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APPLICATION OF PROBABILITY THEORY OF TWO-DIMENSIONS IN DETERMINING DESIGN FLOOD

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Synopsis

In determining a design flood, the social demand and the hydrological regime of a river under consideration should be examined in detail. The hydrological property of the river during a flood is one of the most important factors in the field of natural science. The customary treatment of the hydrological property is the statistical or stochastical estimation of the peak discharge during a flood in the sense of one-dimensional probability theory.

Recently, in Japan, many reservoirs are constructed for the purposes of flood control and water resources. In establishing such a scheme of artificial water control, the hydrological information available should be not only the peak discharge but also the hydrograph itself during a flood under consideration. This paper describes the method of etimation of occurrence probability of the hydrograph during a flood and the computational examples for the Yodo River. First of all, the criterion in determining a design flood is discussed, then the two-dimensional probability of normal distribution and the method of normalization of variables are presented. Secondly, after considering the hydrological quantities appropriate to two-variables, it is shown that the peak discharge and the duration time of hydrograph are desirable for this purpose. Finally several examples of application of this approach to the determination of design flood and the judgement of flood control effect by reservoirs are explained.

1. Introduction

It is one of our urgent problems in Japan to prevent and decrease the damages by disastrous floods. To achieve this end, the project of flood-protection must be established reasonably, and the phenomena of flood runoff must be studied scientifically. Moreover, since the subjective element for this establishment is a design flood, the design flood should be determined most scientifically and rationally.

It is well known that the occurrence probability of floods has been considered and studied in detail by many hydraulic engineers and hydrologists in relation to the determinataion of a design flood. The usual treatment of such probability, however, has been usually limited to application of the probability theory of one-variable. That is, the peak discharge of flood hydrograph has been taken as a variable in the treatment.

In undertaking the water works on a whole river system, it is often found that a reasonable project cannot be established as long as the one-dimensional probability is considered. For example, in the case where a river channel under consideration is composed of two tributaries and one main channel, or has a reservoir for flood control, it is desirable that the occurrence probability of flood hydrograph be evaluated as the joint probability of two or more variables[1]. In this sense, this paper describes application of two-dimensional probability theory to the determination of a design flood and the evaluation of the effect of flood control by a reservoir.

2. Criterion in determination of design flood

Needless to say, the dominant flood losses result from the flooded water overflowing on banks or embankments of a river. Especially in Japan, the main water work for flood-protection was, before the War II, the construction of embankments along a river channel which passed through a highly developed district. Therefore, once a high water during a flood overflows these banks, then the district protected by them suffers disastrous damages. That is to say, whether or not the water during flood flows over the embankments determines the effectiveness of flood-protection. Recently, in accordance with the rapid development of economic and social activities in flood-protected areas, the desired scale of design flood has been larger and larger in Japan. In such a case, enormously high embankments are not always best, but reservoirs or retention pools for flood control are proving to be better. This is the reason why many reservoirs have been constructed for flood control in Japan. Even in this case, however, there will be no essential objection to the conclusion that the embankments are a direct guard of the flood-protected district and that overflowing of water on them means disastrous damages. Therefore, in any case, the effect of flood-protection works can be judged by evaluating the occurrence probability of overflowing on embankments. That is, the criterion in determination of the design flood for a whole river system is to be considered on the basis of this probability.

3. Treatment of two-dimensional probability

Before discussing the determination problem of a design flood, the outline of the probability theory of two-dimensions available and its method of treatment are here explained.

The normal frequency function of two variables, x and y, is given by the following well-known equation[2),],

$$f(x, y)=\frac{1}{2\pi\sqrt{1-\rho^2}}\exp\{-(x^2-2\rho xy+y^2)/2(1-\rho^2)\} \tag{1}$$

where ρ is the theoretical correlation coefficient. Putting,

$$x^2-2\rho xy+y^2=(1-\rho^2)X^2 \tag{2}$$

then, assuming that X is constant, Eq. (2) expresses an ellipse with two symmetrical axes, $x=y$, $x=-y$, in $x-y$ plane. And also this ellipse is the curve on which the probability density is the same. Since the area, A, within the curve is equal to $\pi\sqrt{1-\rho^2}X^2$, the joint probability, $P_r(X)$, of x and y in this domain is given by,

$$P_r(X)=\frac{1}{2\pi\sqrt{1-\rho^2}}\int_0^X\exp\left(-\frac{X^2}{2}\right)dA=1-\exp\left(-\frac{X^2}{2}\right) \tag{3}$$

If the values of $P_r(X)$ for various X are known, the occurence probability within an arbitrary domain can be obtained by means of numerical caluculation. Therefore, in applying the above probability function to estimation of the occurence probability of flood phenomena with two variables, it is enough to find out the normalization functions for the two variables and the correlation coefficient between them. In the practical problems, however, sveral difficulties in normalizing the variables and finding the correlation coefficient will appear, because the satisfactory data are scarcely obtained in this hydrological field.

