# On the Accuracy of Tripartite Method

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#### Abstract

A cooperative observation of microearthquakes was made in the northern part of the Neo Valley Fault in 1964. One of the purposes of this observation was to re-examine various methods for determining the hypocenter. The tripartite observation was carried out at six stations, and 18 earthquakes whose seismograms were available, at least, at 2 stations, were analysed, as for P phase.

In 15 earthquakes, the epicenter was determined pretty well, but the focal depth was reasonably determined only in 9 earthquakes. This result suggests that epicenter is pretty reliable, if we take the observational error into consideration, but for focal depth, further investigation is necessary. Deviation of both approaching direction and apparent velocity caused by some superficial and underground structure may be meaningful, although the detailed discussion was impossible in this analysis because of insufficient data.

#### 1. Introduction

Many seisomologists have done research for a method to determine the hypocenter of earthquakes. No decisive method, however, seems to have been established for determining the hypocenter of, in particular, microearthquakes, in spite of these efforts. In general, if we assume an homogeneous, isotropic medium, we could determine five quantities of hypocenter, namely, the longitude, latitude and focal depth of hypocenter, the time of origin, and the mean velocity, using the arrival time of P phase at more than five stations. It is known, however, that a reasonable solution is not always obtained but the physically meaningless results occasionally come out for some or all of five quantities above-mentioned. No clear explanation seems to have been given for this curious fact, and it is necessary to solve this problem as soon as possible. When we use P-S time, we could get four quantities, the longitude, latitude and focal depth of hypocenter, and the so-called "Omori's factor", using the data at more than four stations. Unreasonableness in any of these four quantities is sometimes seen in this case, too<sup>2)</sup>.

On the other hand, the so-called tripartite method has been used as another method to determine the hypocenter. Needless to say, this method is usefull because we can determine the hypocenter by this method, using the data at fewer stations. From this point of view, many researchers have used this method, but few reports seem to have been so far published regarding the accuracy of this method<sup>3)</sup>.

Fortunately, the second precise observation of microearthquakes was done in the period of August 15 to 30, 1964, under the colaboration of 10 parties

belonging to universities and institutes in Japan\*). The final purpose of this project, which started in 1963 and is expected to continue for three years, is

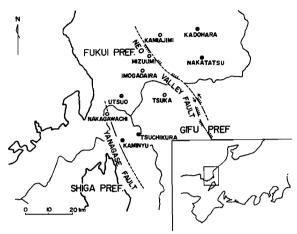


Fig. 1. Locations of ten stations in the microeathquake observation in 1964.

to get a clue for predicting the occurrence of destructive earthquakes. The special research item in 1964 observation was to reexamine various methods for hypocenter-determination, such as those by the arrival time of P phase. P-S time and the tripartite array, and also to get any relations between them. In this observation, tripartite method was carried out at six stations. and 164 earthquakes were recorded at tripartite net-

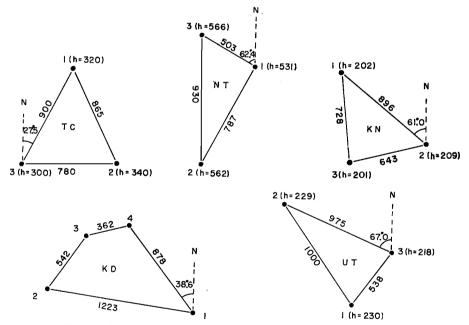


Fig. 2. Outlines of schematic representation of tripartite array. Unit is all in meter.

\*) Tsuchikura : Kyoto Univ.

Kaminyu : Tokyo Univ.

Nakagawachi: Tokyo Univ.

: Tohoku Univ.

Imogadaira : Kyoto Univ.

Utsuo

Mizuumi : Gifu Univ.

Kamiajimi: Meteorological Res. Inst.

Kadohara: Tokyo Univ.

Nakatatsu: Nagoya Univ.

: Osaka Inst. of Technology. Tsuka

TABLE 1. List of tripartite observation networks. Longitude and Latitude are those of Point 1 in Fig. 2.

