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79. Statistical Properties of Earthquake Accelerograms and Equivalent Earthquake Excitation Pattern

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Abstract

In order to analyze the earthquake responses of structures, it is first very important to predict reasonably an earthquake excitation pattern referring to the model of a structural system and the measures of aseismic safety. It is not possible, however, to know the excitation properties of future earthquakes so assuming that future earthquakes will have approximately the same properties as past earthquakes, it is necessary to analyze the properties of the accelerograms of past earthquakes. Most of such accelerograms are likely to be random time functions. And their statistical quantities are, in details, different from each other depending upon the individual past earthquake, the recording place of accelerograms and other conditions.

In this paper, the auto-correlation functions and amplitude probability density distribution functions are estimated for a number of past strong earthquake accelerograms. It is also shown that an earthquake accelerogram has a spectral density with a few peaks and non-Gaussian probability density distribution. On the other hand, the statistical model for the equivalent earthquake excitation pattern is presented by considering the common properties based on the above statistical results. Although it is difficult for the earthquake response analyses to give the dimensional quantities–intensity, frequency characteristics and so on–of earthquake excitations relating to individual soil conditions, the dimensionless parameters which define the statistical model of the equivalent earthquake excitation pattern can be determined in the case of the assumption that the influence of the ground-structure coupling being small. Then it is found that the spectral density of a simplified equivalent earthquake excitation pattern may consist of the band limited white noise spectrum and the delta functions corresponding to a noise component and periodic components.

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Nomenclature

ν, T, τ = sampled nondimensional time, time and nodimensional time.

w, Ω, ω = angular frequency with respect to ν, T, τ.

T_d, ΔT N = sampled duration, sampling interval of time and total sampling number.

$\lambda = \sqrt{\bar{K}/\bar{M}}$ = coefficient of the time transformation.

$\{F^j\}$ = normalized sampling data.

$A(\nu)$, $a(T)$, $\alpha(\tau)$ = wave shape functions.

$\alpha_0(\tau)$ = shifted wave shape function defined by eq. (7).

$\alpha_s(\tau)$ = stationarized wave shape function.

τ_a = nondimensional duration of the wave shape function.
$(\bar{\bar{R}}\nu)$, $\bar{R}(T)$, $R(\tau)$ = auto-correlation functions.
$R_s(\tau)$, $R_{ro}(\tau)$, ${}_nR_{ro}(\tau)$, $R_{fo}(\tau)$ = auto-correlation functions defined by eqs. (9), (10), (12) and (17).
$\bar{\bar{S}}(w)$, $\bar{S}(\Omega)$, $S(\omega)$ = spectral densities.
$S_s(\omega)$, $S_{r0}(\omega)$, $S_{f0}(\omega)$ = spectral densities defined by eqs. (19), (20), (21).
$q=\omega/\omega_0$ = total number of cycles in the duration.
$\omega_0=2\pi/\tau_a$ = reference angular frequency.
Δf, n = sampling interval of amplitude and number of division.
m_j = number of the sampled data included in the jth interval of amplitude.
$d\nu_i$, dT_i, $d\tau_i$ = time intervals contained in the infinitesimal interval df of amplitude.
$\bar{\bar{W}}(f)=\bar{W}(f)=W(f)$ = amplitude probability density distribution function of the wave shape function.
$\alpha_e(\tau)$ = dimensionless wave shape function of the equivalent earthquake acceleration.
$\alpha_{se}(\tau)$ = dimensionless wave shape function of the equivalent stationary earthquake acceleration.
$R_{se}(\tau)$, $S_{se}(\omega)$ = auto-correlation function and spectral density of $a_{se}(\tau)$.
$a_{se}(T)$ = wave shape function of the equivalent stationary earthquake acceleration.
c_0 = level of the band limited white spectrum of $a_{se}(T)$.
c_1 = power of the predominant periodic component of the spectral density of $a_{se}(T)$.
$\Omega_u=2\pi/T_l$, $\Omega_l=2\pi/T_u$ = upper and lower angular frequency limits of the band limited white spectrum of $a_{se}(T)$.
Ω_1 = angular frequency of the predominant periodic component of $a_{se}(T)$.
ω_u, ω_l = upper and lower nondimensional angular frequency limits of the band limited white spectrum of $a_{se}(\tau)$.
ω_1 = nondimensional angular frequency of the predominant periodic cmponent of $a_{se}(\tau)$.
r_p = ratio of the power of the predominant periodic component to that of noise component.
r_{fl} = ratio of the lower angular frequency limit of the band limited white spectrum to the upper angular frequency limit.
r_{f1} = ratio of the angular frequency of the predominant periodic component to the upper angular frequency limit.
p = nondimensional intensity parameter of the dimensionless earthquake acceleration.
ϕ^{-1} = nondimensional frequency parameter of the dimensionless earthquake acceleration.
a = maximum amplitude of the dimensionless earthquake acceleration.
${}_1\Omega=2\pi/{}_1T$ = fundamental natural frequency of a structural system.
${}_1\omega=2\pi/{}_1\tau$ = nondimensional fundamental natural frequency.
A = maximum amplitude of earthquake acceleration.
S_0 = level of the band limited white spectrum of earthquake acceleration.
$\bar{M}$, $\bar{K}$, $\bar{\Delta}$ = reference values of mass, rigidity and deformation of a structural

system.

a_v, v_v, d_v = variances of the dimensionless wave shape function of the equivalent stationary earthquake acceleration, velocity and displacement.

a_{v0}, v_{v0}, d_{v0} = variances of the equivalent stationary earthquake excitation defined by a set of parameters r_p, r_{fl}, r_{f1}, $c_0\bar{\lambda}=1$, $\omega_u=1$.

A_v, V_v, D_v = variances of the earthquake excitation.

c_a, c_v, c_d = dimensionless constants of the frequency characteristics of the level of the band limited white spectrum represented by constant standard deviation of acceleration, velocity and displacement.

C_a, C_v, C_d = constants of the frequency characteristics of the level of the band limited white spectrum of $Aa_{sc}(T)$.

$W'(0)$ = amplitude probability density at zero calculating from $R_{r0}(0)$ on the assumption of the normal distribution.

$R_{r0}'(0)$ = variance calculating from $W(0)$ on the assumption of the normal distribution.

s_1 = ratio of $W'(0)$ to $W(0)$.

s_2 = ratio of $R'_{r0}(0)$ to $R_{r0}(0)$.

s_3 = ratio of the maximum amplitude to the standard deviation.

$s(x)$ = step function, $s(x)=0$ for $x<0$, $s(0)=1/2$, $s(x)=1$ for $x>0$.

$\delta(x)$ = delta function, $\delta(x)=0$ for $x\neq 0$, $\int_{-\infty}^{\infty}\delta(x)dx=1$.

$E(X(Y))$ = mean of X with respect to Y.

$X(X)\supset Y(y)$ = correspondence between Fourier trasform pair $X(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\times Y(y)e^{jxy}dy$, $Y(y)=\int_{-\infty}^{\infty}X(x)e^{-jyx}dx$, $j=\sqrt{-1}$.

$X*Y$ = convolution of X and $Y=\int_{-\infty}^{\infty}X(x)Y(z-x)dx=\int_{-\infty}^{\infty}Y(y)X(z-y)dy$.

1. Introduction

For earthquake response analyses to establish the principle of aseismic design of structures, an earthquake excitation pattern should be given together with a model of structural system and the measures of aseismic safety. To predict the properties of earthquake excitations which will occur in the future, we have to analyze the accelerograms obtained in the past. Most of the accelerograms seem to be random time functions, and their statistical quantities are different in details depending upon various conditions.

In this paper, the statistical properties of several earthquake accelerograms recorded in U.S.A. are estimated as the problem of determining the typical input pattern for the earthquake response analysis of structures[1)~5)]. Auto-correlation function and amplitude probability density distribution function are calculated for each accelerogram, and by inducing common properties in these results, a statistical model for the equivalent earthquake excitation pattern is presented.