(1) Method of normalization of variables

It is well known that many hydrological amounts can be normalized as probability variable by means of the logarithmic transformation and several methods of normalization have been presented. There are many cases, however, where the variables do not cover the whole domain of $x-y$ plane in practical application. In such cases, it is so difficult to determine the normalization function strictly that an approximate method of normalization may be required. Since, however, the lack of data will appear in various figures, it will be impossible to find out the generalized method of normalization. The usual case, where there exist only the flood hydrographs having a peak discharge greater than a definite value, is considered here, and the peak discharge and the duration time of flood hydrograph are taken as two variables for convenience of explanation.

From Eq. (1), the marginal distribution function of normalized variable, x, is written by,

$$f_1(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right) \tag{4}$$

On the other hand, if the number of flood is n in a year and if the flood hydrographs which have a peak discharge Q larger than the definite discharge Q_f are known, the exceeding probability, $1-F(x)$, of occurrence of the peak discharge is given as follows, in the sense of Thomas plot :

$$1-F(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{x^2}{2}\right)dx=\frac{i}{nN+1} \tag{5}$$

Assuming that the marginal distribution function is log-normal, the normalization function is given by,

$$x=\alpha_1\log\frac{Q+b_1}{Q_0+b_1} \tag{6}$$

In Eqs. (5) and (6), i is the order of magnitude of Q from largest to smallest, N the number of years during which the samples exist, and α_1, b_1 and Q_0 numerical constants for normalization. In order to find the constants satisfying the equational relations of Eqs. (5) and (6), the following approximate method may be available for practical purposes in the sense of empirical distribution function.

First of all, the relation between $Q+b_1$ and $\frac{i}{nN+1}$ is plotted on the paper of log-normal for various combinations of b_1 and n. Then final values of b_1 and n are determined so as to arrange the plotted points as straightly as possible. Next, after drawing the most suitable straight line to those points, the value of Q_0 can be found out from that of $Q+b_1$ corresponding to $1-F=50\%$ on the straight line, and the value of α_1 from the gradient of the line.

Figure 1 is an illustration diagram in which two axes represent the valiables x and y in the case of negative correlation. In this figure, x and y correspond to the discharge Q and the duration time T, respectively, and the chain line AA′ is 45° in inclination to the axis through the origin. Assuming that this diagram is normalized in the sense of Gaussian normal, the samples are to be plotted symmetrically about the chain line. It may be permissible, therefore, to assume that the samples within any domain can be replaced symmetrically in other domains about the line AA′. By this characteristic of the normalized density function, the numerical constants for normalization of the duration time T to the normalized variable y can be determined as follows.

The equation of transformation, $y'=\alpha_2 \log (T+b_2)$, is assumed for normalization as first approximation. The samples are plotted on $x-y$ plane for various combinations of α_2 and b_2 in this equation by using the values of x and n determined above. The values of α_2 and b_2 may be determined so as to satisfy the condition that the plotted points included within the domain DCB′ are symmetrical about the line AA′ which is 45° in inclination. After accomplishment of this procedure, the samples plotted in the domain BCD are placed back in their symmetrical position about the line AA′. Then all of the plotted points are to be placed in position under the broken line

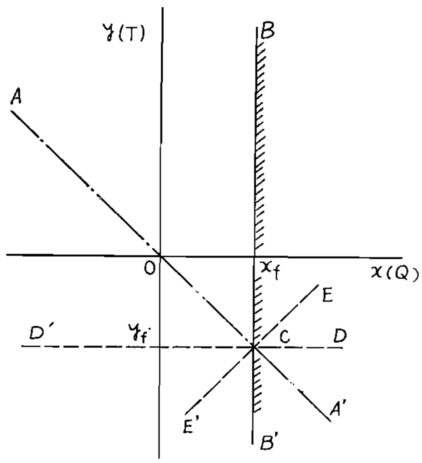

Fig. 1. The illustration diagram for normalization of probability variables in $x-y$ plane.

DD′. Since, therefore, these points can be regarded as samples satisfying the condition $y' \leqq y_r'$, the second approximate values of α_2 and b_2, and that of T_0 in the following normalization function can be determined by the use of a method similar to the one adopted in finding the normalized function of x.

$$y = \alpha_2 \log \frac{T + b_2}{T_0 + b_2} \tag{7}$$

(2) Method of determination of correlation coefficient

It is clear that the line AA′ is the long principal axis of the ellipse which represents the curve of equal density of probability. Generally, the relation between the theoretical correlation coefficient ρ and the theoretical covariance σ_a^2 with respect to the long principal axis is given by,

$$\rho = \pm (1 - \sigma_a^2) \tag{8}$$

where plus and minus signs are applied to the case of positive and negative correlations, respectively. Furthermore, the covariance σ_a^2 is the unvarying quantity for any domain which is bounded by one or two lines perpendicular to the principal axis. Using these theoretical characteristics and replacing the theoretical values ρ and σ_a by the sample's values r and s_a, respectively, the coorrelation coefficient available can be successfully obtained.