Station	Abbre- viation	Longi- tude	Lati- tude	Appa- ratus	Seismo- graph	Damping constant	Geology	Agency	
Tsuchi- kura	тС	136°18′ 16′′.1	35°35′ 30′′.3	Data recorder	1 1c/s 2}3c/s		Paleozoic chert     Paleozoic slate     Younger sediment	D 1 - 1	
Naka- tatsu	NT	136°36′ 22′′.4	35°52′ 31′′. 2	Data recorder				Fuculty of Science, Nagoya Univ.	
Kado- hara	ΚD	136°36′ 26′′.1	35°57′ 50′′, 2	Data recorder	1 c/s	h =2	mesozoic diolite	Earthq. Res. Inst., Tokyo Univ.	
Utsuo	UΤ	136°13′ 43′′.1	35°43′ 30′′, 3	Data recorder	4 c/s	h=0.7	Slate	Fuculty of Science, Tohoku Univ.	
Ka- minyu	KN	136°13′ 37′′. 9	35°34′ 02′′, 0	Data recorder	1 c/s	h=1.3	Paleozoic rock	Earthq. Res. Inst., Tokyo Univ.	

works. Thus, the accuracy of the tripartite method and some results will be discussed in the following sections.

The locations of the ten stations are shown in Fig. 1, and in Fig. 2 the shape of tripartite array at five stations. Detailed descriptions of five tripartite stations used in the analysis are listed Table 1.

(Tsuka tripartite array was not used in the present analysis because of its too short spans.)

#### 2. Estimation of errors

The errors included in the tripartite method are considered to be brought about by various causes, namely the error may come from such items as,

- a) identification of any phase in problem,
- b) sharpness of onset of the phase,
- c) accuracy of time keeping,
- d) frequency characteristies and magnification of instruments used,
- e) S/N ratio,
- f) personal error in reading the seismograms,
- g) spans and shape of the tripartite array,
- h) approaching direction of the seismic wave to the array,
- i) some deviation caused by complexitities of the underground structures. Concerning a), we used only clear P phases which were undoubtedly identifiable on all records, which were sent to the authors by the courtesy of the respective parties, as show in Fig. 6. (The records of Nakatatsu in not shown,) In the present analysis, only the P phase was used. It is necessary,

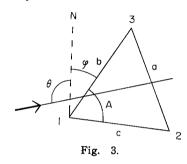
of course, to analyse not only P phase but simultaneously S phase and any later phase, if possible, but this problem will be postponed to a later time.

As for b) and c), we derived the data into two ranks, A and B, allowing for d) and e), where A and B are defined as follows;

A: arrival time difference of P weve between any two seismographs was determined with the accuracy of less than  $\pm 0.01$  sec,

B: that of less than  $\pm 0.02$  sec.

So far as the analysis of P phase in the present observation is concerned, much better condition of seismogram seemed to be seldom available, in which the accuracy was much higher than 0.01 sec. Aki and others stated that higher accuracy may be obtainable by the correlational method<sup>4)</sup>, but at the present time we did not adopt this method. The seismograms with the accuracy worse than B was not used in this analysis. As for f), it was possible



to re-examine all seismograms on the same principle. As to d) we did not use the data of Tsuka by the reason above-mentioned.

Now, it is possible to estimate the errors with respect to the spans and shape of each tripartite array and also to two ranks above-described. If three seismographs are arranged as shown in Fig. 3, the arrival time difference between two seismographs is expressed as follows, for example,

$$t_{31} = \frac{b\cos(\varphi + \theta)}{V},\tag{1}$$

$$t_{21} = -\frac{c\cos(A + \varphi + \theta)}{V} \tag{2}$$

where  $t_{31}$  and  $t_{21}$  are the arrival time difference of any phase observed at point 3 and 2 from point 1, adopted here as the standard point. a, b and c represent the lengths of three sides of triangular network, and  $\varphi$  and  $\theta$  are measured as shown in the figure. V is the apparent velocity of the phase concerned. Accordingly, we get,

$$dt_{13} = \frac{b}{V} \sin(\varphi + \theta) d\theta + \frac{b}{V^2} \cos(\varphi + \theta) dV, \tag{3}$$

$$dt_{21} = \frac{c}{V} \sin(A + \varphi + \theta) d\theta + \frac{c}{V^2} \cos(A + \varphi + \theta) dV. \tag{4}$$

The maximum absolute values of  $d\theta$  and dV are given as follows;

$$\max |d\theta| = \frac{V}{bc |\sin A|} (c|\cos(A + \varphi + \theta) dt_{18}| + b|\cos(\varphi + \theta) dt_{12}|), \tag{5}$$

$$\max |dV| = \frac{V^{2}}{bc |\sin A|} (c |\sin(A + \varphi + \theta) dt_{13}| + b |\sin(\varphi + \theta) dt_{12}|). \tag{6}$$

We may put

$$dt_{12} = dt_{13} = \begin{cases} 0.01 \text{ sec } & \text{for } A \\ 0.02 \text{ sec } & \text{for } B, \end{cases}$$

and substitute these values into equations (5) and (6). Max  $|d\theta|$  and max |dV| are considered to represent the limits of errors in the approaching direction and the apparent velocity of any phase used, which are to be allowed for in the treatment of determination of hypocenter. In Table 2, are shown the analysed data necessary in the following discussion.

