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Direct Measurement of Bottom Shear Stresses in Open Channel Flows

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Synopsis

A high sensitive device for measuring the shearing stresses on the bottom of an open channel by means of the method of a direct balance is described. The noticeable features of the device are that a very small shear to 0.01 dyn/cm^2 in magnitude can be measured, the sensitivity adjustment can be easily made in wide extent and a complete linearity between the shear stress and the displacement of sensing element is assured.

Estimations of the Kármán's constant, those of a resistance coefficient in a flow with very mild hydraulic gradient and measurements of the power spectra of the bottom shear fluctuation are made with success.

1. Introduction

The most significant resistance in the open channel flow is the rigid wall shear like a shearing force acting on a rigid boundary, as well as in other real fluid flows. Obvious effects of the wall shear stress are observed in a mean velocity profile within a boundary layer. Especially, a friction velocity resulting from the wall shear stress is a characteristic velocity in a turbulent boundary layer adjacent to the wall. Besides the effects on the structure of a flow, the wall shear takes part, for instance, in the problems of the surface errosion of a hydraulic structure and of the traction of bed loads on a channel floor. Hitherto, in hydraulics, the resistance of the open channel flow has been mainly estimated from the energy gradient of a steady state and related to a roughness coefficient for the practical use. But this resistance spatially averaged to a fairly wide extent contains many factors such as a bottom shear, a side walls shear, a free surface shear and an internal shear. The intensity of the local wall shear is recently becoming to be required from various demands in hydraulics; for instance, it is necessary to know accurate values of the wall shear in estimating the effects of the side walls and the free surface on the open channel flow. There are a few papers[1] which have treated with local wall shear by the aid of the Preston's method.

Many measurements of the local intensity of the wall shear have been made in meteorology and aeronautics. In the main, the methods available for measuring in an open channel flow comprise direct balance measurements, methods based on the shape of the boundary layer velocity profile close to the wall, and methods based on the rate of heat transfer from the wall. The methods based on the velocity profile represented by the Preston's method[2] and Clauser's method[3] are easy to deal with, but are unsuited for the measurement of a low intensity of the wall shear because of the difficulty in the accurate measurement of a small pressure difference. The methods based on the heat transfer[4] represented by a hot-wire anemometry are suited for measurements of the fluctuation of the small shear, but uneasy in treatment and are difficult in stable operation. The direct balance method will be the most suitable one for the measurement of small shear stresses and that on a rough wall. Recent representative measurements by this method are presented in papers of Hakkinen[5] in aeronautics and of Gurvich[6] in meteorology.

The high sensitive device applied to the measurement on the bottom shear stress of open channel flows has been developed with the aid of the method of the direct balance, which is based on the principle of a simple pendulum. The principle, the structure and the characteristics of the device developed are presented in the former half of this paper, and measurements of the resistance coefficient in the flow with very mild hydraulic gradient, the Kármán's constant in water flows and the power spectra of the turbulent bottom shear in low frequency ranges are in the latter half.

2. Measuring device of the bottom shear in open channel flows

(1) *Principle*

The drag plate, a small portion of the channel floor, is a floating element separated from the channel bottom by a narrow gap. The shearing force acting on the drag plate is directly measured from the displacement of the drag plate. The drag plate is suspended by threads in water as a simple pendulum to have no mechanical friction. Then the device has a high sensitivity and a restoring force in proportion to the displacement of the drag plate. If the horizontal displacement x of the drag plate due to the wall shear is held much smaller than the length of threads for suspension l, the wall shear σ_W is expressed as

$$\sigma_W = \frac{1}{A}\frac{W}{l}x \qquad (l \gg x), \tag{1}$$

where A is the area of the drag plate and W the weight in water. A linear relation exsists between the wall shear and the displacement of the sensing element. An upward displacement of the drag plate in these circumstances is about $x^2/2l$ $(l \gg x)$ and is negligible for the practical use. An adjustment of the sensitivity is easily made by the change of the weight of the drag plate W and it covers a wide range. The small horizontal displacement of the drag plate due to the wall shear is measured by a differential transformer transducer.