In the case under consideration, the samples exist only in the limited domain as shown in Fig. 1. If the samples are assumed to exist in the domain B′CD′ by symmetrical replacement between the domains BCD and D′CB′, the samples' covariance can be easily calculated by the use of the data existing within the right-down domain bounded by the line EE′ which passses through the point C and is perpendicular to the axis AA′.

4. Occurrence probability of design flood

As already considered, in establishing the flood-protection project including flood control by reservoir, it is very difficult to estimate the probability of overflowing on embankments under consideration by using only the peak discharge of flood runoff, because the function of a reservoir for flood control is to store the water during a flood. It is desirable for this purpose to evaluate the occurrence probability of the flood hydrograph itself.

(1) Quantative representation of flood hydrograph

The quantative representation of characteristics of a flood hydrograph is needed in order to estimate its occurrence probability. It is desirable for the treatment based upon the probability theory that representative factors are the original or natural ones which have never been affected by any artificial conditions. That is, these must be necessaiily caused by the meteorological elements which can be, at present, considered as a random process. Generally, the storm rainfalls resulting from the same kind of meteorological cause are inclined to take nearly the same shape in time-distribution diagram, and their intensities may be accorded to that of the meteorological causes. Then, the condition of storm rainfall may be approximately represented by its duration time and intensity.

The duration time and intensity of storm rainfall may be made to directly correspond to those of flood hydrograph, taking account of availability of the unit-hydrograph method in runoff analysis. The peak discharge, Q, of flood hydrograph can be undoubtedly chosen as one probability variable in correspondance to the intensity of storm rainfall. On the other hand, it is very difficult strictly to define how the duration time, T, of flood hydrograph is to be chosen as another probability variable to that of storm rainfall. It seems in a river channel, however, that the duration time while the flood runoff is showing the half or more value of peak in discharge may be conveniently adopted as that

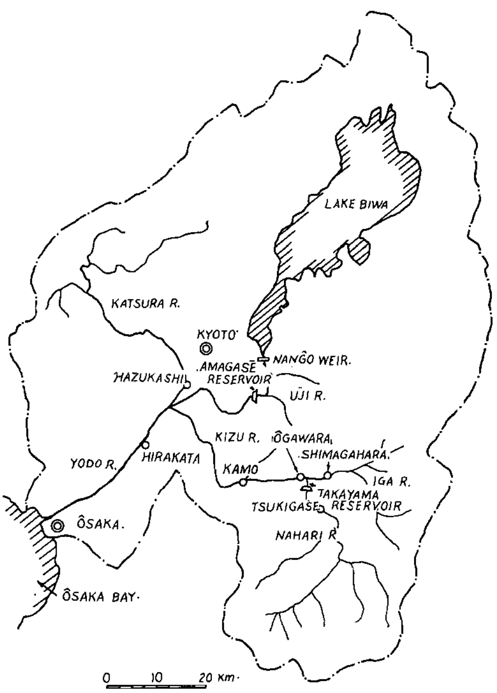

Fig. 2. Map of the Yodo River.

of flood runoff, because such a duration time may correspond to that of the more intense part of storm rainfall and, in many cases, mean the dangerous period of time during a flood.

(2) Design flood in Yodo River

By applying the foregoing treatment of two-dimensional probability to two variables, Q and T, defined above, the estimation problem of occurrence probability of design flood in the Yodo River can be discussed reasonably. The computational example is presented here.

An outline of the channel system in the Yodo River is shown in Fig. 2. The data of flood at Hirakata gaging station, which is of importance in this river, were limited to the flood hydrograph having the peak discharge larger than 3,000 m³/sec from 1926 to 1959. The result obtained by the method of normalization, which is mentioned earlier, is as follows, in m³/sec and hr for Q and T, respectively :