TABLE 2.

Analysed data of tripartite observation used in this paper.

Arrival times of initial P waves are those of point 1 in Fig. 2.

						l		of point 1 in Fig	
No.	d d	h h	and m	Time s	Station	P-S	Rank	Apparent Velocity	Azimuthal Direction
	14	01	49	29, 40	ТC	1.88	A	7.5± 1.2	- 14± 8
1	ļ			27.3	UT	_	A	11.1± 2.1	126± 8
				28. 62	KN	2, 3	A	6.0± 0.6	19± 8
	14	03	27	51, 52	тС	_	A	5.8	- 90± 7
2				50, 20	KN	3, 35	Α	5. 3	$-100 \pm 6$
				51.6	UT	3. 4	A	6.7	−138± 5
3	14	03	00	37. 42	тС	_	A	19.0± 9.0	149±19
Ŭ				37.60	KN	2.14	A	12.4± 3.8	115±10
	14	21	16	54. 44	KN	7. 18	A	4.4± 0.5	9± 9
4				53.04	UT	3. 9	Α	4.8± 0.6	16± 5
•				8.82	КD	2.77	Α	6.0	- 43
				50. 19	NT	_	A	10.4± 2.3	$-28\pm12$
	15	01	27	36. 46	тС	4. 04	В	6.3± 1.9	- 6±23
5				36, 26	KN	4. 92	В	5.7± 0.6	39± 7
				34. 5	NΤ	3.8	A	5.8± 0.6	158± 7
	15	23	28	21. 40	тC	1. 14	A	9.0± 1.5	- 97±15
6				19.83	KN	0.90	Α	$21.4 \pm 10.0$	$-34\pm15$
				22. 1	NΤ	_	A	6.7± 0.8	177±15
7	17	01	45	59.85	тс	1, 85	A	11.6± 3.0	80±15
•				59, 28	KN	2, 42	A	12.0± 2.0	54±20
8	17	02	25	48. 58	TC	5. 73	A	5.6± 0.8	- 78± 7
Ü				46. 52	KN	5, 5 <b>6</b>	A	5.4± 0.7	$-83\pm5$
9	17	03	59	11.42	TС	1, 43	A	8.8± 2.0	-130±10
				09, 83	KN	1, 20	A	12.9± 1.0	$-141\pm20$
	18	03	12	09. 99	TC	4. 59	A	5. 6	- 88± 7
10				07.65	KN	4. 01	A	5. 9	$-105 \pm 7$
				08.97	UT	_	В	4.8	-119± 8

11	19	03	32	52. 10	тС	1. 37	A	7.9± 1.5	-134±10
				50.38	KN	1.14	Α .	11.2± 2.0	-134±15
	20	02	00	26. 39	тс	25.6	Α	38	75
12				25. 54	KN	26. 10	Α	29. 5	- 42
				25, 71	UT	-	В	25, 7	157
	20	03	27	06. 63	ТC	4. 58	A	6.8± 0.4	- 3±10
13				03.41	UΥ	2.3	Α	4.4± 1.0	19± 6
				04. 45	NT	3.09	Α	10.1± 4.5	- 77±17
_	20	03	36	23. 03	ТC	2. 92	В	8.1	91±21
14				22.92	KN	3.78	Α	6.8	43±14
				23. 26	UT	3.6	Α	6. 2	122± 8
_	22	01	06	28. 94	ТC	1, 33	Α	7.0± 1.0	-155±11
15				27.37	KN	1.01	Α	10.3± 1.3	$-176\pm16$
				30. 44	UΥ	3. 04	Α	6.0± 0.6	177± 8
	22	02	06	27.21	тС	0.96	A	7.6± 1.2	- 7±11
16				26.68	KN	1.47	Α	8.6± 1.0	39±15
				27.06	UΤ	-	В	5.3± 0.8	145±12
17	24	01	40	07.00	ТC	1.39	A	7.5± 1.0	-153±20
				05,36	KN	1,11	Α	7.9± 1.3	$-174\pm12$
18	24	02	57	58,23	ТC	_	A	9.5± 2.7	79±13
				58.78	KN	2.58	Α	12.6± 3.0	64±20

#### 3. Process of determining the hypocenter

In the following treatment, it is assumed that the error should be allowed for only in relation to both the shape of tripartite array and the rank of accuracy of time, in such a manner as expressed by equations (5) and (6), and that all other errors are safely neglected. Therefore, the epicenter and hypocenter are estimated graphically by the aid of Table 2.