2. Wave shape function of accelerogram

The wave shape function is obtained by normalizing the original accelerogram so that the mean is zero and the maximum value is unity. At first, the wave shape function $A(\nu)$ is defined as the following piecewise linear type function by using the time series $\{F^j\}$, for $j=0, 1, \cdots\cdots, N$ sampled for the interval $\Delta T = T_d/N$ from the original accelerogram having the time duration T_d.

$$A(\nu)=\sum_{j=1}^{N}\{F^{j-1}+(F^j-F^{j-1})(\nu-(j-1))\}\{s(\nu-(j-1))-s(\nu-j)\} \quad \cdots\cdots\cdots(1)$$

$$E(A(\nu))=\frac{1}{N}\int_0^N A(\nu)d\nu=E(\{F^j\})=0, \quad |A(\nu)|_{max}=|F^j|_{max}=1 \quad \cdots\cdots\cdots\cdots(2)$$

$$\{F^j\}=\frac{\{\bar{F}^j-E(\{\bar{F}^j\})\}}{|\bar{F}^j-E(\{\bar{F}^j\})|_{max}}, \quad E(\{\bar{F}^j\})=\frac{1}{N}\left\{\frac{\bar{F}^0+\bar{F}^N}{2}+\sum_{k=1}^{N-1}\bar{F}^k\right\} \quad \cdots\cdots\cdots\cdots(3)$$

The wave shape function $a(T)$ with respect to the time T, and the nondimensional wave shape function $\alpha(\tau)$ can be obtained easily from $A(\nu)$, if we perform the following transformation related to the independent variables.

$$\alpha(\tau)=a(T)=A(\nu) \quad \text{for} \quad \tau=\bar{\lambda}T=\bar{\lambda}\Delta T\nu \quad \cdots\cdots\cdots\cdots(4)$$

$$|\alpha(\tau)|_{max}=|a(T)|_{max}=1, \quad E(\alpha(\tau))=E(a(T))=0 \quad \cdots\cdots\cdots\cdots(5)$$

where τ, T and ν are the nondimensional time, time and the sampled nondimensional time variable respectively. $[0, \tau_d]$, $[0, T_d]$ and $[0, N]$ are the corresponding intervals. $\bar{\lambda}$ is the coefficient of the time transformation. $s(\nu)$ is a step function defined as follows:

$$s(\nu)=0 \text{ for } \nu<0, \quad s(\nu)=\frac{1}{2} \text{ for } \nu=0, \quad s(\nu)=1 \text{ for } \nu>0 \quad \cdots\cdots\cdots\cdots(6)$$

The stationarized wave shape function $\alpha_s(\tau)$ is now defined by using the above obtained wave shape function $\alpha(\tau)$ as follows:

$$\alpha_s(\tau)=\sum_{\mu=-\infty}^{\infty}\alpha_0(\tau-\mu\tau_d), \quad \alpha_0(\tau)=\alpha\left(\tau+\frac{\tau_d}{2}\right) \quad \cdots\cdots\cdots\cdots(7)$$

were

$$\alpha_0(\tau)\not\equiv 0 \text{ for } |\tau|\leqq\tau_d/2, \quad \alpha_0(\tau)\equiv 0 \text{ for } |\tau|>\tau_d/2$$

and

$$|\alpha_s(\tau)|_{max}=|\alpha_0(\tau)|_{max}=|\alpha(\tau)|_{max}=1 \quad \cdots\cdots\cdots\cdots(8)$$

3. Statistical quantities of wave shape function

The statistical quantities are defined here only with respect to the abovementioned wave shape function of accelerogram, assuming that it is a part of the ergodic stationary time functions. The quantities determining the statistical properties of the wave shape function for the finite interval can be reduced to the auto-correlation function or the spectral density and the amplitude probability density distribution function of the stationarized wave shape function.

3.1. *Auto-correlation function*

The auto-correlation function of $\alpha_s(\tau)$ is defined as follows:

$$R_s(\tau)=\lim_{\lambda\to\infty}\frac{1}{2\lambda}\int_{-\lambda}^{\lambda}\alpha_s(t)\alpha_s(t+\tau)dt=\sum_{\mu=-\infty}^{\infty}R_{T0}(\tau-\mu\tau_d) \quad \cdots\cdots\cdots\cdots(9)$$

$$R_{r0}(\tau)=\frac{1}{\tau_d}\int_{-\tau_d/2}^{\tau_d/2}\alpha_0(t)\sum_{\mu=-1}^{1}\alpha_0(t+\tau-\mu\tau_d)dt\left\{s\left(\tau+\frac{\tau_d}{2}\right)-s\left(\tau-\frac{\tau_d}{2}\right)\right\} \quad \cdots\cdots(10)$$

$$R_{r0}(\tau)={}_hR_{r0}(\tau)+{}_hR_{r0}(-\tau) \quad \cdots\cdots\cdots\cdots(11)$$

$${}_hR_{r0}(\tau)=R_{r0}(\tau)\left\{s(\tau)-s\left(\tau-\frac{\tau_d}{2}\right)\right\} \quad \cdots\cdots\cdots\cdots(12)$$

Among the auto-correlation functions corresponding to the variables τ, T and ν, the following relation exists:

$${}_hR_{r0}(\tau)={}_h\bar{R}_{r0}(T)={}_h\bar{\bar{R}}_{r0}(\nu) \quad \text{for} \quad \bar{\lambda}T=\bar{\lambda}\Delta T\nu \quad \cdots\cdots\cdots\cdots(13)$$

Herein, ${}_h\bar{\bar{R}}_{r0}(\nu)$ can be calculated from the following formula:

$${}_h\bar{\bar{R}}_{r0}(\nu)=\frac{1}{N}\sum_{\kappa=1}^{N}A_\kappa\{A_{\underline{\kappa+\nu}}+A_{\underline{\kappa+\nu-N}}\}\left\{s(\nu)-s\left(\nu-\frac{N}{2}\right)\right\} \quad \cdots\cdots\cdots\cdots(14)$$

where

$$A_\kappa=\frac{1}{2}\{A(\kappa-1)+A(\kappa)\} \quad \text{for} \quad \kappa=1,\ 2,\ \cdots\cdots,\ N \quad \cdots\cdots\cdots\cdots(15)$$

$$A_\kappa\neq 0 \text{ for } 0<\kappa\leqq N, \quad A_\kappa=0 \text{ for } \kappa\leqq 0 \text{ and } \kappa>N,$$

$\underline{x}=\mu$, means such an integer as $x-1/2<\mu\leqq x+1/2$. Introducing the auto-correlation function $R_{f0}(\tau)$ corresponding to $\alpha_0(\tau)$, $R_{r0}(\tau)$ is expressed by the following formula

$$R_{r0}(\tau)=\sum_{\mu=-1}^{1}R_{f0}(\tau-\mu\tau_d)\left\{s\left(\tau+\frac{\tau_d}{2}\right)-s\left(\tau-\frac{\tau_d}{2}\right)\right\} \quad \cdots\cdots\cdots\cdots(16)$$

$$R_{f0}(\tau)=\frac{1}{\tau_d}\int_{-\tau_d/2}^{\tau_d/2}\alpha_0(t)\alpha_0(t+\tau)dt \quad \cdots\cdots\cdots\cdots(17)$$

$$R_{f0}(\tau)\neq 0 \text{ for } |\tau|<\tau_d \text{ and } R_{f0}(\tau)\equiv 0 \text{ for } |\tau|\geqq\tau_d.$$

Particularly, the following relation is valid among the mean squares of the various wave shape functions.

$$R_s(0)=R_{r0}(0)=2{}_hR_{r0}(0)=R_{f0}(0) \quad \cdots\cdots\cdots\cdots(18)$$

3.2. *Spectral density*

The spectral densities of $\alpha_s(\tau)$ and $\alpha_0(\tau)$ are defined as the Fourier transforms of the corresponding auto-correlation functions.