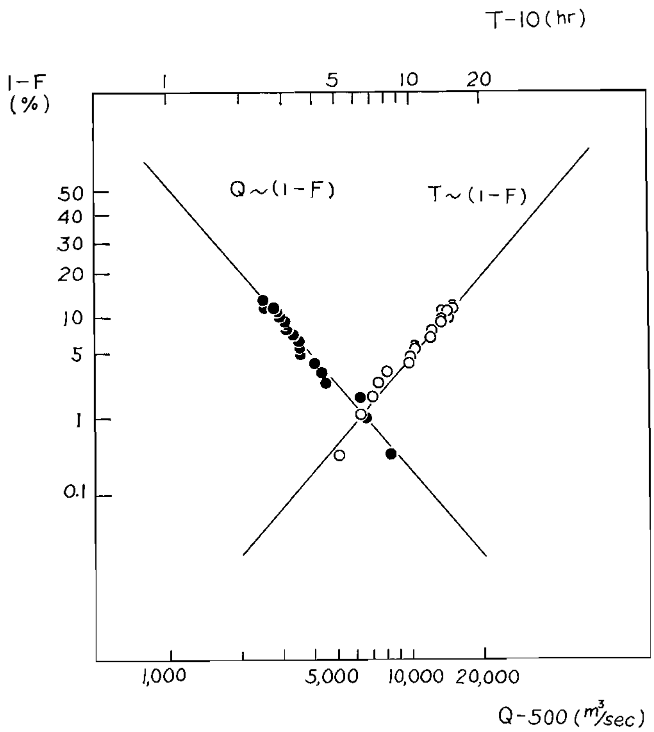

Fig. 3. The marginal distribution function of probability.

$$\left.\begin{aligned} x &= 2.04 \log (Q-500) - 6.155 \\ y &= 2.06 \log (T-10) - 3.277 \end{aligned}\right\} \qquad (9)$$

And the figure of fitness of these equation to the data is Fig. 3, for the marginal distribution of probability. Introducing the sample's correlation coefficient $r=-0.95$, which is obtained by using the limited data, into Eq. (1), the density function of occurrence probability becomes,

$$f(x,\ y)=0.511 \exp \{-5.13(x^2+1.90xy+y^2)\} \qquad (10)$$

The result obtained is summarily shown by Fig. 4, in which the small circles, the chain line, and the elliptic curve express the plotted points of past data, the principal axis of probability ellipse, and the equi-density curve of occurrence probability of flood hydrograph, passing through the sample which has the maximum peak discharge in the past, respectively.

Since the maximum discharge permitted to flow for the existent embankments, which it is not desirable to make any higher from the viewpoint of social activities in the flood-protected districts, is 6,950 m³/sec, several reservoirs for flood control are now being constructed to remove the danger of flooding caused by a larger discharge of runoff during a

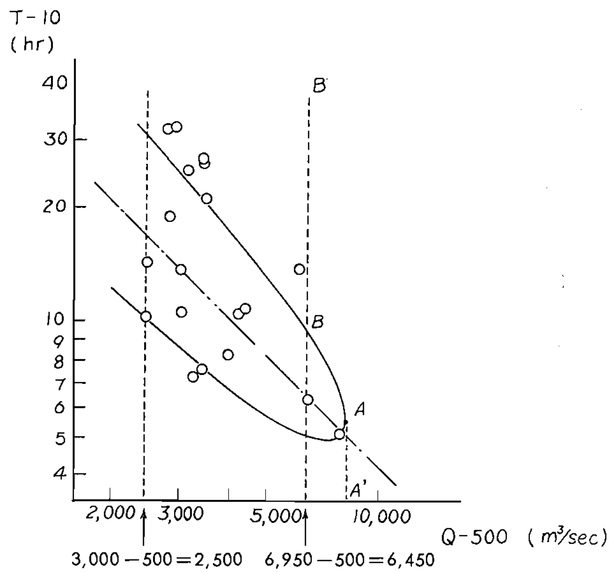

Fig. 4. The two-dimensional representation of occurrence probability of flood hydrographs at the Hirakata gaging station.

flood. In Fig. 4, therefore, the flood on the left side of the line B′B can be considered to have no danger of overflowing.

Now, let us assume that the design flood of this river is to be considered so as to prevent damages resulting from the largest flood in the past. The maximum peak discharge of the flood having the equal density of occurrence probability to the past largest flood is shown by point A on the elliptic curve in Fig. 4. If the reservoir for flood control is planned as to the flood at point A, the flood in the domain between the lines BB′ and AA′, and at least under point A can be controlled adequately by it. The peak discharge of the controlled flood flow is to be decreased below 6,950 m^3/sec, because the peak discharge and the duration time of the flood flow are smallar than those at point A. However, there remains a problem for the floods in the part above point A in the domain. Considering that a design flood is to be based upon the concept of the occurrence probability of floods, it is necessary to plan the water works to the floods at least on the curve AB, which have the same density of probability as that at point A. That is, the control operation of the reservoirs under consideration should be examined to find the way by which the floods on the curve AB are controlled so as not to create danger.

