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74. On the Design Wind Force of Steel Stacks

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1. Introduction

It is well known now that the vibrations of stacks are mainly caused perpendicular to the wind direction, but research on this problem had not been made until ten or twenty years ago. After the war, many all-welded tall steel stacks were constructed and some of them were partially damaged due to the vibrations induced by wind. For this reason a number of papers on this problem have been published recently and although this phenomenon has not yet been clarified in detail, we now have more data about it. In the following, we discuss the wind resistant design of welded steel stacks referring to the researches in our country and abroad.

2. The vibrational character of steel stacks induced by wind

(1) *The vibrational character*

To find out a method of wind resistant design of steel stacks, we must first study what vibrations are caused by wind. So it is important to investigate the vibrational character of steel stacks, the wind force properties, and the relation between them.

It is not difficult to measure the vibrations of actual stacks, and comparatively many experiments on the vibrations have been made.

The fundamental natural periods due to bending vibrations are shown in

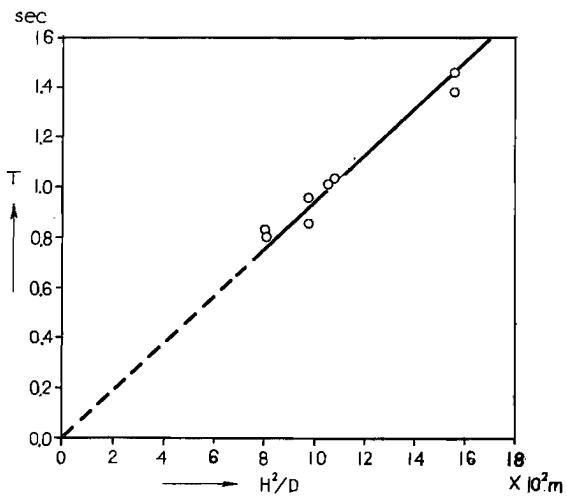

Fig. 1. Natural periods of steel stacks and H^2/D where H is the height and D is the diamter of stack.

Table 1. In the case of self supported stacks, the fundamental natural periods of many stacks are proportional to H^2/D as shown in Fig. 1, where H is the height of a stack from the ground and D, the diameter of the stack.

The natural periods of stacks on buildings are a little longer than the values in Table 1.

TABLE 1.

Stack	Height (m)	Diameter		Fundamental natural period (sec)	Logarithmic decrement
		Top (m)	Bottom (m)		
A	76	4.5	7.0	1.01	0.04
B	76	5.0	7.5	0.86	0.05-0.07
C	76	5.0	7.5	0.94	0.03-0.05
D	90	4.37	6.86	1.47	0.02-0.04
E	90	4.37	6.86	1.39	—
F	69	5.2	7.2	0.82	—
G	69	5.2	7.2	0.81	0.03-0.05
H	76	4.61	6.24	1.02	0.03-0.04

The logarithmic decrement of vibration varies over a wide range depending upon the amplitude, the lining thickness, and the condition of the foundation of a stack. Some examples of decrements are also shown in Table 1 that were obtained from the free oscillations caused by artificial forces.

The vibrational modes or deflections are the vibrational figures of cantilever with variable sections and the calculated deflection curves coincide well with the results of measurements on real stacks.

In some cases, the ovalling vibrations were observed. We have not so much information about them as on the bending vibrations but the periods obtained on some all welded tall stacks are 1.0~2.0 sec.

(2) *The wind force on stacks*

When the vibrations of stacks are caused by wind, the amplitude perpendicular to the wind direction is larger than that in the wind direction. If we are concerned with the vibration in wind direction, we must consider the variation of wind velocity and wind direction, or the buffeting by wind. But here the vibrations perpendicular to the wind direction are important, so we shall consider the wind as a uniform flow of air and the wind force on a stack, as the same as that applied to the circular cylinder in a uniform stream. In this case, it is well known that Karman vortices behind the cylinder produce alternating periodic forces on it, but this phenomenon or the formation and discharge of vortices has not yet been investigated in detail.

The periodicity of Karman vortices is most marked over the range where Reynolds number is from 40 to 1000 as may be seen by model experiments, but when the wind blows the stack, Reynolds mumber is about 10^7 or more.

Therefore in such a case, it is questionable whether periodic vortices

are shed behind the stack in natural wind or not.

It is difficult to obtain the Strouhal number on an actual stack precisely, but we can estimate it from the wind velocities and the frequencies of the stack when the violent oscillations occur.

According to many observations of this kind on actual stacks, the Strouhal number is about 0.2. In model experiments where Reynolds numbers are comparatively small as mentioned above, the Strouhal number is about 0.2, but when Reynolds number is large, the Strouhal number will become larger. Recently A. Roshko indicated the Strouhal number as 0.27 when Reynolds number was near 10^7 by wind tunnel test. We consider this difference between these values of Strouhal numbers as follows.