First, we shall estimate the epicenter by the approaching directions at the respective networks. Fig. 5a shows this process, in which a sector spreading out from each station represents the approaching direction allowed for the errors derived from Table 2. The hatched area in the figure represents the "epicentral area" in which the true epicenter is to be inculuded.

In order to estimate the hypocenter, we have to assume an underground structure. We shall tentatively adopt a structure based on one of the profils along Kurayoshi-Hanabusa line, which were presented in the Kurayoshi and Hanabusa Explosions in 1963 and 1964. It is convenient to construct a nomograph in which a vertical section is divided by two families of lines. One of them consists of lines on which P-S time is the same, and another is that of equal apparent velocity. Fig. 4 shows this graph in the case of the crustal structure adopted here, in which the Poisson's ratio is assumed as 1/4. On this nomograph, we can determine the respective hypocenter from the ap-

parent velocity and P-S time observed at each tripartite network, and then the corresponding epicenter is determined on the ground surface. The position of epicenters thus determined is shown by the white circle ( $\bigcirc$ ) in Fig. 5a. These epicenters should lie, naturally, in the hatched area in Fig. 5a. The most probable epicenter ( $\diamondsuit$ ) was determined, as shown in Fig. 5a, considering the group of epicenters above-obtained.

Again, measuring epicentral distances of respective networks from the most probable epicenter, the position of networks is given on the abscissa of Fig. 5b. We can now estimate a range of variation of focal depth by the aid of Fig. 4 in the same way as the "epicentral area" derived in Fig. 5a, taking the errors into consideration. Fig. 5b shows this process. The part of vertical line through the epicenter intercepted by the hatched area in Fig. 5b represents a variation range of focal dept. P-S time is considered, of course, to contain some errors, but the effect of this error does not seem so great as to give a serious distortion to the result. Therefore, the error of P-S time is tentatively neglected in the above procedure.

These procedures gave reasonable results for some cases, but not for others. The results will be given in the following section.

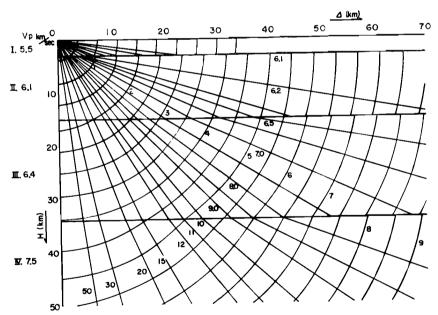


Fig. 4. The model of crustal structure adopted in the determination of hypocenters from apparent velocity and P-S time.

# 4. Results and Some Discussions

### (i) Results of analysis

18 earthquakes, for which at least 2 networks were available, were analysed, and as follows:

(a) Case in which 4 stations were available.	
<ol> <li>Hypocenter was determined well:</li> </ol>	0
2. Hypocenter was determined, if	
one station was omitted:	4 (No. 4.)
3. Epicenter was determined, but forcal	
depth was not:	1 (No. 5,)
(b) Case in which 3 stations were available.	
<ol> <li>Hypocenter was determined well:</li> </ol>	4 (Nos. 1, 6, 12, 15.)
2. Epicenter was determined, but focal	
depth was not:	2 (Nos. 13, 16.)
3. Both epicenter and focal depth	
were not determined:	3 (Nos. 2, 10, 14.)
(c) Case in which 2 stations were available.	
<ol> <li>Hypocenter was determined well:</li> </ol>	4 (Nos. 3, 8, 9, 17.)
2. Epicenter was determined, but	
focal depth was not:	3 (Nos. 7, 11, 18.)

# (ii) Deviations proper to each Station

As recognized in the list, 15 earthquakes of 18 gave resonable solutions for the epicenter within the limit of the error. On the other hand, the hypocenter, including the focal depth, was derived reasonably for 9 earthquakes of 18. Therefore, we may say that the determination of epicenter is considerably reliable, if the error consideration is made adequately. As for the estimation of focal depth this method does not seem so good, and further investigation is needed. As mentioned in the following, however, the consideration of the errors having physical meaning may be cocasionally necessary. If this effect is clarified by many data, it seems possible to remove any unexplained deviation in the above results and the tripartite method will be much improved.