$$R_s(\tau)\supset S_s(\omega)=S_{r0}(\omega)(1+2\sum_{\mu=1}^{\infty}\cos\mu\tau_d\omega)=S_{f0}(\omega)(1+2\sum_{\mu=1}^{\infty}\cos\mu\tau_d\omega) \quad \cdots\cdots(19)$$

$$R_{r0}(\tau)\supset S_{r0}(\omega)=2\int_0^{\tau_d/2}R_{r0}(\tau)\cos\omega\tau d\tau \quad \cdots\cdots\cdots\cdots(20)$$

$$R_{f0}(\tau)\supset S_{f0}(\omega)=2\int_2^{\tau_d}R_{f0}(\tau)\cos\omega\tau d\tau=\frac{1}{\tau_d}|F_0(j\omega)|^2 \quad \cdots\cdots\cdots\cdots(21)$$

where

$$\alpha_0(\tau)\supset F_0(j\omega)=\int_{-\tau_d/2}^{\tau_d/2}\alpha_0(\tau)e^{-j\omega\tau}d\tau \quad \cdots\cdots\cdots\cdots(22)$$

Among the angular frequencies ω, Ω and w corresponding to variables τ, T

and ν respectively, the following relation is valid,

$$\omega=\Omega/\bar{\lambda}=w/(\bar{\lambda}\Delta T) \qquad \cdots\cdots\cdots\cdots(23)$$

so that, the spectral densities are related to each other by the following expression.

$$S_p(\omega)=\bar{\lambda}\bar{S}_p(\Omega)=\bar{\lambda}\Delta T\bar{\bar{S}}_p(w) \quad \text{for} \quad p=s,\ ro,\ fo \qquad \cdots\cdots\cdots\cdots(24)$$

The spectral density $\bar{\bar{S}}_{r0}(w)$ with respect to ν is given by the following formula.

$$\bar{\bar{R}}_{r0}(\nu)\supset\bar{\bar{S}}_{r0}(w)=2\int_0^{N/2}{}_h\bar{\bar{R}}_{r0}(\nu)\cos w\nu d\nu \qquad \cdots\cdots\cdots\cdots(25)$$

Therefore, $\bar{\bar{S}}_{r0}(w)$ can be calculated by the following formulae corresponding to the total number N of the sampled data.

$$\bar{\bar{S}}_{r0}(w)=2\sum_{\mu=0}^{N/2-1}{}_h\bar{\bar{R}}_{r0}(\mu)\frac{\sin\frac{w}{2}\cos\mu w}{\frac{w}{2}} \qquad \cdots\cdots\cdots\cdots(26)$$

$$+2{}_h\bar{\bar{R}}_{r0}\left(\frac{N}{2}\right)\frac{\sin\frac{w}{4}\cos w\left(\frac{N}{2}-\frac{1}{4}\right)}{\frac{w}{4}} \qquad \text{for } N \text{ is even,}$$

and

$$\bar{\bar{S}}_{r0}(w)=2\sum_{\mu=0}^{(N-1)/2}{}_h\bar{\bar{R}}_{r0}(\mu)\frac{\sin\frac{w}{2}\cos\mu w}{\frac{w}{2}} \qquad \text{for } N \text{ is odd.} \cdots\cdots\cdots(27)$$

Instead of w defined in the interval $(-\infty,\ \infty)$, the total number of cycles in the duration,

$$q=\frac{w\tau_d}{2\pi}=\frac{\Omega T_d}{2\pi}=\frac{wN}{2\pi} \qquad \cdots\cdots\cdots\cdots(28)$$

may be used in the same interval $(-\infty,\ \infty)$. It can be shown that the following relations exist between $S_{r0}(\omega)$ and $S_{f0}(\omega)$.

$$S_{r0}(\omega)=\frac{1}{\pi}S_{f0}(\omega)(1+2\cos\tau_d\omega)*\frac{1}{\omega}\sin\frac{\tau_d\omega}{2} \qquad \cdots\cdots\cdots\cdots(29)$$

$$\sum_{\mu=-\infty}^{\infty}\{S_{r0}(\omega)-S_{f0}(\omega)\}^{-j\mu\tau_d\omega}=0\subset\sum_{\mu=-\infty}^{\infty}\{R_{r0}(\tau-\mu\tau_d)-R_{f0}(\tau-\mu\tau_d)\}=0 \quad \cdots\cdots(30)$$

$$S_{r0}(\mu\omega_0)=S_{f0}(\mu\omega_0) \quad \text{for} \quad \mu=0,\ \pm1,\ \pm2,\ \cdots\cdots,\ \omega_0=\frac{2\pi}{\tau_d} \qquad \cdots\cdots\cdots\cdots(31)$$

and

$$\left.\begin{aligned}S_{r0}(\omega)&=\frac{\omega_0}{\pi}\sum_{\mu=-\infty}^{\infty}S_{f0}(\mu\omega_0)\frac{1}{\omega-\mu\omega_0}\sin\frac{\tau_d(\omega-\mu\omega_0)}{2}\\&=\frac{\omega_0}{\pi}\sum_{\mu=-\infty}^{\infty}S_{r0}(\mu\omega_0)\frac{1}{\omega-\mu\omega_0}\sin\frac{\tau_d(\omega-\mu\omega_0)}{2}\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(32)$$

Since $S_s(\omega)$, $S_{r0}(\omega)$ and $S_{f0}(\omega)$ are even functions and the wave shape functions $\alpha_s(\tau)$ and $\alpha_0(\tau)$ have the zero mean values, the following relation is valid:

$$S_s(0)=S_{r0}(0)=S_{f0}(0)=0 \qquad \cdots\cdots\cdots\cdots(33)$$

and also, the following relation exists in regard to the total power.

$$\frac{1}{\pi}\int_0^\infty S_s(\omega)d\omega=\frac{1}{\pi}\int_0^\infty S_{r0}(\omega)d\omega=\frac{1}{\pi}\int_0^\infty S_{f0}(\omega)d\omega=R_{r0}(0) \qquad \cdots\cdots\cdots\cdots(34)$$

3.3. *Amplitude probability density distribution function*

Assuming that the wave shape function $\alpha_s(\tau)$ is ergodic and stationary, the amplitude probability density distribution function is defined by the following equation as the time ratio:

$$W(f)df=\frac{\sum_{i=1}^{m_f}d\tau_i}{\tau_a}=\frac{\sum_{i=1}^{m_f}dT_i}{T_a}=\frac{\sum_{i=1}^{m_f}d\nu_i}{N} \qquad \cdots\cdots\cdots\cdots(35)$$

where $\{d\tau_i\}$, $\{dT_i\}$ and $\{d\nu_i\}$ for $i=1, 2, \cdots\cdots, m_f$ relating to the variables τ, T and ν, respectively, are the successively arranged infinitesimal time intervals contained in the infinitesimal interval $(f, f+df)$ in the common coordinate for the normalized amplitude of $\alpha(\tau)$, $a(T)$ and $A(\nu)$. $W(f)$ is an invariable quantity with respect to time transformation and has the integral over the defined interval $[-1, 1]$ contributed to unity.

$$W(f)=\bar{W}(f)=\bar{\bar{W}}(f), \quad \int_{-1}^{1}W(f)df=1 \qquad \cdots\cdots\cdots\cdots(36)$$

This quantity is expressed by the following formula by using the previously defined time series $\{A_\kappa\}$ for $\kappa=1, 2, \cdots\cdots, N$:

$$W(f)=\sum_{j=-n}^{n}W^j\left\{s\left(f-\left(j-\frac{1}{2}\right)\Delta f\right)-s\left(f-\left(j+\frac{1}{2}\right)\Delta f\right)\right\} \qquad \cdots\cdots\cdots\cdots(37)$$

where

$$W^j=m_j/(N\Delta f)$$

and m_j for $j=0, \pm1, \cdots\cdots, \pm n$ is the number of the sampled data from $\{A_\kappa\}$ included in the jth interval

$$\left(j-\frac{1}{2}\right)\Delta f\leqq f<\left(j+\frac{1}{2}\right)\Delta f \quad \text{for} \quad j=0, \pm1, \cdots\cdots, \pm n \qquad \cdots\cdots\cdots\cdots(38)$$

$$\Delta f=\frac{1}{n}$$

where Δf is the equally devided sampling interval of the amplitude and $2n+1$ is the total number of division.