If the flood hydrograph, corresponding to such a point A as expresses the largest flood hydrograph in discharge on the ellipse having the equal density of occurrence probability, is taken account of as the design flood, it seems very appropriate in the sense of considering mainly a peak discharge during a flood and in the convenience of comparison with those of other rivers. For example, the probability of a flood being larger in peak discharge than the design flood can be easily obtained from the marginal distribution function for the discharge given by Eqs. (5) and (6). In order to estimate the effect of flood-protection works, the probability of overflowing on embankments can be also calculated as the volume under the surface of the normal function of joint probability within the semi-infinite domain, bounded by the semi-infinite lines A′A and BB′, and the elliptic curve AB which will show the limit of no dangerouness.

(3) Occurrence probability of floods in a net of river channels

Next, it is necessary to examine the occurrence probability of floods in a net of river channels, because the Yodo River is composed of three tributaries, the Kizu River, the Katsura River and the Uji River, and one main channel, the Yodo River, as shown in Fig. 2, and there are important districts to be protected along each river channal. For this purpose, the treatment of joint probability can be applied as follows.

Strictly speaking, even if the peak discharge is unchanging while the flood flow flows down in each tributary during a flood, the peak discharge of flood in the main channel after confluence will be influenced by the difference of arrival times of the flood peaks in tributaries to the confluence point. The resultant hydrograph, therefore, may be considered as a function of the flood hydrographs themselve in the tributaries and the time difference of their occurrence. In this river, however, the stream flow of the Uji River is controlled in nearly constant discharge by operation of the Nango Weir during not only a low flow but also a flood. Furthermore, the time difference of confluence of flood peaks from the Kizu and the Katsura rivers can be assumed empirically to be nearly constant for every flood.

Therefore, the condition of confluence in this river is given by the method of least squares as follows :

$$Q=0.884Q_1+1.035Q_2+70 \tag{11}$$

where Q_1 and Q_2 are the peak discharges in the Kizu and the Katsura rivers, respectively, and Q that in the Yodo River after confluence, in m^3/sec. Applying these properties of confluence of floods, it may be possible to estimate the occurrence probability of floods in each river channel by the use of the theory of joint probability.

Here, the data of floods available were the yearly maximum in each channel from 1912 to 1961. Since these data are considered as the whole sample for yearly maximum flood during this period, the normalization from Q_1, Q_2 to x_1, x_2, respectively, and the calculation of correlation coefficient between them can be easily done as a special case of that explained earlier. By using the customary method of normalization for log-normal,[3)4)5)] x_1 and x_2 are given by,

$$\left.\begin{aligned} x_1&=\frac{1}{0.5355}\left\{\log(Q_1-262)-3.100\right\} \\ x_2&=\frac{1}{0.4524}\left\{\log(Q_2-113)-2.884\right\} \end{aligned}\right\} \tag{12}$$

The correlation coefficient between x_1 and x_2 is equal to -0.11. And this value is so small that the assumption of no correlation can be reduced by the so-called rejection test.

In the flood-protection works of this river, the embankments along every river channel have been already constructed for the following discharges[6) :

1) $Q_0 = 6{,}950\ m^3/sec$, at Hirakata gaging station in the Yodo River,
2) $Q_{10} = 4{,}650$ 〃, at Kamo gaging station in the Kizu River,
3) $Q_{20} = 2{,}850$ 〃, at Hazukashi gaging station in the Katsura River.

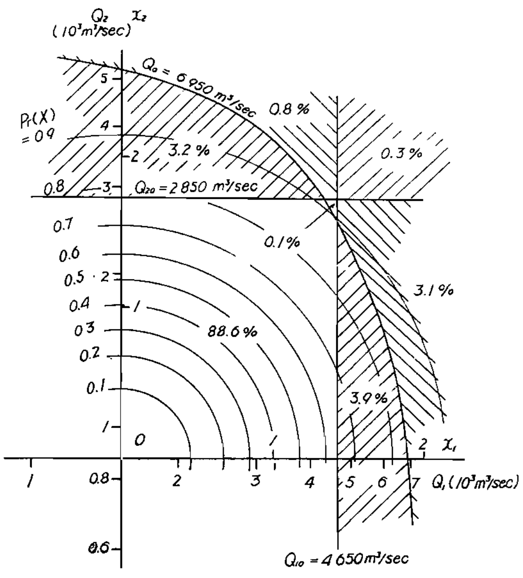

Fig. 5 Numerical example for the net of channels in the Yodo River.

Figure 5 is the two-dimensional representation of occurrence probability of the peak discharges obtained by a method of computation similar to that mentioned before. In this figure, the design peak discharge for each river channel are shown by the three lines : $Q_0 = 6{,}950$, $Q_{10} = 4{,}650$ and $Q_{20} = 2{,}850$, in m^3/sec. From this figure, it is seen after numerical integration that the occurrence probability of overflowing on embankments anywhere through the whole net of the river channels is equal to 11.4%, which is computed as the percent of volume to unity under the surface of the normal density function of joint probability within the right-upper domain bounded by these three lines. It can be said, therefore, that there might be a danger of overflowing in any channel at the rate of once in nearly ten years, as long as the present embankments remain unchanging.