In general, there are some differences between observational values and those estimated from the most probable hypocenter obtained above, in respect to both the approaching direction and the apparent velocity. These deviations are not only caused by various errors mentioned above but also should be brough about by some other causes, for instance, miceo-geological structure, topographic feature, inclination of crustal structure and others. Therefore, if we treat many data statistically, it may be possible to abstract the deviations caused by the latter effects, and it will contribute to exact interpretation of the result of tripartite observation. Unfortunately, since the data are so few and the treatment of errors in the previous section is insufficient to do such a statistical consideration because of the simplification, only the preliminary discussion will be given here.

We shall consider the deviations at each station, on the basis of three criteria. We take the earthquakes which occurred in two opposite sectors, one of which has a range of angle of  $0^{\circ}\pm45^{\circ}$  and the other  $180^{\circ}\pm45^{\circ}$ , where  $0^{\circ}$  and  $180^{\circ}$  are taken as North and South directions respectively.  $n_{1.4}$  is the number of earthquakes the deviation of with exceeds  $10^{\circ}$  regarding the most probable approaching direction, and  $n_{1}$  is the total number of earthquakes in

these two sectors. This is called criterion A, and listed in Table 3 for five stations, where + and - signs represent clockwise and counterclockwise senses seen from the station, respectively. In criterion B, two other sector,  $90^{\circ}\pm45^{\circ}$  and  $270^{\circ}\pm45^{\circ}$  are taken. By criterion C, the deviation of the apparent velocity is examined.  $n_{30}$  is the number of earthquake whose apparent velocity deflects more than  $10\,\%$ , where + and - signs mean the shallower and deeper deviations, respectively.

TABLE 0.							
Station	1	1 <u>A</u> _	ı – –	2 <u>B</u> _	T -	3 <u>0</u> _	
TС	1/7	1/7	2/6	1/6	0/9	2/9	
NΤ	1/3	1/3	0/1	0/1	0/4	1/4	
KD	0/2	0/2			1/2	0/2	
UΤ	0/5	0/5	0/1	0/1	3/6	0/6	
ΚN	4/9	0/9	0/4	1/4	1/9	0/9	

TABLE 3

# (iii) Tripartite observation in the first Hanabusa Explosion

The tripartite method was carried out by one of the authors at Tsuchikura in the case of the first Hanabusa Explosion on November 18, 1964, for the purpose of examining the accuracy of this method<sup>8)</sup>. Obtained seismogram is shown in Fig. 7. The results are as follows:

21.155 km,

Azimuth of explosion point seen from Tsuchikura;

N83°E,

Approaching direction of P phase;

 $N80^{\circ}E \pm 3^{\circ}$ .

Apparent velocity;

 $5.68 \pm 0.3 \, \text{km/sec}$ ,

(Time accuracy is taken as 5 msec.)

These results are very satisfactory. Although this is only one example, it is highly significant to use the artificial explosion for discussing the accuracy of hypocenter-determination<sup>9)</sup>.

- (iv) The earthquake, No. 12, was found to be of a rather deep focus, about 200 km. This result was ascertained by the tripartite method and also by the wave forms on the seismograms. Katsumata proposed a method to estimate the magnitude of deep focus earthquakes, the magnitude of which is larger than  $5^{10}$ . If this method is assumed to be valid for microearthquakes, the magnitude of this earthquake is calculated as 2, using the maximum amplitude (200  $\mu$  kine) and frequency (5 c/s) observed at Tsuchikura. It is interesting to note that such a micro-earthquake of deep focus can be clearly detected by the tripartite observation<sup>11)</sup>.
- (v) For reference, the hypocenters determined by Kayano using P-S time are listed in Table 4, and plotted in Figs. 5 a and b by the symbol,  $+^{12}$ .

TABLE 4.

Hypocenters determined from P-S time by Kayano
Co-ordinates are taken as TC: 50.0E, 50.0N and eastward and northward plus. Unit is all in kilo meter. K is Omori's constant obtained in this process.