3.4. *Numerical results*

Accelerograms of four strong-motion earthquake records furnished by the U. S. Coast and Geodetic Survey are analysed statistically[1]. The sampling intervals are selected as $\Delta T=0.02$ sec and $\Delta f=0.05$, and the auto-correlation functions and the amplitude probability density distribution functions for the

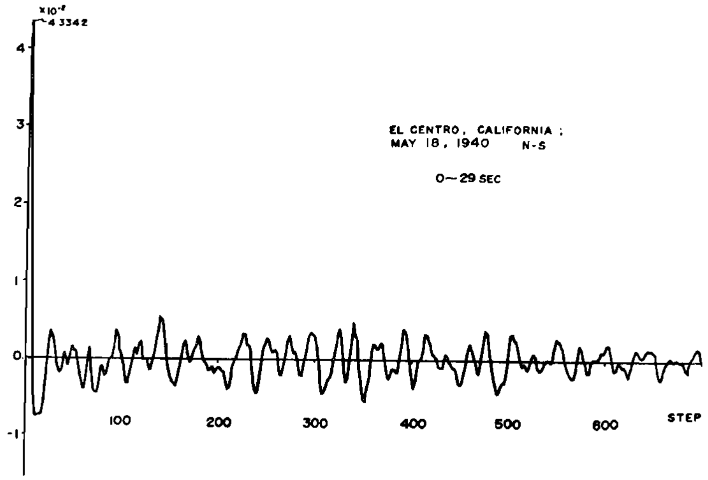

Fig. 1. Auto-correlation function of normalized accelerogram.

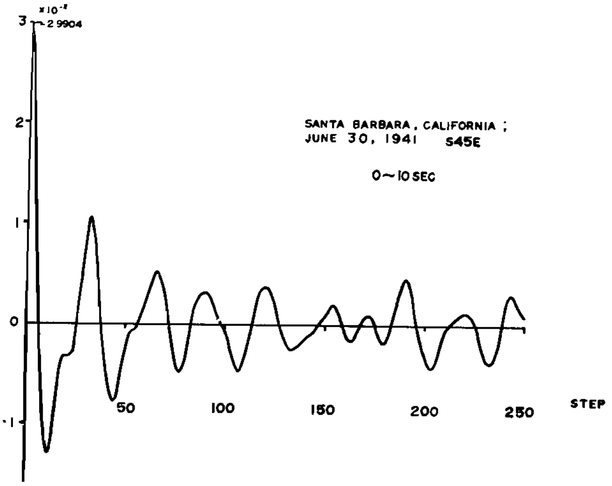

Fig. 2. Auto-correlation function of normalized accelerogram.

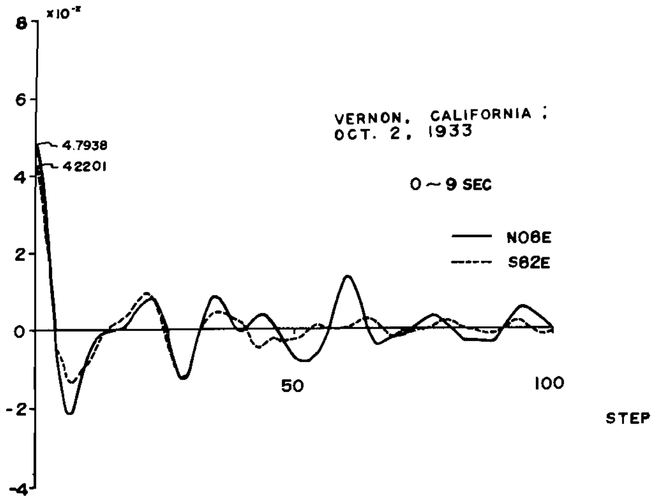

Fig. 3. Auto-correlation function of normalized accelerogram.

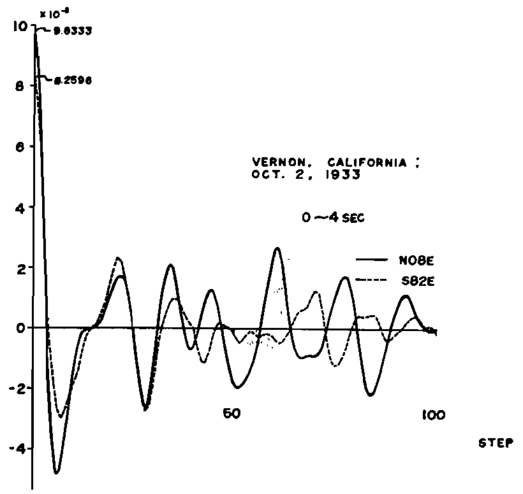

Fig. 4. Auto-correlation function of normalized accelerogram.

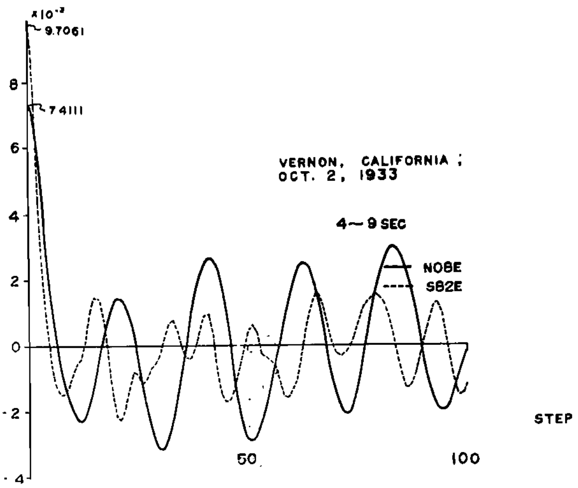

Fig. 5. Auto-correlation function of normalized accelerogram.

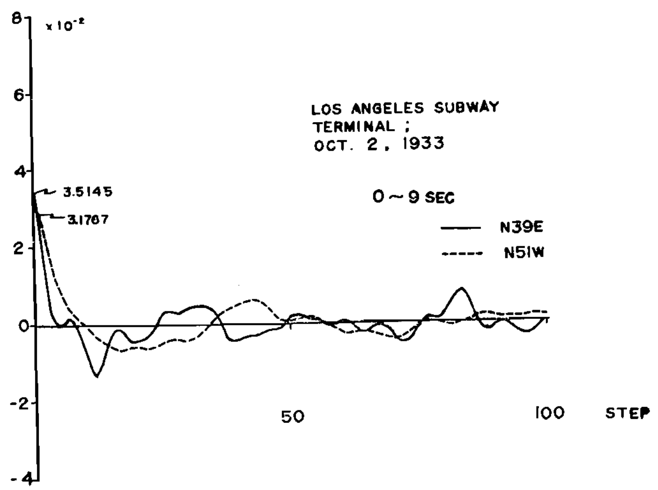

Fig. 6. Auto-correlation function of normalized accelerogram.

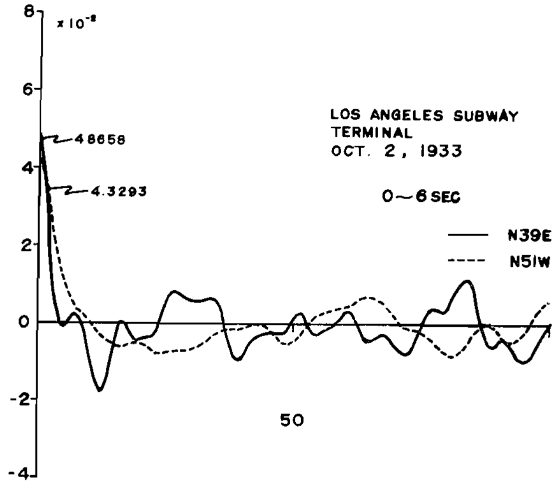

Fig. 7. Auto-correlation function of normalized accelerogram.