Moreover, it is possible to estimate by using Fig. 5, the occurrence probability with which the high water during a flood overflows on embankments along one river channel but does not along other channels.

Such a probability is 3.9% for the Kizu River, 3.2% for the Katsura River, and 0.1% for the Yodo main river. It can be found from the result obtained that the probability of overflowing is larger in the tributaries than in the main channel. Therefore, in order to decrease the peak discharges in the tributaries during floods, it would seem that the reservoirs for flood control ought to be constructed in the upstream parts of the tributaries. Such reservoirs are now being constructed in this river.

The next desired calculation is to estimate the effect of flood control by such a reservoir in decreasing the peak discharge downstream during floods.

5. Evaluation of effect of flood control by reservoir

(1) General remarks

As a matter of course, a reservoir for flood control is designed to prevent or decrease the flood losses in the downstream district by using its storage capacity available. The effect of such flood control may be reasonably evaluated by the consideration of the following two items.

The first, with respect to the main object of a flood-control reservoir, is whether or not the sum of the outflow from the reservoir and the runoff from the residual drainage basin is in danger of exceeding the design peak discharge in the downstream channel, that is, in the viewpoint of outflow. Usually, the outflow in the downstream channel may be related more or less to the runoff from the residual drainage basin, but mainly determined by the inflow into the reservoir, the effective storage capacity of the reservoir for flood control, and the method of control operation. The second, with respect to a flood-control reservoir itself, is whether or not the desired effect of flood control can be obtained by the use of a definite storage capacity. For example, when the duration time of flood hydrograph is rather long, the desired effect of flood control cannot be expected in the viewpoint of the stored water in the reservoir, even if its peak discharge is not so large as the one for which the capacity of the reservoir is designed. In such a case, the storage capacity of the reservoir should be determined by the water volume of inflow into the reservoir and the method of control operation.

In order to make the following consideration clear, let us assume a rule of flood control in which the inflow is controlled with the constant ratio,

p, between inflow and outflow during the period while the discharge of inflow is larger than Q_0 defined adequately, and the outflow is held in $(1-p)Q_p$ after the discharge of inflow reaches the peak, Q_p, as shown in Fig. 6. In the case where such a rule of control operation is adopted, the scale of flood hydrograph may be approximately represnted by the discharge, $Q'=Q_p-Q_0$, and the duration time, T, of larger discharge than Q_0, which are shown in Fig. 6.

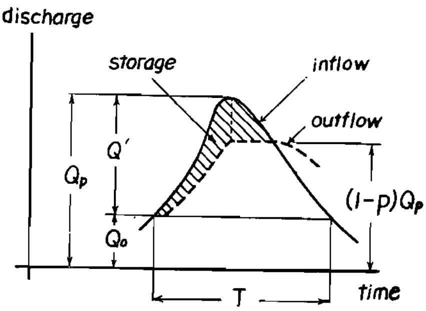

Fig. 6. The rule curve of operation for flood control and the notation.

Therefore, the density function of the occurrence probability of flood hydrograph in this case is symbolically written by using two variables, Q' and T, as follows :

$$f=f(Q', T) \tag{13}$$

On the other hand, in the case where the inflow during a flood is controlled with the constant ratio through the reservoir, the peak discharge of the controlled outflow and the volume of stored water in the reservoir are also expressed, respectively.

$$Q_a=Q_a(p\ ;\ Q', T) \tag{14}$$

$$V_T=V_T(p\ ;\ Q', T) \tag{15}$$

In Fig. 7, which is the illustation diagram of the two-dimensional representation of probability, the two relations of Q_a=const. and V_T= const. are generally represented by two curves. These curves divide the $T-Q'$ plane into four domains, I, II, III and IV. If the discharge, Q_a, is the design peak discharge in the downstream channel and the volume, V_T, the maximum effective storage capacity of the reservoir for flood control, then these domains have the following concrete meaning.

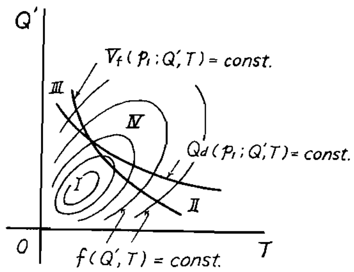

Fig. 7. The two-dimensional representation of the occurrence probability of floods flowing into a reservoir.

