics.

2) Peak flow

a) peak flow

Next we will discuss the relationship between peak discharge and its lag.

According to the approach in previous sections, the time of peak t_p at a gauging station must be in the time domain $(D.D.)_I$, if $(D.D.)_I$ arises. If $(D.D.)_I$ does not arise, t_p is to be in $(U.D.)_{II}$, and may appear near the boundary of $(U.D.)_{II}$ and $(D.D.)_{II}$ because the potential energy of water stored in channels may become maximum at this time point. The peak for the pattern of NTI_c or $NTI_c \sim NTV$ is always the former. The latter seldom arise in NTI pattern and the significance of the concentration time usually used as a criterion of the lag of peak flow becomes suitable in this case

only.

The peak t_p for NTI$_e$~NTV pattern must be in (D.D.)$_I$ for (NTI$_e$+NTV) system, and the occurrence area F of overland flow on the surface stratum varies with time. It is convenient for practical runoff analysis to assume that the area is constant and equal to the whole area A of a river basin and that the average intensity of side-seepage from the surface stratum, being the maximum value $\bar{r}_a$ of the effective rainfall r_f in NTI$_e$, is converted to the variant quantity r_v. This conversion can be given by the condition that the same discharge appears at the same time in both gauging stations of the variant field F and the constant field A. Using Eqs. (11) and (12), r_v becomes approximately, in the neighbourhood of the peak of runoff,

$$r_v = \bar{r}_a - L_r(r_e + \bar{r}_a) \tag{23}$$

In NTI$_e$~NTV pattern, replacing the effective rainfall r_f by the equivalent effective rainfall r_f* after puting $F=A$, the peak discharge of this pattern can be evaluated easily.

$$r_f^* = r_e + r_a \tag{24}$$

The avove replacement of the effective rainfall can be applied to NTI$_e$, because the change of the occurrence area of overland flow in NTV is equivalent physically to that of the depth of stored water in the surface stratum in NTI$_e$. In this case, however, L_r is not the one determined by Eq. (7) but the one representing the variation of stored water depth. Under these considerations, L_r applicable to the two patterns for $D>0$ is expressed as follows[1], from the continuity condition and simple assumption on the configuration of stored water depth,

$$L_r \doteqdot \varphi(t)^{-1}\left(\int_0^t b\varphi(s)ds\right) \tag{25}$$

in which $\varphi(t)=\exp\left(-\int_0^t a\cdot dt\right)$, $a=2(1/rD)(r+\bar{r}_a)$, $b\doteqdot 2(1/rD)(\bar{r}_a+i)$. Although the above treatment of the equivalent effective rainfall r_f* is theoretically not strict, it seems practically convenient, especially in the analysis of the runoff from a rainfall which has a complicated hourly distribution. The values of $\bar{r}_a$ and rD of basin can be estimated by the following relations derived by the hydraulic procedure on the recession curve of subsurface runoff.

$$\left.\begin{aligned} rD &= \{e^{\lambda_2(t_2-t_1)} i_c/\lambda_2 \\ \bar{r}_a &= (1-\delta)\lambda_2 rD \end{aligned}\right\} \tag{26}$$

in which λ_2 is the constant, t_2 and t_1 the cessation times of subsurface runoff and surface runoff, respectively ($t_1 = t_d$, the cessation time of rainfall, for no surface runoff), i_c the final infiltration capacity and $\delta = A_0/A$. In NTI, $r_f = r_e$.

Using Eqs. (11) and (12) and inserting the equivalent effective rainfall r_f^*, the occurrence condition of the peak of runoff for $(\mathrm{D.D.})_I$ is obtained,

$$(dT_c/dt)_{t=t_p} = 0 \tag{27}$$

The relation between initial time τ_p and arrival time t_p of the characteristics, as

$$r_f^*(\tau_p) = r_f^*(t_p) \tag{28}$$

Fig. 7 is the schematic representation given by Eq. (28). Accordingly, the arrival time interval T_{pc} on the characteristics concerning the peak discharge becomes, from Eq. (17), as follows.

$$\left.\begin{aligned} T_{pc} &= KL^p/r_{mp}{}^{1-p}, \\ r_{mp} &= \int_{T_{pc}} r_f^* dt/T_{pc} \end{aligned}\right\} \tag{29}$$