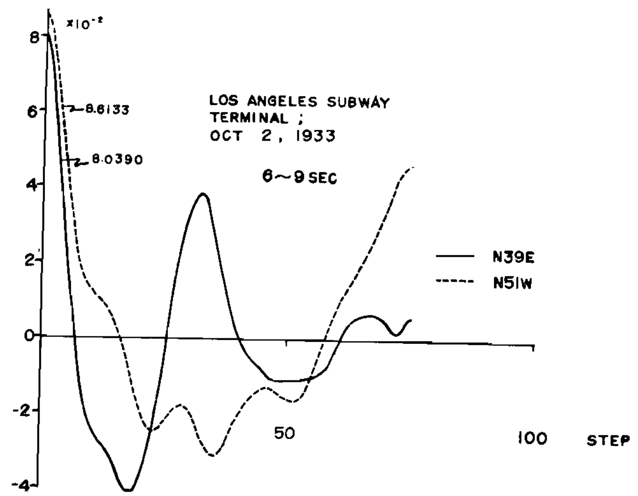

Fig. 8. Auto-correlation function of normalized accelerogram.

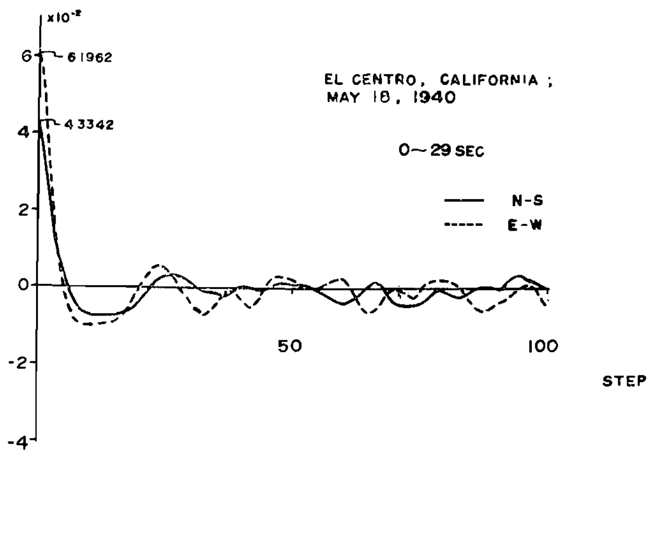

Fig. 9. Auto-correlation function of normalized accelerogram.

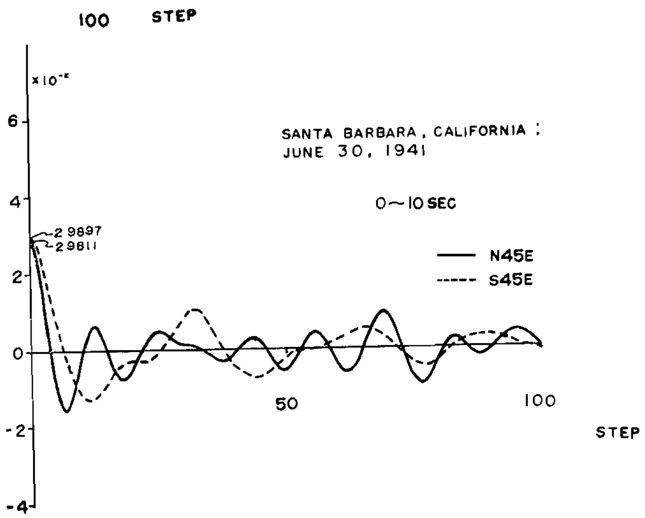

Fig. 10. Auto-correlation function of normalized accelerogram.

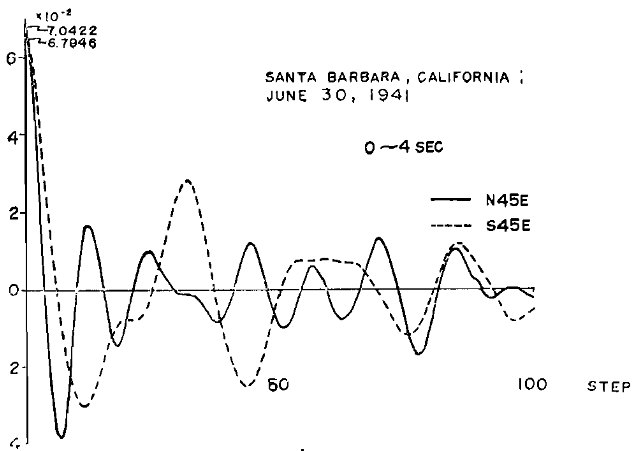

Fig. 11. Auto-correlation function of normalized accelerogram.

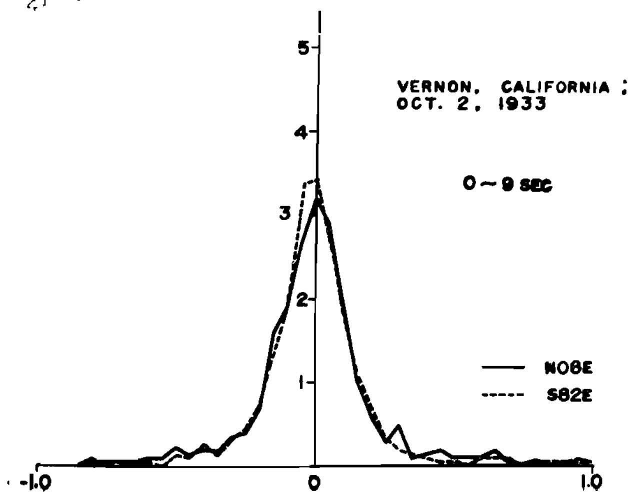

Fig. 12. Amplitude probability density distribution function of normalized accelerogram.

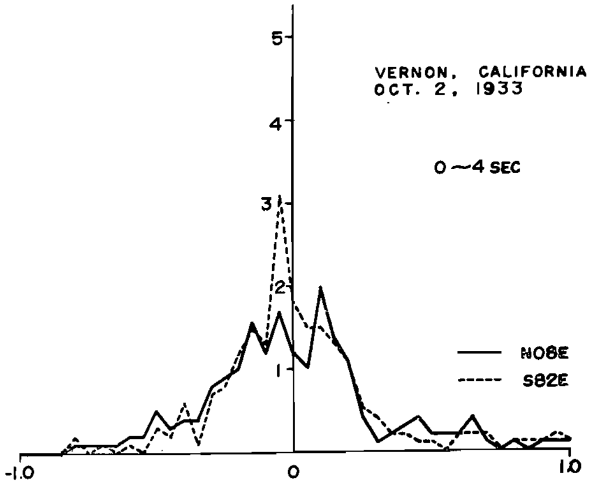

Fig. 13. Amplitude probability density distribution function of normalized accelerogram.

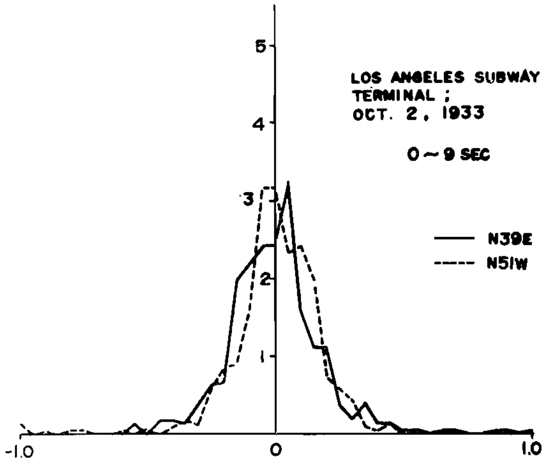

Fig. 14. Amplitude probability density distribution function of normalized accelerogram.

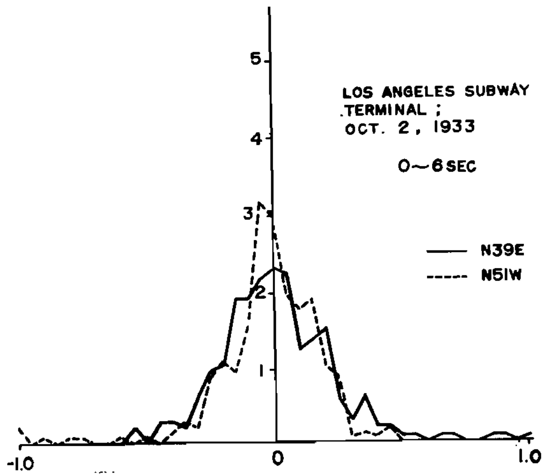

Fig. 15. Amplitude probability density distribution function of normalized accelerogram.