(2) *Structure*[7]

A sketch of the device is shown in Fig. 1. The sensing element consists of a Π-shaped rectangular frame of 3 mm thick aclylic plates, of which the top surface is the rectangular drag plate with 6 cm width and 15 cm length ②. The sensing element is suspended by four parallel nylon threads ③ less than 0.3 mm in diameter within a fixed brass box of the size 14×18×6.6 cm ① mounted under the 3 mm thick steel channel ⑬, and is so arranged that a part of the channel floor is just replaced with the drag plate as a measuring portion. The length of the thread for suspension is 14.3cm. The streamwise displacement of the drag plate due to the wall shear is measured from the relative displacement between a ferrite core ⑤ fixed to the sensing element and the bobin of the differential transformer ④ fixed to the brass box. The drag plate can be adjusted by the four screws ⑦ to hold the elevation at the same level of the channel bottom. A clearance between the both sides of the drag plate and the cut ends of the channel floor is 1mm. Clearances before and behind the drag plate arising from its displacement due to the wall shear is made as small as possible not to disturb the flow with the aid of two clearance-adjusting screws ⑧ which are connected to the movable plates for the clearance adjustment ⑨. The extent of this adjustment is about 5 mm. The sensitivity adjustment is easily made with the exchange of the weight ⑩. Various weights are prepared. One side of the brass box is made of a transparent aclylic plate as a window in order to observe the motion of the sensing element and to adjust the core in a proper position. The side part has also a round window, 5 cm in diameter to remove the core. The differential transformer is completely water-proofed by an aclylic resin, because the interior of the fixed brass box is filled with water during experiments. The extent of the linearity is ± 5 mm and the sensitivity 200 mV/mm. A bore of the bobin is made large enough not to be rubbed with the core.

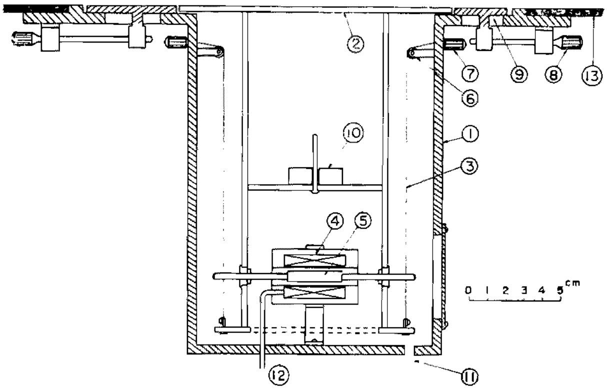

Fig. 1. Sketch of the device.
① Fixed brass box
② Drag plate
③ Nylon threads for suspension
④ Bobin of differential transformer
⑤ Core of differential transformer
⑥ Pulley
⑦ Suspension adjusting screws
⑧ Clearance adjusting screws
⑨ Movable plate for clearance adjust
⑩ Weights for sensitivity adjust
⑪ Drain cock
⑫ to displacement meter
⑬ Floor of flume

(3) *Static characteristics*

The resistance against the static displacement of the sensing element is only an electromagnetic attractive force acting on the core. But the attractive force can be made negligibly small with a high frequency of a carrier wave and a low current of the primary coil of the differential transformer. An estimation of the attractive force is as follows: The electromagnetic

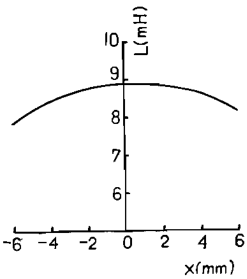

Fig. 2. The variation of the inductance of the primary coil of the differential transformer due to the displacement of the core from the neutral position.

attractive force acting on the core is given

$$F = i^2 \frac{dL}{dx}, \tag{2}$$

where x is the displacement of the core, L the inductance of a primary coil and i the magnetization current. For instance, if the displacement of the core from the neutral position is 1 mm, dL/dx of this position obtained from Fig. 2 is of the order of 10^{-2}(H/cm). Then the attractive force F becomes of the order of 10^{-4} dyn with the magnetization current 150 mA. The displacement of the drag plate is restricted as small as possible not to disturb the flow. On the other hand, the total shear at the 1 mm displacement of the drag plate is obtained as of the order of 10^2 dyn from the following equation (3). Therefore the attractive force becomes negligible.

As the weight of the sensing element in water is 36.2 g and the length of the threads of the pendulum is 14.3 cm, the wall shear is given from eq. (1) as

$$\sigma_W = 27.6x.$$

However, most experiments are made with the lower sensitivity than the above equation, adding the weight 84.6 g in water, i.e.

$$\sigma_W = 92.0x. \tag{3}$$

All runs of measurements described in this paper are based on eq. (3).