The Strouhal number of the cylinder moving in the stream will be smallar than that of the cylinder standing still or stationary, even if Reynolds numbers are the same in both cases. This is verified by the experiments in wind tunnel. When Reynolds number is about 10^5, the Strouhal number on a stationary cylinder is about 0.2 as mentioned above, but under the same Reynolds number the Strouhal number on a movable cylinder which is elastically restrained is 0.13~0.15.

In analogy to this fact it is not strange that the Strouhal number obtained from the state of the resonant vibrations of actual stacks was comparatively small.

Usually the Strouhal number S is defined as $S=ND/V$, where N is the frequency of vortex shedding, D the diameter of a cylinder and V the wind veloicty. But since S will be variable with Reynolds number, A. Roshko proposed that we should take the width of wake D_W instead of the diameter of the cylinder D. The width of wake behind the moving cylinder will be wider than that of the wake behind the stationary one, so the Strouhal number on the former will be reduced.

When the Reynolds number is very large, as in the case of the actual stacks, the periodic vortices in a regular manner may not be seen in the wake but the vortices will work upon the stacks at the instant when they are formed or discharged.

Therefore the vortices have an effect on the stacks even if the so-called Karman vortices are not seen. Furthermore the vortices are more likely to be shed into the wake when the stacks are moving in wind. This phenomenon will be called a self-excited motion.

The drag coefficient of a stationary cylinder in fluid is well known and the maximum lift coefficient has obtained as $C_D=0.6\sim1.1$ by several investigators. Since moving of the cylinder will reduce the Strouhal number as mentioned above, the drag and lift coefficients will also be changed. In general the drag coefficient grows as the Strouhal number becomes smaller, for widening wake. So the drag coefficient of the moving cylinder must have a larger value than that of the stationary one. However here we are considering the oscillations of stacks perpendicular to the wind direction and the lift coefficient is important for the present problem.

We have obtained some data on the lift coefficient of the moving cylinder in the stream, but we presumed it to be smallar than that of the statio-

nary cylinder from the wake behind the cylinder. K. Nakagawa indicated the similar matter. Y. G. Fung obtained the lift coefficients of the cylinders which were forced to oscillate, but their values are scattered over a wide range. At present, the value of the lift coefficient of the moving cylinder is not accurately fixed. Probably it will be varied not only with Reynolds number but also with the frequency and the amplitude of the cylinder.

(3) *The wind-induced vibration of actual stacks*

Several papers have been already published concerning the phenomena of the wind-induced vibration of steel stacks. Here only a brief description will be needed. We believe that the violent oscillation of stacks is a self-excited motion and not a forced vibration, but that it resembles closely a resonant vibration, so here we will call it a resonant vibration.

The amplitude of most steel stacks grew with wind velocity and at critical wind velocity it came up to a peak value. On the other hand the amplitude of other stacks grew uniformly with wind velocity. These stacks appear to have larger stiffness, damping or weight. In model tests, there were more than two resonant states at the critical velocities.

As has been already stated above, it is difficult to obtain the value of the Strouhal number precisely when the actual stacks are in the resonant state by wind.

Since, as is well known, the wind velocity grows with the height from ground, and most stacks in our country are tapered off, there is much difference between the wind velocities together with the diameters of a stack at the top and the bottom. Taking the values of the wind velocity and the diameter near the top, Strouhal numbers of about 0.2 were obtained for many tall stacks from the resonant states. In such cases, the frequencies of stacks coincided with the natural frequencies. The maximum amplitudes of some stacks grew up to approximately 60~100 cm and some other several stacks were partially damaged.

The ovalling vibration of stacks were also observed in some cases but they were almost unlined. Every stack in which the vibration problem was caused, was slender and E. Durand suggested that the present policy was to avoid stacks with a diameter to height ratio greater than 20.

3. The wind force for the design of steel stacks

The wind force to design the steel stack was estimated from the values of the velocity pressure of the maximum design wind and the usual drag coefficient before the vibrational problem had come into question. Since the vibration problems of stacks occured, this method was developed to take the lift coefficient in place of the drag coefficient, the resonant state of stacks considered and a magnification factor taken into account. In this method, we have to estimate the values of the lift coefficient and the magnification factor.

E. J. Stankiewicz recommeded the following formula to estimate the equivalent lateral load at resonance for the steel stacks in 1955, assuming the lift force coefficient as 0.66.

$$Pr = \frac{8.0\, m V_r^2}{10^4} \quad \text{(1b per sq ft)}$$

Here V_r is the resonant wind velocity, and m is the magnification factor. He suggested that for down-wind stacks spaced $3\frac{1}{2}$ to $5\frac{1}{2}$ diameters apart, a magnification factor of 15 for lined and 20 for unlined, and for up-wind stacks a magnification factor of 10 for lined and 15 for unlined, should be required.

However the value of the lift coefficient assumed here seems to be too large from the experimental facts afore-mentioned. If the stack is supposed to be a one mass system, the magnification factor is $m=\pi/\delta$, where δ is the logarithmic decrement. Then the magnification factor can be calculated from the value in Table 1. The logarithmic decrement obtained in this way are much greater than the values indicated by Stankiewicz.