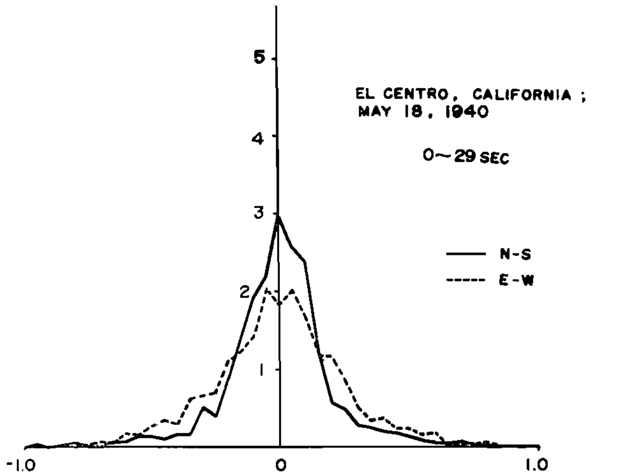

Fig. 16. Amplitude probability density distribution function of normalized accelerogram.

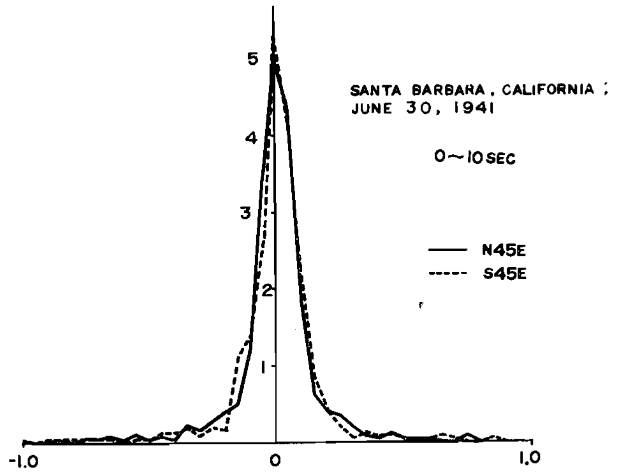

Fig. 17. Amplitude probability density distribution function of normalized accelerogram.

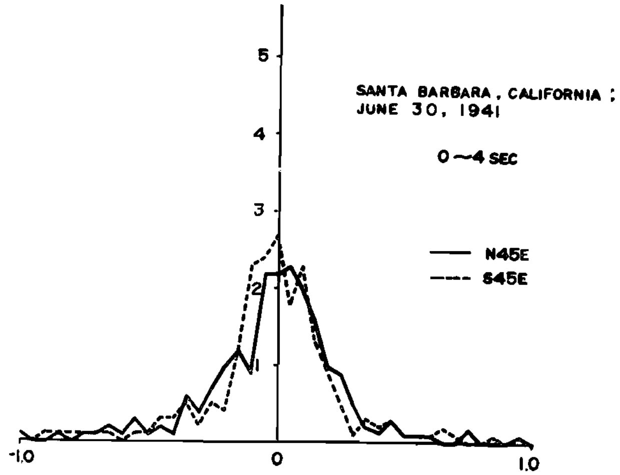

Fig. 18. Amplitude probability density distribution function of normalized accelerogram.

wave shape functions are computed using the Kyoto University Digital Computer—KDC-I—and, the results are shown in Figs. 1 to 11 and in Figs. 12 to 18, respectively[4),5)]. In Figs. 1 to 11, the abscissa denotes $\nu = T/\Delta T = \tau/(\bar{\lambda}\Delta T)$, the coordinate does ${}_h\bar{\bar{R}}_{r0}(\nu) = {}_h\bar{\bar{R}}_{r0}(T) = {}_h\bar{R}_{r0}(\tau)$ for $\tau = \bar{\lambda}T = \bar{\lambda}\Delta T\nu$, and the number inscribed are the values $2{}_h\bar{\bar{R}}_{r0}(0) = {}_h\bar{\bar{R}}_{r0}(0+0) = \bar{\bar{R}}_{r0}(0)$. In Figs. 12 to 18, the abscissa and the coordinate represent the amplitude and the probability density, respectively.

4. Equivalent earthquake excitation pattern

Equivalent earthquake excitation pattern for the earthquake response analysis can be inferred from the above obtained statistical quantities of the wave shape function by the following procedure. At first, stationarized quantities $R_{r0}(\tau)$ and $S_{r0}(\omega)$ are approximated as the sum of known statistical quantities of typical stationary random processes.

$$\left.\begin{aligned} R_{r0}(\tau) &\cong \sum_{\lambda=0}^{m} R_{r0\lambda}(\tau)\left\{s\left(\tau+\frac{\tau_d}{2}\right)-s\left(\tau-\frac{\tau_d}{2}\right)\right\} \\ S_{r0}(\omega) &\cong \frac{1}{\pi}\sum_{\lambda=0}^{m} S_{r0\lambda}(\omega) * \frac{1}{\omega}\sin\frac{\tau_d\omega}{2} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(39)$$

where

$$R_{r0}(\tau) \supset S_{r0}(\omega),\ R_{r0\lambda}(\tau) \supset S_{r0\lambda}(\omega) \quad \text{for} \quad \lambda = 0, 1, \cdots\cdots, m$$

Although the wave shape function of the accelerogram is assumed to be represented by the finite part of a stationary random function $\alpha_{se}(\tau)$, the following approximate relation may be valid if the duration τ_d is long enough to compare with τ.

$$\left.\begin{aligned} R_{r0}(\tau) &\cong R_{se}(\tau)\left\{s\left(\tau+\frac{\tau_d}{2}\right)-s\left(\tau-\frac{\tau_d}{2}\right)\right\} \\ S_{r0}(\omega) &\cong \frac{1}{\pi} S_{se}(\omega) * \frac{1}{\omega}\sin\frac{\tau_d\omega}{2} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(40)$$

where

$$R_{se}(\tau) \supset S_{se}(\omega)$$

From the above two approximate relations, we have the auto-correlation function and spectral density of stationarized equivalent earthquake excitation pattern.

$$R_{se}(\tau) = \sum_{\lambda=0}^{m} R_{r0\lambda}(\tau), \quad S_{se}(\omega) = \sum_{\lambda=0}^{m} S_{r0\lambda}(\omega) \qquad \cdots\cdots\cdots\cdots(41)$$

Therefore, the equivalent earthquake acceleration pattern can be obtained by cutting off the finite interval τ_d from the equivalent stationary random function $\alpha_{se}(\tau)$ having the above obtained statistical quantities.

$$\left.\begin{aligned} \alpha_{0e}(\tau) &= \alpha_{se}(\tau)\left\{s\left(\tau+\frac{\tau_d}{2}\right)-s\left(\tau-\frac{\tau_d}{2}\right)\right\} \\ \alpha_e(\tau) &= \alpha_{0e}\left(\tau-\frac{\tau_d}{2}\right) = \alpha_{se}(\tau)\{s(\tau)-s(\tau-\tau_d)\} \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots(42)$$

As an example of such equivalent stationary statistical quantities, the following may be considered with wide application.

$$\left.\begin{aligned}R_{se}(\tau)&=\frac{2\bar{\lambda}c_0}{\pi\tau}\sin\frac{\omega_u-\omega_l}{2}\tau\cos\frac{\omega_u+\omega_l}{2}\tau+\frac{c_1}{\pi}\cos\omega_1\tau\\ S_{se}(\omega)&=\bar{\lambda}c_0\{s(\omega+\omega_u)-s(\omega+\omega_l)+s(\omega-\omega_l)-s(\omega-\omega_u)\}+c_1\delta(|\omega|-\omega_1)\end{aligned}\right\}\cdots\cdots(43)$$

where the first term corresponds to the band limited white noise[2),3)] component, and the 2nd term is related to predominant periodic component. The parameters contained in this expression $c_0\bar{\lambda}$, c_1, ω_u, ω_l, ω_1 are all nondimensional quantities. On the other hand, the original statistical quantities with physical dimensions are easily obtained by the transformation of independent variable $\tau=\bar{\lambda}T$.