No.	E	N	Z	K
2	23.4± 36.8	45.7± 11.4	13.9± 39.7	7.60
4	60.4± 16.3	117.0± 49.3	3.0±170.2	10.36
5	67.8± 1.8	78.3± 1.4	9.8± 2.7	8,35
8	24.0± 21.5	66.5± 6.2	24.5± 8.9	6.81
10	10.1± 33.1	37.3± 21.1	18.2± 49.0	9.89
13	$37.0 \pm 175$	106.3±231	15.3±192	10.47
16	52.3± 1.7	54.3± 1.4	6.5± 1.9	8.61

### 5. Conclusion

18 earthquakes, including one intermediate deep one, in which the tripartite observation was available at two to four stations, were analysed as to P phase, for the purpose of examining the accuracy of tripartite method.

The epicenter was determined pretty well for 15 of 18 earthquakes within the limit of error. On the other hand, the focal depth was less determined, 9 examples of 18. These results suggest that the determination of epicenter derived from the tripartite method is considerably reliable, but the focal depth leaves some problems. The unreasonable results seen in some examples may be solved by considering the systematic deviations of approaching direction and/or apparent velocity caused by, mainly, topographic and underground structures. Further investigation is needed on this problem.

### Acknowledgment

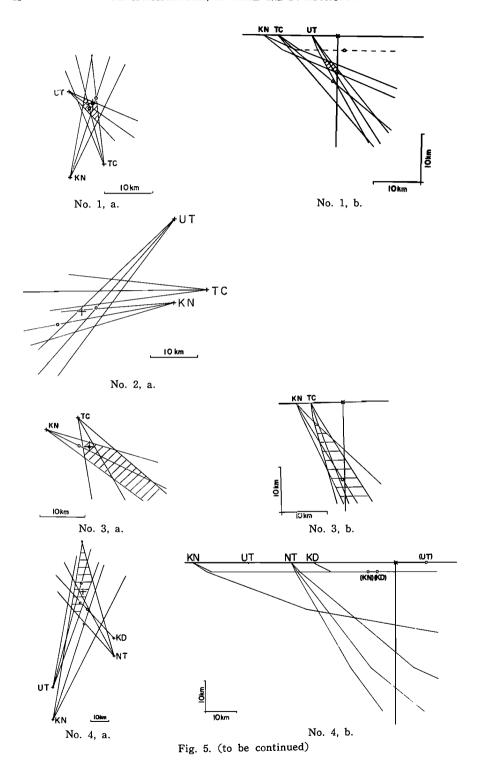
The authors are greatly indebted to participants of this observation for their generous permission for us to analyse their seismograms, and also to the members of "Research Group for Explosion Seismology" who kindly sent us their data.

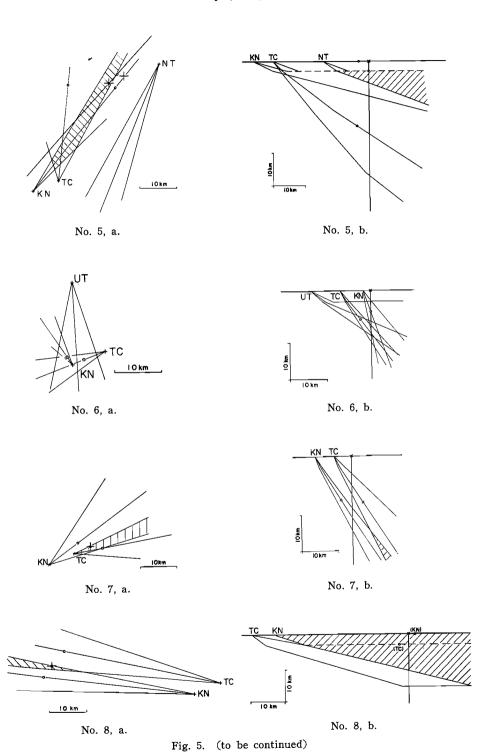
The authors also wish to thank Dr. T. Mikumo for his valuable discussion on this problem, and to Messrs. K. Mino and S. Takemoto for their help in observation and analysis.

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10km

I O km

No. 10, b.

No. 9, b.

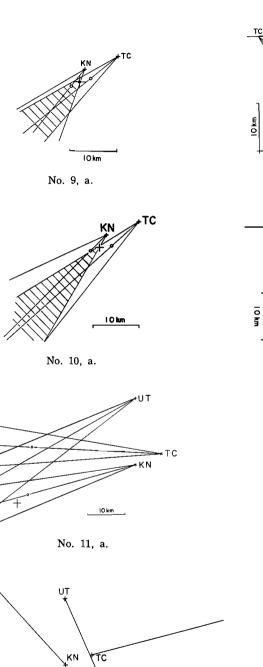


Fig. 5. (to be continued)

No. 12, a.