An effect of the clearance around the drag plate is not known quantitatively, but is assumed to be very small from an observation of trajeclories of ink near the clearance and from a comparison between the scale of the drag plate and that of clearances.

(4) Dynamic characteristics

The forces taking part in a motion of the sensing element are the wall shear acting on the drag plate as an external force, the electromagnetic attractive force acting on the core of the differential transformer and the viscous damping force. There exists no solid friction. Since the electromagnetic attractive force is negligible as previously stated, the equation of motion for the sensing element is assumed that

$$\ddot{x} + 2\varepsilon\dot{x} + n^2 x = \sigma_W(t), \tag{4}$$

where $x(t)$ is the displacement of the sensing element from the neutrally stable location, $\dot{x} = dx/dt$, ε and n are positive real constants. The wall shear $\sigma_W(t)$ is generally a stationary random function of time t in the turbulent boundary layer, and the variation of $x(t)$ will be likewise random. It is clear that the stationary random variation of $\sigma_W(t)$ produces a similar variation of $x(t)$ since the relation between $\sigma_W(t)$ and $x(t)$ is linear as shown in eq. (4). Therefore the statistical behaviour of $\sigma_W(t)$ is determined by means of that of $x(t)$. Let assume that the spectral functions of $\sigma_W(t)$

and $x(t)$ are $\Sigma(\omega)$ and $\Xi(\omega)$, i.e.,

$$\Sigma(\omega)=\int_{-\infty}^{\infty}<\sigma_W(t)\sigma_W(t+\tau)>e^{-i\omega\tau}d\tau,$$

$$\Xi(\omega)=\int_{-\infty}^{\infty}<x(t)x(t+\tau)>e^{-i\omega\tau}d\tau,$$

where ω is an angular frequency, then from eq. (4) the following relation[8] between $\Sigma(\omega)$ and $\Xi(\omega)$ is obtained:

$$\Sigma(\omega)=[(n^2-\omega^2)^2+4\varepsilon^2\omega^2]\Xi(\omega). \quad (5)$$

Hence the determinations of ε and n make the calculation of $\Sigma(\omega)$ from $\Xi(\omega)$ possible. Figure 3 shows an example of a damping free oscillation of the sensing element in water. A period of the damping oscillation and the logarithmic decrement are determined as $T'=3.21$ sec and $\Lambda=0.256$ (damping ratio 1.80), respectively, from the averaged ten records such as Fig. 3. The determinations of both constants ε and n from T' and Λ are made with the aid of the relations[9] as follows:

$$\varepsilon=4.61\frac{\Lambda}{T'}, \qquad n=\frac{2\pi}{T'}\sqrt{1+0.537\Lambda}.$$

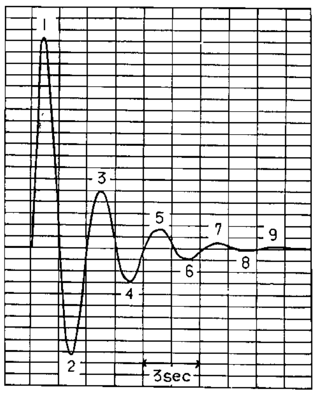

Fig. 3. An example of the record of the damping free oscillation of the drag plate in water.
A period of the damping oscillation is 3.21 seconds and damping ratio is 1.80.

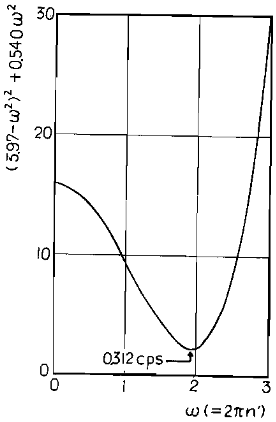

Fig. 4. The relation between the spectrum of the wall shear and that of the displacement of the drag plate. In the figure, n' is the frequency.

The results are $\varepsilon=0.367$ and $n^2=3.97$. Then the spectrum of the wall shear can be obtained from that of the displacement of the drag plate through eq. (5). Figure 4 shows the relation of eq. (5) with $\varepsilon=0.367$ and $n^2=3.97$, the state of the damping and the resonance is well understood, in which the frequency of the proper oscillation is 0.312 cycle/sec as indicated by the arrow in Fig. 4.