$$\left.\begin{aligned}\bar{R}_{se}(T)&=\frac{2c_0}{\pi T}\sin\frac{\Omega_u-\Omega_l}{2}T\cos\frac{\Omega_u+\Omega_l}{2}T+\frac{c_1}{\pi}\cos\Omega_1 T\\ \bar{S}_{se}(\Omega)&=c_0\{s(\Omega+\Omega_u)-s(\Omega+\Omega_l)+s(\Omega-\Omega_l)-s(\Omega-\Omega_u)\}+c_1\delta(|\Omega|-\Omega_1)\end{aligned}\right\}\cdots\cdots(44)$$

where c_0, $\bar{\lambda}\omega_u=\Omega_u$, $\bar{\lambda}\omega_l=\Omega_l$ are the level, upper and lower limits of angular frequency of the band limited white spectrum with respect to the original wave shape function $a_{se}(T)$, respectively, and c_1, $\bar{\lambda}\omega_1=\Omega_1$ are the power of predominant component and its angular frequency. Besides, the level of the noise component and the power of the periodic component of the equivalent spectral density of original acceleration with the maximum amplitude A are expressed by the following, respectively.

$$S_0=c_0A^2,\quad S_1=c_1A^2 \qquad\cdots\cdots\cdots\cdots(45)$$

Among the nondimensional parameters which define the equivalent stationary statistical quantities, the following three are most important.

$$\left.\begin{aligned}r_p&=\frac{c_1\int_0^\infty\delta(\omega-\omega_1)d\omega}{\bar{\lambda}c_0(\omega_u-\omega_l)}=\frac{c_1\int_0^\infty\delta(\Omega-\Omega_1)d\Omega}{c_0(\Omega_u-\Omega_l)}=\frac{c_1A^2\int_0^\infty\delta(\Omega-\Omega_1)d\Omega}{c_0A^2(\Omega_u-\Omega_l)}\\ r_{f1}&=\frac{\omega_1}{\omega_u}=\frac{\Omega_1}{\Omega_u},\quad r_{fl}=\frac{\omega_l}{\omega_u}=\frac{\Omega_l}{\Omega_u}\end{aligned}\right\}\cdots\cdots\cdots(46)$$

where r_p is the ratio of the power of predominant periodic component to that of noise component, and r_{f1}, r_{fl} are the ratios of the predominant angular frequency and the lower angular frequency limit of the noise component to the upper angular frequency of the noise component respectively. If the parameters r_p, r_{f1}, r_{fl} are appropriately selected corresponding to the properties of the earthquake and the soil conditions, c_1, ω_1, ω_l are determined in reference to $c_0\bar{\lambda}$ ω_u which are containable in the nondimensional intensity parameter p and the frequency parameter ψ^{-1} of the earthquake excitation, respectively.

$$\left.\begin{aligned}p&=\alpha\sqrt{\bar{\lambda}c_0}=S_0^{1/2}\bar{M}^{3/4}/\bar{K}^{3/4}\bar{A}\\ \psi&=\frac{{}_1\omega}{\omega_u}=\frac{{}_1\Omega}{\Omega_u}=\frac{\tau_l}{{}_1\tau}=\frac{T_l}{{}_1T}\end{aligned}\right\}\qquad\cdots\cdots\cdots\cdots(47)$$

where $\alpha = A\bar{M}/\bar{K}\bar{\Delta}$ is the maximum amplitude of nondimensional earthquake acceleration with zero mean, and ${}_1\omega(=\Omega/\bar{\lambda}=2\pi/{}_1\tau=2\pi/\lambda_1 T)$ is the nondimensional fundamental natural angular frequency, where $\bar{\lambda}=\sqrt{\bar{K}/\bar{M}}$. ${}_1\Omega=2\pi/{}_1T$ is fundamental natural angular frequency of a structural system. $T_l=2\pi/\Omega_u$ is the lower limit of period of the noise component, and $\bar{M}$, $\bar{K}$, $\bar{\Delta}$ are the reference values of mass, rigidity and deformation of the structural system, respectively. Then, the parameters which define the equivalent stationary statistical quantities of an earthquake excitation with the nondimensional intensity parameter α are given as follows:

$$\left.\begin{array}{l}\omega_u={}_1\omega/\psi, \quad \omega_l={}_1\omega r_{fl}/\psi, \quad \omega_1={}_1\omega r_{fl}/\psi \\ \alpha^2 c_0\bar{\lambda}=p^2, \quad \alpha^2 c_1\int_0^\infty \delta(\omega-\omega_1)d\omega=p^2{}_1\omega r_p(1-r_{fl})/\psi\end{array}\right\} \qquad \cdots\cdots\cdots\cdots(48)$$

A group of earthquake excitations for the response analyses is supposed to be determined by the frequency characteristic of the level of spectral density S_0 of the noise component. This frequency characteristic, for instance, may be given by the constant standard deviation of acceleration, velocity of displacement. The variance of the dimensionless wave shape function of the equivalent stationary earthquake acceleration, and those of the corresponding velocity and displacement obtained by integrations are expressed by the following formulae.

$$\left.\begin{array}{l}a_v=E(a-E(a))^2=\dfrac{1}{\pi}\displaystyle\int_0^\infty S_a(\omega)d\omega=\dfrac{\bar{\lambda}c_0}{\pi}(\omega_u-\omega_l)+\dfrac{c_1}{\pi} \\ v_v=E(v-E(v))^2=\dfrac{1}{\pi}\displaystyle\int_0^\infty S_v(\omega)d\omega=\dfrac{\bar{\lambda}c_0}{\pi}\dfrac{\omega_u-\omega_l}{\omega_u\omega_l}+\dfrac{c_1}{\pi}\dfrac{1}{\omega_1^2} \\ d_v=E(d-E(d))^2=\dfrac{1}{\pi}\displaystyle\int_0^\infty S_d(\omega)d\omega=\dfrac{\bar{\lambda}c_0}{3\pi}\dfrac{\omega_u^3-\omega_l^3}{\omega_u^3\omega_l^3}+\dfrac{c_1}{\pi}\dfrac{1}{\omega_1^4}\end{array}\right\} \cdots\cdots\cdots\cdots(49)$$

where

$$\left.\begin{array}{l}a=\alpha_{se}(\tau), \quad E(a)=E(\alpha_{se}(\tau))=0 \\ v=v(\tau)=\displaystyle\int_{-\infty}^{\tau}\alpha_{se}(\tau)d\tau \\ d=d(\tau)=\displaystyle\int_{-\infty}^{\tau}(v-E(v))d\tau\end{array}\right\} \qquad \cdots\cdots\cdots\cdots(50)$$

$$S_a(\omega)=S_{se}(\omega), \quad S_v(\omega)=S_a(\omega)/\omega^2, \quad S_d(\omega)=S_a(\omega)/\omega^4 \qquad \cdots\cdots\cdots\cdots(51)$$

The variances of nondimensional earthquake excitation are expressed in terms of the variances of the equivalent stationary earthquake excitation which is defined by a set of parameters r_p, r_{fl}, r_{f1}, $c_0\bar{\lambda}=1$, $\omega_u=1$, namely

$$\left.\begin{array}{l}\tilde{a}_v=\dfrac{p^2{}_1\omega}{\psi}a_{v0}, \quad \tilde{v}_v=\dfrac{p^2\psi}{{}_1\omega}v_{v0}, \quad \tilde{d}_v=p^2\left(\dfrac{\psi}{{}_1\omega}\right)^3 d_{v0} \\ \tilde{a}_v=\alpha^2 a_v, \quad \tilde{v}_v=\alpha^2 v_v, \quad \tilde{d}_v=\alpha^2 d_v\end{array}\right\} \qquad \cdots\cdots\cdots\cdots(52)$$

where

$$\left.\begin{aligned} a_{v0} &= \frac{1}{\pi}(1-r_{fl})(1+r_p) \\ v_{v0} &= \frac{1}{\pi}(1-r_{fl})(r_{fl}^{-1}+r_p r_{fl}^{-2}) \\ d_{v0} &= \frac{1}{\pi}(1-r_{fl})[\{(r_{fl}^{-1}+r_{fl}^{-2}+r_{fl}^{-3})/3\}+r_p r_{fl}^{-4}] \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(53)$$