10km

(UT) 10 km No. 13 b, No. 13, a. 10 km No. 14, a.

Fig. 5. (to be continued)

No. 15, a.

No. 15, b.

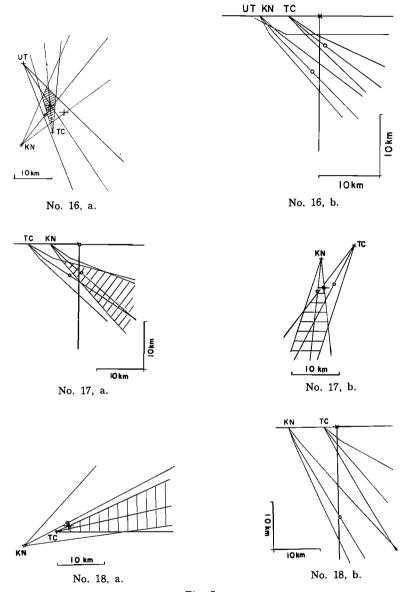
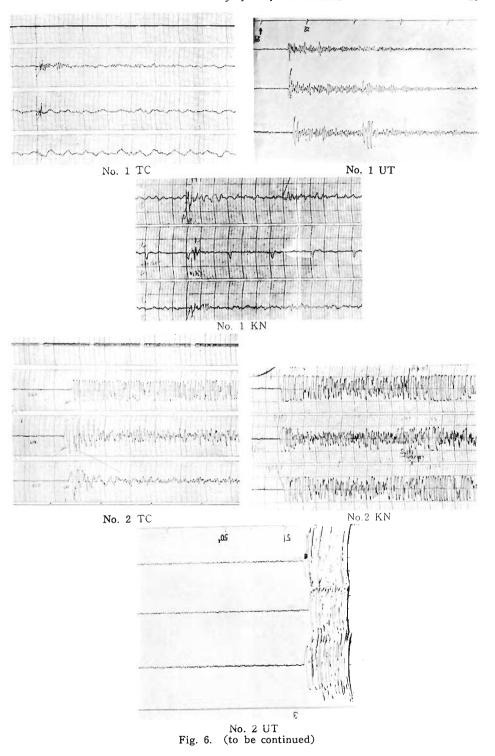


Fig. 5.



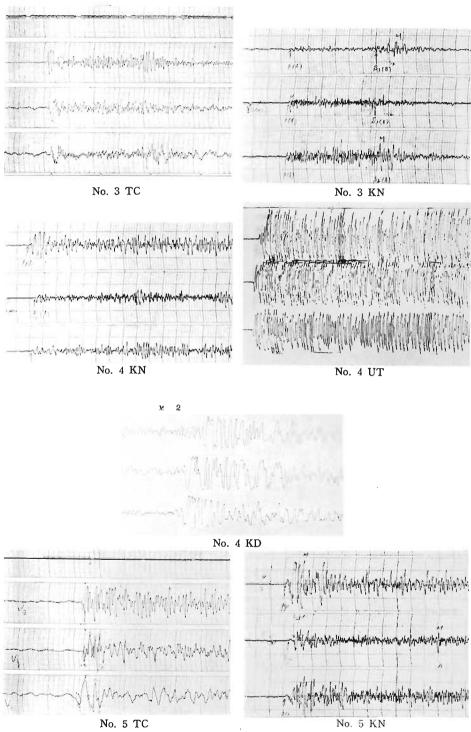
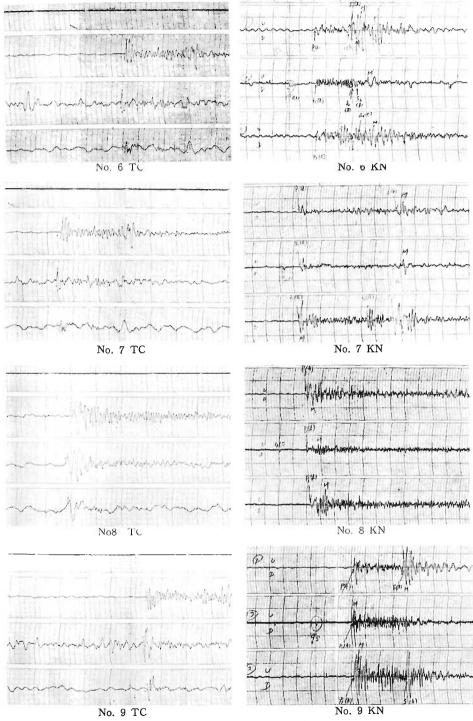
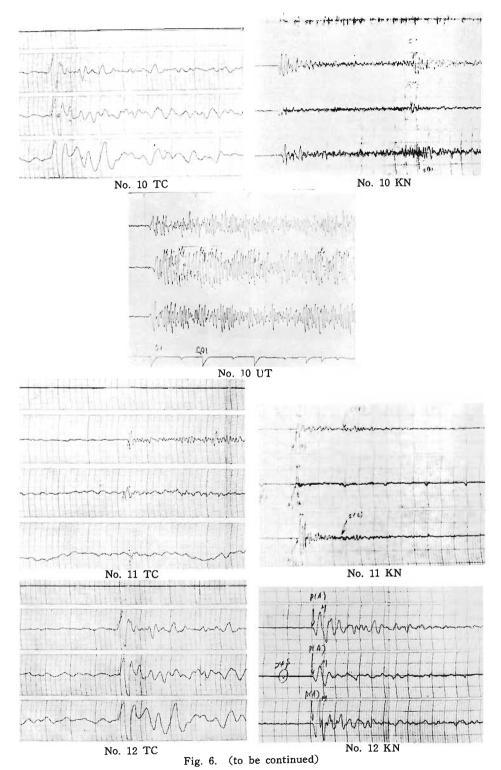
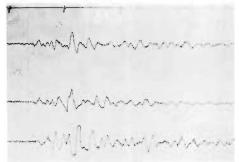