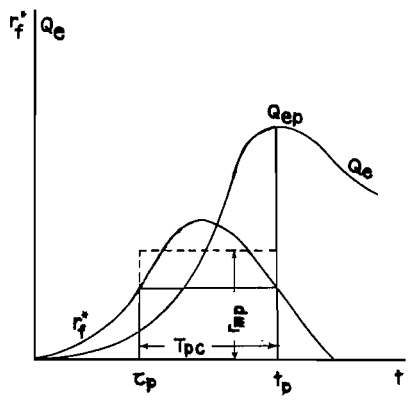

Fig. 7. Schematic representation illustrating the relation between equivalent rainfall, the occurring time of maximum discharge, and the propagation time of peak flow T_{pc}.

Discontinuity of $T_{pc} \sim r_{mp}$ relation may arise in the neighbourhood of $r_{mp} = \bar{r}_a$, because the signigicance of L_r is different for NTI_c and (NTI_c+NTV). The maximum discharge rate of runoff Q_{ep} is given by

$$Q_{ep} = r_{mp} A \tag{30}$$

When the time of peak t_p is in $(\mathrm{U.D.})_{II}$ of NTI system, the lag corresponding to peak flow is equal to the concentration time. Using Eq. (18), putting $\tau = 0$ and $\xi_0 = 0$, t_p is given by

$$t_p = pLK^{1/p}/R_e(t_d)^{1/p-1} + (1-p)t_d \tag{31}$$

in which t_d is the duration time of excess rainfall. The maximum discharge is expressed as follows, ignoring the final infiltration capacity.

$$Q_{ep}=AR_e(t_d)/t_d \tag{32}$$

b) application

Some applications of the theory on peak flow are shown in this article. Example I is an application to many flood runoffs in one basin, and Example II to runoffs in many basins in which runoff patterns seem to be the same.

(i) Example I (Peak flow of Yura River basin ; at ONO)

The upstream part of the Yura River basin above ONO gauging sation (Kyoto Prefecture) is 346 km^2 in area, 40 km in length of main channel and is well vegetated. From these basin characteristics, S_t is less than unity and the runoff pattern shows NTI$_c$ or NTI$_c$~NTV for any rainfall condition.

Using the relation of Eq. (26), the values of $\bar{r}_a$ and rD in this basin are estimated as 6 mm/hr, and 120 mm, respectively[4]. Applying these values to Eq. (25), r_f* are calculated from Eqs (23) and (24). Applying the occurrence condition of peak flow Eq. (28) to r_f*, the relations of T_{pc}~r_{mp} and Q_{ep}~r_{mp} are obtained as shown in Fig. 8 and 9. The calculated values are in good agreement with the theoretical relation expressed by the solid line in the figures. It is remarkable that the discontinuity of T_{pc}~r_{mp} relation arises in the neighbourhood of $r_{mp}=\bar{r}_a$, and the theoretical estimation of the system transition stated in the previous article is verified.

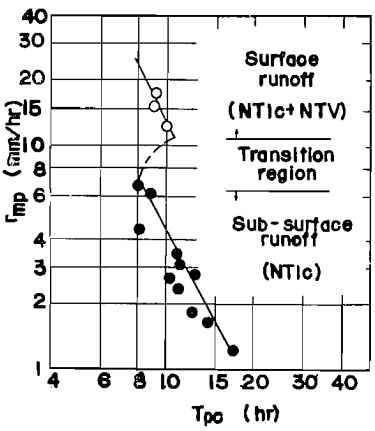

Fig. 8. Relation between the propagation time of peak flow and mean intensity of equivalent rainfall at ONO in Yura River basin.

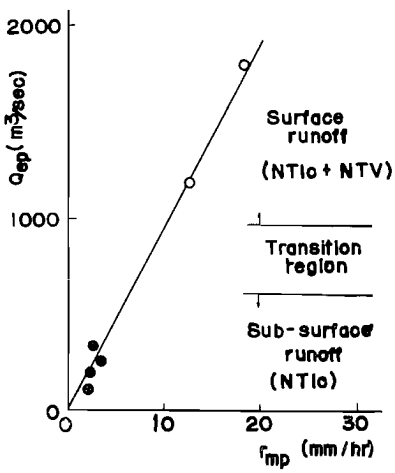

Fig. 9. Relation between the maximum discharge of Direct runoff and the mean intensity of equivalent rainfall, at ONO in Yura River basin.