In terms of the original physical quantities, the variances are expressed as follows:

$$\left.\begin{aligned} A_v &= S_0\Omega_u a_{v0} = 2\pi S_0 T_l^{-1} a_{v0} \\ V_v &= S_0\Omega_u^{-1} v_{v0} = (2\pi)^{-1} S_0 T_l v_{v0} \\ D_v &= S_0\Omega_u^{-3} d_{v0} = (8\pi^3)^{-1} S_0 T_l^3 d_{v0} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(54)$$

The frequency characteristics represented by constant standard deviation of acceleration, velocity and displacement are given by the following equations corresponding to the nondimensional system and the physical system, respectively.

$$\left.\begin{aligned} &p\psi^{-1/2}=c_a, \quad p\psi^{1/2}=c_v, \quad p\psi^{3/2}=c_d \\ &S_0^{1/2}T_l^{-1/2}=C_a, \quad S_0^{1/2}T_l^{1/2}=C_v, \quad S_0^{1/2}T_l^{3/2}=C_a \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(55)$$

where, c_a, c_v, c_d are dimensionless constants and C_a, C_v, C_d are constants which have the dimensions corresponding to acceleration, velocity and displacement, respectively. Now, we consider the equivalent stationary statistical quantities simplified by substituting $\omega_l=0$ in equation (43), and $\Omega_l=0$ in equation (44).

$$\left.\begin{aligned} R_{se}(\tau) &= \frac{\bar{\lambda}c_0}{\pi\tau}\sin\omega_u\tau+\frac{c_1}{\pi}\cos\omega_1\tau \\ S_{se}(\omega) &= \bar{\lambda}c_0\{s(\omega+\omega_u)-s(\omega-\omega_u)\}+c_1\delta(|\omega|-\omega_1) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(56)$$

$$\left.\begin{aligned} \bar{R}_{se}(T) &= \frac{c_0}{\pi T}\sin\Omega_u T+\frac{c_1}{\pi}\cos\Omega_1 T \\ \bar{S}_{se}(\Omega) &= c_0\{s(\Omega+\Omega_u)-s(\Omega-\Omega_u)\}+c_1\delta(|\Omega|-\Omega_1) \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(57)$$

The variances of velocity and displacement tend to infinity in this case, and this figure seems to be unreal. However, we should be permitted to make practical use of this simplified formula, because the earthquake response of ordinary structural systems may not be affected by the very low frequency component in the neighbourhood of zero and the consideration of this range is marginal as far as engineering safety is concerned. If we introduce the finite duration τ_d, the spectral density of the stationarized wave shape function may be obtained by the following equation.

$$S_{r0}(\omega)=\frac{\bar{\lambda}c_0}{\pi}\int_{\tau_d(\omega+\omega_u)/2}^{\tau_d(\omega+\omega_u)/2}\frac{\sin x}{x}dx+\frac{c_1}{\pi}\left\{\frac{1}{\omega+\omega_1}\sin\frac{\tau_d(\omega+\omega_1)}{2}+\frac{1}{\omega-\omega_1}\sin\frac{\tau_d(\omega-\omega_1)}{2}\right\} \quad \cdots\cdots\cdots\cdots(58)$$

The parameters which define the simplified stationary statistical quantities given by equation (57) are calculated from the previously obtained auto-correlation functions of typical earthquake accelerograms, shown in Table 1.

Finally, we shall consider on the amplitude probability density distribution function of the equivalent stationary earthquake excitation pattern $\alpha_s(\tau)$. In general, the previously obtained amplitude probability density distributions of the wave shape functions are not purely Gaussian. The distortion of the probability density distribution function from the normal distribution may be approximately estimated by the following nondimensional parameters.

$$s_1=\frac{W'(0)}{W(0)},\quad s_2=s_1{}^2=\frac{R_{r0}'(0)}{R_{r0}(0)},\quad s_3=\frac{1}{\sqrt{R_{r0}(0)}} \qquad \cdots\cdots\cdots\cdots(59)$$

where, $W'(0)$ is the density at zero calculating from $R_{r0}(0)$ on the assumption of the validity of normal distribution, and $R'_{r0}(0)$ being the variance calculating from $W(0)$ under the same assumption. Therefore, s_1 or s_2 represents the distortion of density in the small amplitude of the wave shape function. On the other hand, s_3 indicates the distortion in large amplitude

TABLE 1.
Parameters of the statistical quantities of the wave shape function for the equivalent stationary earthquake acceleration.

parameter / accelerogram	c_0(sec)	c_1	$\Omega_u\left(\frac{\text{rad}}{\text{sec}}\right)$	$\Omega_1\left(\frac{\text{rad}}{\text{sec}}\right)$	r_p	r_{fl}
El Centro, May 18, 1940, N-S, 0~29 sec	4.94 $\times10^{-3}$	1.06 $\times10^{-2}$	2.55 $\times10$	1 37 $\times10$	8.39 $\times10^{-2}$	5.36 $\times10^{-1}$
Santa Barbara, June 30, 1941, N45°E, 0~4 sec	4.76 $\times10^{-3}$	3.46 $\times10^{-2}$	3.91 $\times10$	2.67 $\times10$	1.85 $\times10^{-2}$	6.81 $\times10^{-1}$
Santa Barbara, June 30, 1941, N45°E, 0~10 sec	2.09 $\times10^{-3}$	2.41 $\times10^{-2}$	3.88 $\times10$	2.67 $\times10$	1.56 $\times10^{-1}$	6.87 $\times10^{-1}$

TABLE 2.
Parameters of the amplitude probability density distribution function of accelerograms referring to the Gaussian distribution.

parameter / accelerogram	$W(0)$	$R_{r0}(0)$	$W'(0)$	$s_1=\frac{W'(0)}{W(0)}$	$s_2=\frac{R'_{r0}(0)}{R_{r0}(0)}$	$s_3=\frac{1}{\sqrt{R_{r0}(0)}}$
El Centro, May 18, 1940, N-S, 0~29 sec	3.09	4.33 $\times10^{-2}$	1.92	6.21 $\times10^{-1}$	3.85 $\times10^{-1}$	4.81
Santa Barbara, June 30, 1941, N45°E, 0~4 sec	2.19	7.04 $\times10^{-2}$	1.51	6.88 $\times10^{-1}$	4.72 $\times10^{-1}$	3.77
Santa Barbara, June 30, 1941, N45°E, 0~10 sec	4.99	2.98 $\times10^{-2}$	2.31	4.63 $\times10^{-1}$	2.15 $\times10^{-1}$	5.79

because it is defined as the ratio of the maximum amplitude of the wave shape function (i.e. unity) to the standard deviation. These parameters calculated from the data of the previously mentioned typical earthquake accelerograms are shown in Table 2. As a result, it should be noticed that the amplitude probability density distribution function of equivalent stationary earthquake acceleration is not exactly normal, since the density at small amplitude is quite higher than that of normal distribution and the density at the amplitude larger than the several times of standard deviation is always zero. However, to find the analytical expression of the density distribution, the higher central moments will be estimated in detail.

5. Conclusion

As a result of the statistical analyses of earthquake accelerograms, it has been shown that an earthquake accelerogram has spectral density with some peaks and non-Gaussian probability density distribution function, and that the statistical properties of the wave shape function of accelerograms are different in details, depending on each earthquake, the direction of the component and the sampled duration of the accelerogram. However, from the standpoint of earthquake engineering, a statistical model of the equivalent earthquake excitation pattern should be inferred to common statistical properties of accelerograms. To define it the nondimensional parameters have been presented. Assuming the ground-structure coupling is small, the spectral density of the equivalent stationary earthquake acceleration pattern will consist of the "band limited white noise" spectrum and the delta-function corresponding to a noise component and predominant periodic component, respectively. As regards the amplitude probability density distribution function of the equivalent stationary earthquake acceleration pattern, it should be set up remembering that the maximum amplitude is several times as large as the standard deviation, and that the density at small amplitude is quite higher than that of the normal distribution having the same standard deviation. The above mentioned statistical model of the equivalent stationary earthquake excitation pattern can be used as the input pattern for the earthquake response analyses to use analog or digital computer.

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