(5) *Measurements*

The measurements are made in a steel open channel which is 39 m long, 50 cm wide and 20 cm deep. The device is mounted at the midstream section of a third from the down stream end of the channel. The output voltage from the differential transformer is lead to the displacement gauge and indicated by the 0.5 mA amperemeter. Measurable extents are changed by a switch in four stages, i.e., ± 5, ± 2.5, ± 1 and ± 0.5 mm/± 100 mV. The voltage at the maximum deflection of the gauge is 100 mV. The gauge has the accuracy of 1%, and the minimum graduation is 0.01 mm. The adopted displacement gauge comprises a CR-oscillator generate 1.2 kc carrier wave for the magnetization, automatic compensating circuits to eliminate the errors of a sourse voltage, temperature, frequency, resistance of leads and linearity, and the circuits to rectify the output voltage. The wall shear is recorded by the electronic self-balancing recorder in practice. As the full scale, 20 cm, in the recorder corresponds to 10 mV, the displacement of the drag plate is magnified four thousand times on the recording paper, namely, the wall shear of 0.01 dyn/cm^2 is observed as the displacement of 1.5 cm on the record. In measurements, a good selection in the ranges of the displacement gauge is required simultaneously with a proper adjustment of the clearance before and behind the drag plate and the sensitivity of the device. An example of the record is shown in Fig. 5. When the mean value is required promptly, recording is made with a high damping and a low speed in records as shown in Fig. 6.

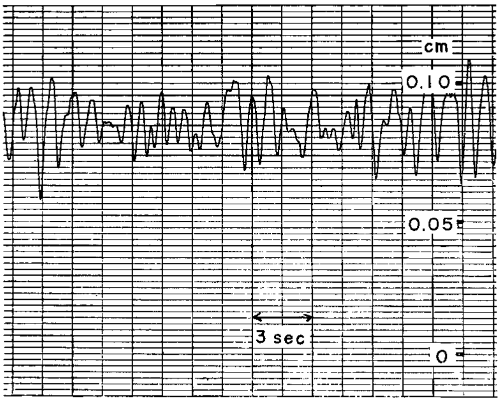

Fig. 5. An example of the wall shear for a dynamic analysis. The vertical scale is the displacement of the drag plate.

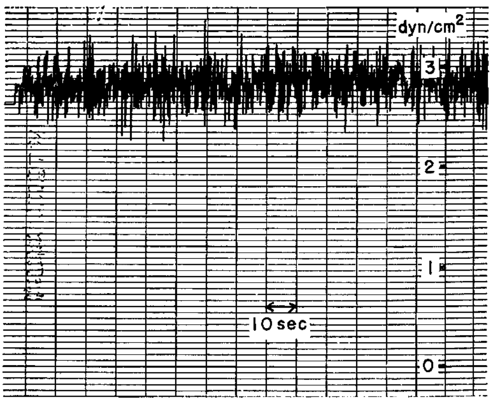

Fig. 6. Record of the wall shear to obtain the mean value. The vertical scale is the shear stress.

3. Experiments made with the device

(1) *Estimation of the Kármán's constant*

The universal constant of Kármán in wall turbulence is familiar in the formulae of the logarithmic velocity profile. It is an important proportional constant which determines the relation between the distance from the wall and the size of the representative eddy at this distance in a turbulent boundary layer. The determination of the constant is easily made by simultaneous measurements of the wall shear and the velocity profile above the wall surface. Figure 7 shows the results, i.e., the dependence of the friction velocity $u_* = \sqrt{\sigma_W/\rho}$ on the velocity gradient $d\bar{u}(z)/d \log z$:

$$u_* = k\frac{d\bar{u}(z)}{d\log z}, \qquad (6)$$

where ρ is the density of water, k the Kármán's constant and $\bar{u}(z)$ the longitudinal mean velocity at the height z. In Fig. 7, the straight line of gradient 0.4 of Nikuradse is drawn for reference. The scattering of points in Fig. 7 are assumed due to the result in rough measurements of the mean velocity. More precise measurements are needed to determine the accurate value of the Kármán's constant in open channel flows.

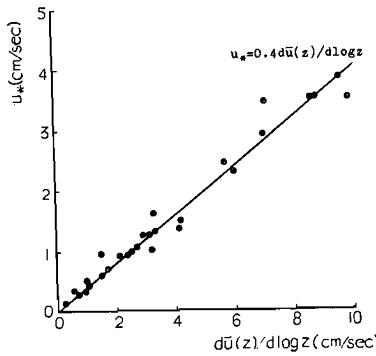

Fig. 7. Dependence of friction velocity on velocity gradient.