We supposed the lift coefficient to be 0.1~0.2 and the value of the magnification factor that was obtained from the damping factor. Consequently as a result the value of $m\ C_L$ which we have obtained, coincides approximately with the value which Stankiewicz suggested. Therefore it is convenient to consider m and C_L together as mC_L. We need not fix the value of m and C_L separately to design steel stacks.

Now it is difficult to define the value mC_L from model tests in a wind tunnel due to the difference between Reynolds numbers, and we have to fix that value from our experience on actual stacks.

Many tall steel stacks that were made recently in our country were designed in the way which Stankiewicz had suggested, or by a similar method, and some of these stacks that caused vibrational problems were slender. If we take the diameter D at the 2/3 point of the stack from the ground as the equivalent diameter, we can show the slenderness of the stack by the H/D. The stacks which have the value $H/D=15\sim16$ or less, have been safe up to date, so the value mC_L which Stankiewicz had indicated must be a proper one for these stacks. What value should be taken as mC_L for more slender stacks is a complicated problem, but we propose here to take the simplest way, that mC_L should be proportional to H/D according to the following consideration.

If we assume the stack as a cantilever with a uniform cross-section, the displacement y at the top is given by

$$y = \frac{PDH^4}{8EI}, \qquad P = \frac{1}{2}\, mC_L V_r^2,$$

and the maximum stress σ of the section at the bottom due to the bending moment M is

$$\sigma = \frac{M}{Z} = \frac{PDH^2}{2Z},$$

where E is the Young's modulus, I is the moment of inertia of the section, and Z is the section modulus.

As a rule the thickness of the steel plate of the stack is far smaller than

the diameter. Therefore we have

$$Z=\frac{\pi}{4}D^2t, \qquad I=\frac{\pi}{8}D^3t,$$

and

$$y=\frac{\frac{1}{2}mC_LV_r^2H^4}{\pi ED^2t}, \quad \sigma=\frac{mC_LV_r^2H^2}{\pi Dt}.$$

The maximum stress should be the allowable stress, and approximately the same value for every stack, so we assume it as constant. Then

$$y=\frac{H^2\sigma}{2ED} \quad \text{or} \quad y\propto\frac{H^2}{D}$$

If we take the wind force P for design to be independent of the dimensions of stacks, the displacement of the top of a stack is proportional to H^2/D. To fix the displacement for all stacks, the wind force P should be proportional to H^2/D. To make the displacement proportional to height of the stack H, P is required to be proportional to H/D.

In this consideration, we have assumed that the lift coefficient and the magnification factor are not influenced by the figure or the dimensions of the stack.

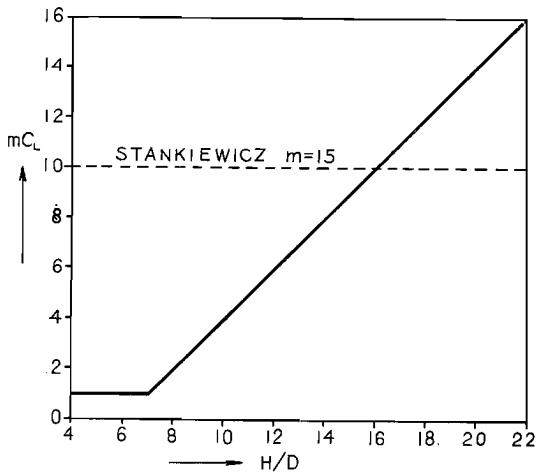

Fig. 2. Resonant wind force coefficient and H/D.

Actually, the problem may not be so simple as above, but we believe that it is a conventional way at present. In Fig. 2, the value of mC_L is shown that is proportional to H/D and $mC_L=10$ when $H/D=16$.

The value $mC_L=10$ is equivalent to $m=15$ due to the value which Stankiewicz has shown. The minimum value of mC_L is 1, because the maximum value of the lateral force coefficient of the stationary circular cylinder is about 1. So in the range where H/D is less than 7, the value mC_L is con-

stant and equal to unity. The value mC_L shown in Fig. 2 should be applied to independent self-supported and all welded large stacks.

Different values of mC_L must be required for stacks which have different figures, linings, or thicknesses of steel plates, from the one we have discussed. The Strouhal number when resonant vibrations of actual stacks occured, was about 0.2 as mentioned above, but for the design the number 0.18 will be required for safety.

In model tests, it is assured that there were more than 2 resonant wind velocities, but it will be sufficient to take just one Strouhal number, because one resonant wind velocity alone to each actual stack has been found for the bending vibration.

4. Conclusions

We have considered the lateral forces resulting from vortex shedding to design independent self-supported steel stacks. The problems described in this paper do not include the buffeting, the ovalling vibration and the wind force reduction by some devices. Recently, new styled stacks, or several stacks combined by frames, and stacks stiffened by steel towers, have been erected. The problems on these stacks are more complicated and further research is required for the determination of wind forces.