(ii) Example II (Peak flows of many basins in Illinois)

Morgan and Johnson selected twelve river basins located in Illinois, ranging area from 10 square miles to 101 square miles, and obtained the runoff data in order to determine the unit graph. The following statement on the runoffs in the basins selected may be possible. ① S_t is less than unity. ② The basins have surface stratum as in a usual mountainous basin. ③ As the value of rD of a usual basin is in the range from 100 mm to 150 mm, the patterns of the runoffs shown in Table 4 are NTI_c, and they are simple cases because of $ft_d \ll L$.

Table 4. Characteristics of selected basins and runoffs (After Morgan and Johnson).

Basin No.	Name	A, in square miles	L_c, in miles	t_d, in hours	t_p, in hours	Q_{ep}, in cubic feet per second
1.	Hickory Creek	10.1	4.9	1	2.0	1040
2.	Canteen Creek	22.5	10.1	1	3.0	2530
3.	Big Creek	32.2	15.6	2	10.0	1570
4.	Indian Creek	37.0	18.8	2	12.0	1700
5.	Eay Creek	39.6	11.7	1	5.0	4010
6.	Money Creek	45.0	23.8	2	16.0	1280
7.	Farm Creek	60.9	17.3	1	3.0	9350
8.	Mill Creek	62.5	19.9	1	8.0	4920
9.	W. Br. Salt Fork	71.4	16.8	2	12.0	2140
10.	Hadley Creek	72.7	16.8	1	4.0	9430
11.	W. Bureaue Creek	83.3	19.6	2	4.0	5730
12.	E. Bureau Creek	101.0	23.3	2	4.0	5740

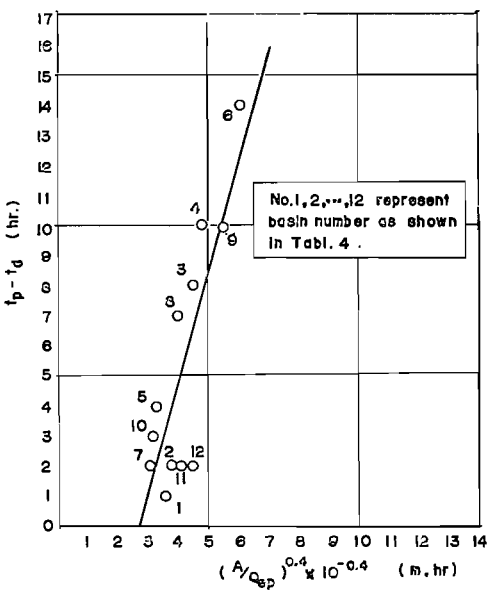

Fig. 10. Relation between the maximum discharge of direct runoff and its occurence time, for twelve river basins in Illinois.

From these assumptions, the time interval $t_p - t_d$ can be

expressed as follows, modifying Eq. (17).

$$t_p - t_d = KL^p (Q_{ep} A^{-1})^{1-p} \tag{33}$$

Assuming the uniform distribution of mountain slope characteristics, the values of KL^p for the basins selected are not so different. Fig. 10 shows the relation between $t_p - t_d$ and $(Q_{ep} A^{-1})^{0.4}$, using the values in Table 4. This result indicates that the assumptions mentioned above and Eq. (33) are good.

4. Conclusions and Discussion

In this paper, the storm runoff during a flood has been discussion as the transformation system from rainfall to discharge, and it has been pointed out that three fundamental patterns exist in runoff process and their characteristics are mainly determined by the non-linearity and the variation properties of occurrence area of runoff phenomena. The results obtained in this paper play an important role in runoff analysis of a river basin as small as several hundred square kilo-meters in area.

In larger basins, we still face the problem of the effects of channels distributed in a river basin on runoff process, in an even quasi-deterministic sense. Futhermore, the statistical treatment of the effects of stochastic variables on runoff process, which is impossible to determine by the deterministic approach, is of importance. The theory of non-linear statistical sequences has, however, not yet been succesfully established and further we do not know clearly the choice of statistical variables concerning runoff process and its interactions.

The sudy of these problems will constitute an interesting and necessary research project whose object will be to derive a generalized theory of runoff, based on the approach and the results outlined in this paper.

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