(2) *Resistance coefficient of a flow with very mild hydraulic gradient*

Most of the drainage channels are of a very mild slope. However, the knowledge on hydraulic characteristics of the flow with very mild hydraulic gradient is little. At the experiments of such flow, the resistance coefficient increases extraodinarily compared with that of Blasius according as the downstream end of the channel is damed up. In order to make the above facts clear, more accurate measurements of the boundary shear have been desired earnestly. The resistance coefficient of the open channel flow may be calculated from the following equation with the measurements of the mean velocity and the gradient of the free surface[10] as

$$f = 2\frac{u_{*R}^2}{u^2_m} = \frac{2gR}{u_m^2}\left(\sin\theta - \cos\theta\frac{dh}{dx}\right) + \frac{2R}{h}\left(\beta\frac{dh}{dx} - h\frac{d\beta}{dx}\right), \qquad (7)$$

where u_{*R} is the friction velocity due to the boundary shears without free surface, u_m the mean velocity, g the acceleration of gravity, R the hydraulic mean depth, θ the inclination angle of the channel bed, h the depth of the flow, x the streamwise coordinate and β the correction factor of momentum. Experiments are made in a broad open channel to compare the resistance coefficient directly measured on the bottom by the device with that calculated from eq. (7). But in the comparison, u_* obtained from the direct measurement is assumed to be equal to u_{*R}. A dependence of the resistance coefficient on the Reynolds number is shown in Fig. 8, in which white circle and black one on the same Reynolds number are simultaneously obtained. The fact that black circles

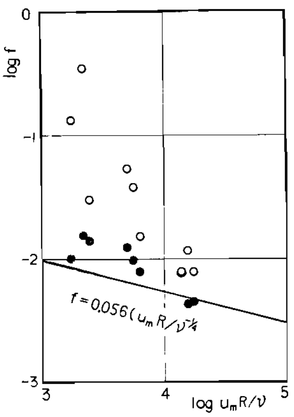

Fig. 8. Resistance coefficient of the flow of very mild slope.
○ Calculated from eq. (7)
● directly measured

are much closer than white ones to the Blasius' resistance law of turbulent flow, which is

$$f=0.056(u_m R/\nu)^{-1/4},$$

where ν is the kinematic viscosity, gives us the instructive knowledges to make clear the apparent increasing of the resistance coefficient in the damed up flow. The detailed studies of this problem will be presented in another occasion.

(3) *Statistical behaviours of a fluctuation of the wall shear*

The turbulent wall shear can also be expressed as the sum of the time averaged value and the fluctuating component alike another turbulent quantities. However, an investigation of the statistical characteristics of the turbulent wall shear is not yet made. The statistical nature of the fluctuation of the wall shear can be regarded as a manifestation of the motion of the eddies moving contact with the wall surface. The flow of a broad open channel contains the eddies of various size, from largest eddies, whose size is of the order of the depth of the flow, to smallest eddies, turbulent viscosity of which is of the order of the kinematic viscosity of water. Since the fluctuations of the wall shear are thought to result from the motion of each eddy, they form a band spectrum. An investigation of the spectrum of the wall shear are usefull not only to promote our knowledge on the structure of the wall shear but to enrich our experience on the various turbulent phenomena in a turbulent boundary layer.

The power spectrum of the wall shear is obtained from that of the displacement of the drag plate with the aid of eq. (5). The power spectrum of the displacement of the drag plate is computed by the electronic digital computer (KDC-I) from a series of equi-spaced values of time which are obtained from a continuous record such as Fig. 5 by the A-D converter. The method used for the spectral analysis is that developed by Tukey[11] for the analysis of a noise problem in electrical communications.

A choice in the length of the record and the sampling time interval is important. It is generally said that the length of the record is necessarily ten times larger than the largest period in a phenomenon and the sampling time interval is a half of the smallest period. Since it is believed that the vertical length of the largest eddies is of the order of the depth of flow h and the longitudinal length of the largest eddies is ten times its vertical length[12], the time interval which the largest eddies pass on the sensing element of the device is $10h/U$, where U is the mean velocity. Then the necessary length of the record becomes $100h/U$.

Since the size of the smallest eddies sensible to the drag plate is of order of the length of the drag plate L, the motion of the eddies smaller than L will not be measured in the record. Then the sampling time interval is $L/2U$, which is far larger than the smallest eddies in the water flow.