Domain I corresponds to that of such floods as, in carrying out an operation of the flood control mentioned before, the peak discharge of outflow is smaller than the design one in the downstream channel and as the stored water in the reservoir is less than its effective storage capacity for flood control. In this case, the flood losses will not occur from the two viewpoints of the outflow discharge and the stored water. In domain II, the outflow from the reservoir does not lead to any flood loss in the downstream district, but the stored water in the reservoir exceeds its effective storage capacity. That is, some trouble will occur on operation of control. Domain III corresponds to that of such floods in which the storage capacity of reservoir has more space for flood control but the outflow discharge exceeds the design one in the downstream channel, and then flood damages will occur. For floods in domain IV, the flood losses will appear from both viewpoints of the outflow discharge and the stored water in the reservoir. Accordingly, it can be said that the flood damages will appear for floods not in domain I but in damains II, III and IV. This is the basic property in determination of the most suitable ratio between inflow and outflow and evaluation of the effect of flood control by the reservoir.

(2) Determination of controlling ratio between inflow and outflow

The domains explained above vary with changes in the ratio of flood control. Assuming that, for example, the ratio of flood control is changed from p_1 to p_2, where p_2 is larger than p_1, the two curves, $Q_d(p_2)=$const. and $V_r(p_2)=$const., as shown in Fig. 8. Then, the domain which means no damage by flood in the downstream district varies from I to I+II′−III′, and the domains of II, III and IV vary similarly. Correspondingly, the occurrence probability of floods within the varied domains can be easily found for various values of the ratio. In other words, the dangerousness of overflowing on embankments by the large outflow over the design discharge in the downstream channel, the occurrence of dangerous surge wave resulting from the insufficiency of the storage capacity of reservoir, and so on, can be evaluated as their occurrence probabilities. Therefore, when the design peak discharge, Q_d, of the downstream channel during a flood has already been determined, and when the operation of flood conrol is to be done by the use of the storage capacity, V_r, of a reservoir defined in advance due to the topographical and geological features of the dam site, as is usual in most cases in Japan, it becomes possible by the estimation of such probabilities to find out the most suitable ratio of flood control. That is, the ratio between inflow and outflow in the operation of flood control should be determined so as to minimize the occurrence probability of overflowing on embankments.

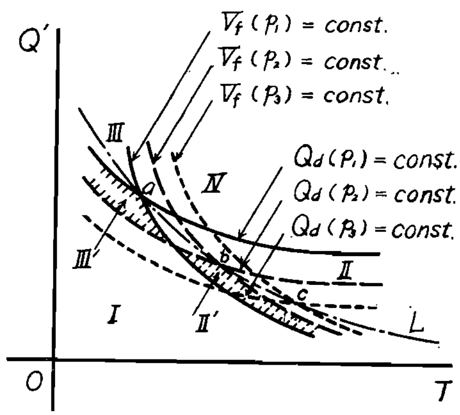

Fig. 8. The illustration diagram when the ratio of flood control is changed.

Next, the intersection point of the two curves, Q_d=const. and V_f= const., moves from a to b, c …… by changing the ratio from p_1 to p_2, p_3, ……, as shown in Fig. 8. Under the consideration of the meaning of the curve L joining these intersection points, it can be understood that the right-upper domain bounded by the curve L corresponds to that of the occurrence of floods, which cannot be desirably controlled by applying any ratio under the operation of flood control mentioned before. On the other hand, the left-under domain corresponds to that of floods which can be successfully controlled not to exceed the design discharge of the downstream channel by the use of the design storage capacity of the reservoir under the condition of changing suitably the ratio of flood control. Applying such an efficient method of control operation, the occurrence probability of non-damage in the downstream district can be computed as the volume under the surface of the probability function within the domain.

If precise flood forecasting is expected, the probability of non-damage may be brought close to this maximum value by the adoption of the suitable ratio of flood control to the flood. More effectiveness of flood control by the reservoir cannot be anticipated, so long as the rule of operation of flood control is not changed.

(3) Computational example

The results obtained by the above consideration can be applied to evaluate the effect of flood control by the Takayama reservoir, under construction in the Nahari River, which is the largest tributary in the basin of the Kizu River. The data of floods available are during the period from 1949 to 1962 at the Tsukigase gaging station near the dam site, and have peak discharges larger than 330 m³/sec.

The discharge of 330 m³/sec is adopted as the discharge over which the flood flow is controlled by the reservoir, and corresponds to the appointed water stage which is considered as an index of beginning to cause some kind of flood damage in the downstream channel. Then, taking the discharge $Q'=Q_p-330$ and the duration time, T, of inflow over the discharge $Q_0=330$ as two probability variables, the normalized variables, x_1 and x_2, are obtained as was done before, as follows :

$$x_1=\frac{\log(T+30)-1.663}{0.0583}, \quad x_2=\frac{\log(Q'+234)-3.058}{0.294} \tag{16}$$

where the unit is m³/sec for discharge and hr for time.