Fig. 6. (to be continued)

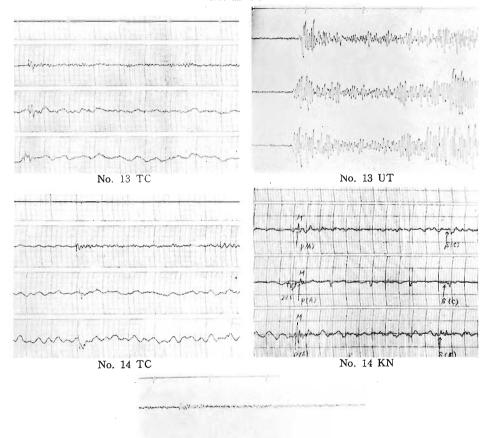


Fi.g 6. (to be continued)



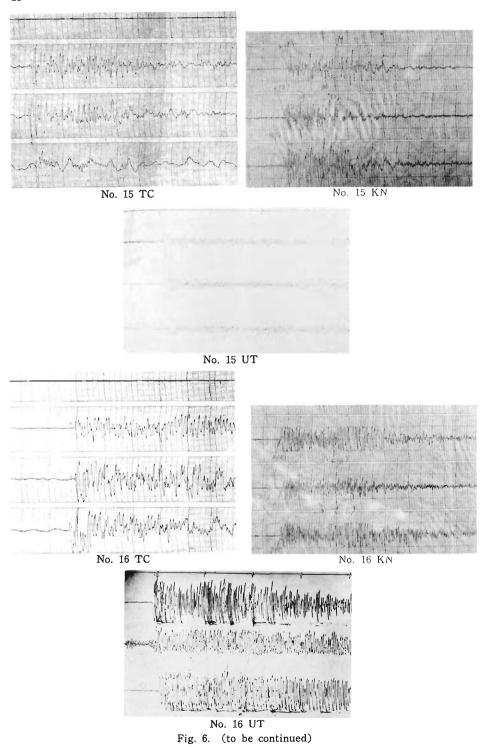


No. 12 UT



No. 14 UT Fig. 6. (to be continued)

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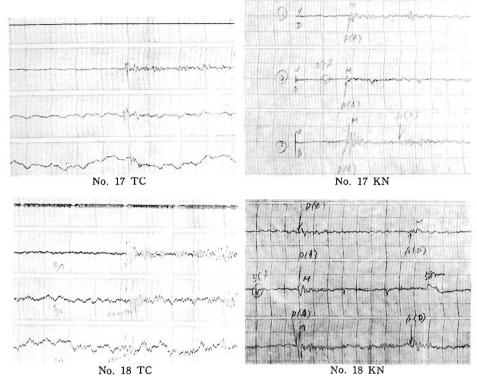


Fig. 6.

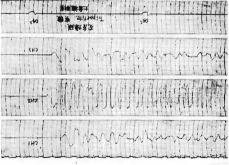


Fig. 7