A response of the self-balancing type recorder is generally not so quick, but the recorder used is sufficient to this experiment. A recording pen moves full scale 20 cm in 0.4 sec.

The choice in number of lags is also important. For the resolution of the spectrum into narrow bands the number of lags m should be as large as possible, whereas large m will make the computational work involved laborious and, more importantly, the accuracy of the estimate itself decreases. Tukey[11] suggests that m should be small enough in relation to the number of data N to make the number of degress of freedom

$$F=\frac{2(N-m/4)}{m} \tag{8}$$

satisfactorily large, and the values which have been commonly used are $6<m<30$ and $F>30$.

The following is one run from many experiments[13]. The depth of flow, which restricts the scale of eddies, is held constant; $h=5$ cm and the mean velocity U which influences on the sweeping speed of the phenomena, is changed within the range of 12–63 cm/sec. In this case, the slope of the channel is 1/600, the Reynolds number $Uh/\nu=6.2\times10^3-3.1\times10^4$ and the friction velocity $u_*=0.46-2.7$ cm/sec. The power spectra of the wall shear obtained are shown in Fig. 9. They are made in dimensionless form, after the normalization, by h which restricts the scale of the eddies and U which is concerned with a passing frequency of the eddies. The mean wall shear is denoted by $\bar{\sigma}_W$. The sampling time interval $\Delta t=0.3$ sec, the sample size $N=600$, the length of the record $N\Delta t=3$ minutes and the degree of freedom $F=40$. The resistance coefficients measured in this experiment are shown in Fig. 10, which agree well with Blasius' formulae.

It becomes evident from Fig. 9 that the spectrum of the turbulent wall shear in a lower frequency range

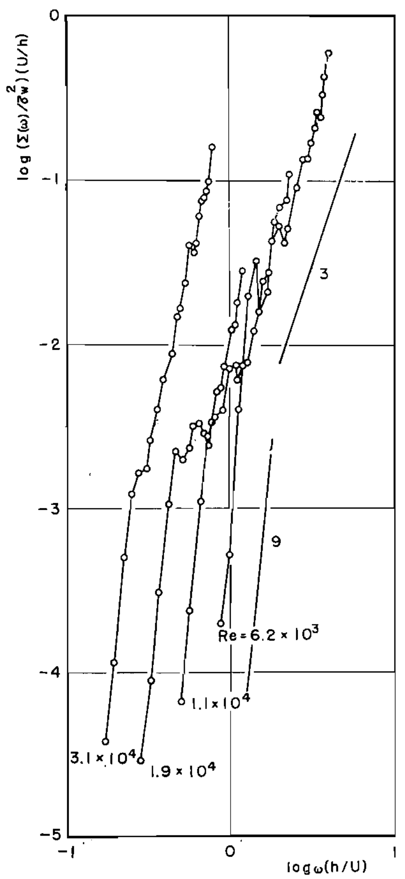

Fig. 9. Dimentionless normalized power spectra of the turbulent wall shear.
$N=600$, $\Delta t=0.3$ sec, $m=30$, $F=40$

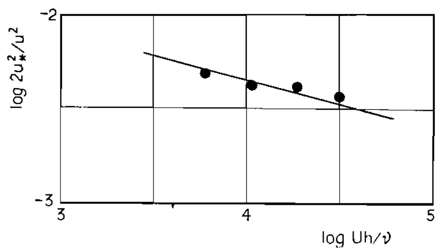

Fig. 10. Resistance coefficient in the case of Fig. 9.
The straight line shows the Blasius' resistance law.

consists of two parts where the spectrum seems to be proportional to ω^3 and ω^9. This is also supported by other cases of the experiment. Brief considerations about this matter are given below. A variation of the wall shear, similarly to that of velocity, will be related to the motions of eddies of various sizes. Furthermore, it is supposed to be also applicable to the relation between the fluctuating wall shear and the velocity that the mean wall shear is in proportion to the square of the mean velocity through a resistance coefficient. Assuming that λ is of the order of magnitude of the size of the given eddy and v_λ, $\sigma_{W\lambda}$ of the velocity and the wall shear, respectively, then,

$$\sigma_{W\lambda} \sim v_\lambda^2. \tag{9}$$

On the other hand, as $\Sigma(\omega) \sim \omega^3$ in Fig. 9 leads to $\sigma_{W\lambda} \sim \omega^2$, the relation