On the other hand, the correlation coefficient is given by $r=0.69$. Under consideration of the runoff from the residual drainage basin during a flood, the maximum allowable limit to the discharge at the Kamo gaging station can be represented by the following relation :

$$(1-p)(Q'+330)+Q_r=4{,}650, \text{ in m}^3/\text{sec} \tag{17}$$

where Q_r is the peak discharge of flood hydrograph from the residual drainage basin. Moreover, since it is ascertained empirically that the contribution of the residual drainage basin to the peak flow of the design flood at the Kamo station is given by the relation of $Q_r=0.79Q_p=0.79\times(Q'+330)$ in m³/sec, the above relation is rewritten as follows :

$$(1.79-p)(Q'+330)=4{,}650, \text{ in m}^3/\text{sec} \tag{18}$$

Therefore, the design peak discharge of the downstream river channel may be considered by replacing with the peak discharge at the dam site through the above relationship.

It is seen, after several examinations of flood control about the actual data of floods, that the relation between the storage capacity of the reservoir and the scale of the flood is approximately given by the expression, $V_f=\alpha(Q'T)^\beta$, where α and β are numerical constants defined by the ratio p. Since the storage capacity of this reservoir for flood control is designed as 3.54×10^7 m³, the above relation becomes,

$$\alpha(Q'T)^{\beta} = 3.54 \times 10^7, \text{ in } m^3. \tag{19}$$

Under these conditions, the result obtained is summarized in Fig. 9.

It can be found after several trial computations by using this figure that the most proper ratio of flood control is $p = 0.4$ under condition of minimizing the occurrence probability of flood losses, and the probability becomes 5.2%. That is, applying $p = 0.4$, the flood flow at the Kamo gaging station can be successfully controlled by the Takayama Reservoir under the design peak discharge. It seems, however, that there arises the trouble resulting from the insufficiency of the storage capacity for the flood, shown in Fig. 4, of which the scale is the maximum in the past. From this reason, two other reservoirs for multiple purposes, including flood control, are now being planned in the upstream parts of this river.

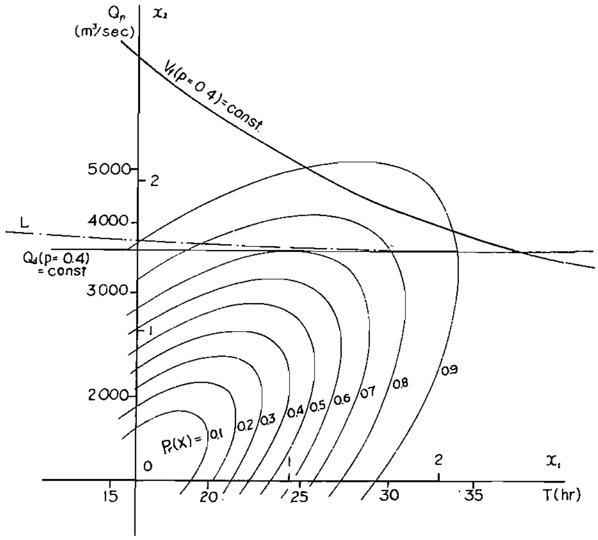

Fig. 9. The numerical example for the Takayama Reservoir in the Nahari River.

The curve L in Fig. 9 shows the result obtained in the case where the ratio of flood control is adequately changed so as to use the storage capacity of the reservoir for flood control as much as possible. The probability in this case is 4.6%. In the case of floods having single peak, however, it is seen by comparing these probabilities that the additional effect by changing the ratio of flood control is not so valuable.

6. Summary

This paper has described the application of the joint probability theory of two dimensions in determining a design flood. The discussions were done for three different problems in the Yodo River. Although the result obtained are related to each other, a sufficiently satisfactory relationship between them cannot be obtained yet. The main conclusions are as follows.

1. In establishing the flood control scheme in a river, the occurrence probability of overflowing on embankments along the river channel should be adopted as the most basic criterion on the evaluation of its effect.

2. In such a case, the flood hydrograph itself becomes important, and its probability should be expressed by the joint probability function. Since many hydrological quantities can be normalized by the logarithmic transformation, the two-dimensional and normal distribution function can be applied for this purpose ; the practical treatment of normalization has been shown.

3. The design flood should be defined as the flood which has the defined density of occurrence probability and the maximum peak discharge.

4. The design peak discharge in a net of river channels or a river should be determined so as to co-ordinate the expected flood losses in each river channel, and in a river channel where the expected flood losses are larger, a reservoir for flood control should be planned.

5. The ratio between inflow and outflow in operation of the flood conrol by a reservoir should be determined so as to minimize the occurrence probability of floood losses in the downstream district.

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