$$v_\lambda \lambda \sim \text{const.} \tag{10}$$

is obtained with the aid of eq. (9), where the transformation $\omega \sim 1/\lambda$ is used. The relation (10) reminds us of Kármán-Lin's hypothesis[14] in the theory of an isotropic turbulence that the turbulent viscosity of the eddies larger than the largest eddy is held equal to the turbulent viscosity of the largest eddy. A similar deduction gives that

$$v_\lambda \lambda^5 \sim \text{const.}, \tag{11}$$

since $\Sigma(\omega) \sim \omega^9$ in Fig. 9. The relation (11) is also analogous to the hypothesis used in an isotropic turbulence that the right-hand side of eq. (11) is equal to the Loitsiansky's invariant[14] which is the 4th moment of correlation.

From the above considerations, the hypotheses used in the theory of an isotropic turbulence seem to be well applicable to the eddies of larger scale at the region close to the wall, where the degree of nonisotropy of the turbulence is extreme.

In these experiments, knowledges on the scale corresponds to the peak of the spectrum and the spectral form in an inertial subrange are not found in spite of their importance in practice, because the size of the sensing element is much larger than the energy containing eddies. But the spectral form of Reynolds stress in an inertial subrange measured by meteorologists at a wall adjacent region shows the $-7/3$ power law, which agrees well with that derived from the Kolmogorov and Obukhov's law and eq. (9)[13]. Therefore, techniques used in an isotropic turbulent theory seem to be applicable with a good approximation to an explanation of several turbulent phenomena near the wall.

4. Conclusion

The device developed for the measurement of mean bottom shear of open channel flows achieved sufficiently the expected purposes. The problems still unsolved are that influences of gaps before and behind the drag plate on flow, the adequate scale of the drag plate for many purposes, the proper period of the device, the measurement on side walls of the channel, the measurement of small scale fluctuation of the shear and others. Most of the problems above mentioned will be solved fairly well with the newly developed instrument which is based on the hot-film anemometry[15].

References

1) Hwang, L. S. and Laurren, E. M. : Shear measurement technique for rough surface. Proc. A.S.C.E., HY 89-2, 1963 p. 19.
2) Preston, J. H. : The determination of turbulent skin friction by mean of pitot tubes. J. Roy. Aero. Soc. **58**, 1954, p. 109.
3) Clauser, F. H. : Turbulent boundary layer in adverse pressure gradients. J. Aero. Sci. **21**. 1954, p. 91.
4) Liepmann, H. W. and Skinner, G. T. · Shearing-stress measurements by use of a heated element. NACA TN 3268, 1954.
5) Hakkinen, R. J. : Measurements of turbulent skin friction on a flat plate at transonic speeds. NACA TN 3486, 1955.
6) Gurvich, A. S. : Measurement of frictional stresses in the near-ground layer of the atmosphere. Bull. (Izv.) Acad. Sci. USSR, Geophys. Ser. No. 3, 1961, p. 295.
7) Yokosi, S. and Kadoya, M. : Direct measurement of wall shear —Studies on flow with very mild hydraulic gradient (2)—. Annuals of Disas. Prev. Res. Inst. of Kyoto Univ. No. 7, 1964, p. 311 (in Japanese).
8) Batchelor, G. K. : *The theory of homogeneous turbulence*. 1953, p. 56.
9) Hagiwara, T. : *Measurements of oscillation*. 1960, p. 42. (in Japanese)
10) Kadoya, M. and Imao, A.: Studies on flow with very mild hydraulic gradient (1). Annuals of Disas. Prev. Res. Inst. of Kyoto Univ. No. 6, 1963, p. 200 (in Japanese).
11) Blackman, R. B. and Tukey, J. W. : *The measurement of power spectra*. Dover, 1959.
12) Townsend, A. A.: The structure of the turbulent boundary layer. Proc. Camb. Phil. Soc., **47**, 1951, p. 375.
13) Yokoshi, S.: Measurements of spectra of turbulent wall shear in open channel flow. Annuals of Disas. Prev. Res. Inst. of Kyoto Univ. No. 8, 1965 (in Japanese).
14) Kármán, Th. v. and Lin, C. C.: On the concept of similarity in the theory of isotropic turbulence. Rev. of Mod. Phys. **21**, 1949, p. 516.
15) Yokoshi, S. and Kadoya, M.: An instrument for measuring the wall shear in open channel based on the hot-film anemometry. Proc. 19th Conf. Agr. Engr. Soc. Japan, 1964, p. 39 (in Japanese).