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A Study on the Variation of Low Flow

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1. Introductory Statement

Today in Japan and in many other countries, water shortage is becoming a problem of deep concern and we are facing the necessity of building a complete projection of water usages to meet the demand in the near future. It is very important to consider the effective usage of stream water which flows out to the sea unused. For such consideration, firstly we should study the natural states of flow in the streams.

In Japan, we have considerable rainfall each year during the rainy season and from typhoons between early summer and fall, but less in winter and spring. Hence river discharge during a year is very variable.

As we know, stream water consists of two runoff components. One is the direct runoff which supplies much discharge but is limited to a short period and the other is the indirect runoff, namely the ground-water runoff, which supplies the stream water unceasingly during extended periods. This low flow should, therefore, be especially examined in regard to the water demand. One of the most important problems is to clarify the runoff mechanisms.

For the purpose of disaster prevention there has been much effort to clarify the behaviour of floods in Japan in the past and problems of ground-water runoff have been treated only in respect to floods. Hence, there is little research on ground-water runoff itself.

In this paper, the authors discuss the several characteristics of low flow phenomena from the view point of the kinematical mechanism of runoff.

2. General Remarks

Many investigators have studied the characteristics of ground-water runoff and low flow, discussing the behaviour of their recession characteristics which appear in the data of natural river discharge and have proposed some equations for recession curves[1~6]. Most of them, however, are only empirical and the significances of those equations are left theoretically unexplained.

Now, considering that the ground-water runoff is one phase of the movement of ground-water itself, we should study it from the mechanical point of view in order to clarify its characteristics and behaviours.

Phenomenological treatment assumes that the state of ground-water runoff to the ground surface is classified in two categories, I: the seepage of water from the unconfined aquifer, II: the leakage from the confined aquifer. And there are very different characters in the kinematical phase between those two components.

The movement of unconfined water is kinematically the same as diffusion but it may be considered that the water movement through confined strata is due to a pressure stronger than the atmospheric one.

Although there may be complicated combinations of strata in natural river basins and we cannot see their states in detail, stream water is certainly supplied by their runoff components. Hence, the low flow may be characterized by these characteristics of the two components. So, this paper deals with the some characteristics of low flow, by means of the discussion of the movement of flow in mathematical runoff models for these components.

For brief treatment in this paper we use the mathematical models shown in Fig. 1 for the two runoff components and our discussion deals with the movement of water for the unit width of these models. In this figure, the symbols denote

q_c, q_u : runoff discharge per unit width of these models,

k, k' : permeability coefficients of unconfined and confined strata respectively,

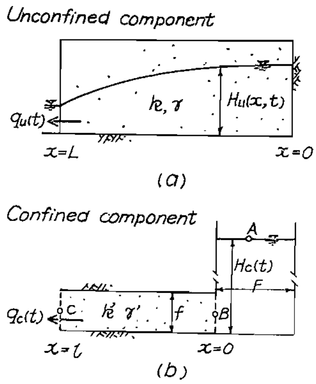


Fig. 1. Runoff models.

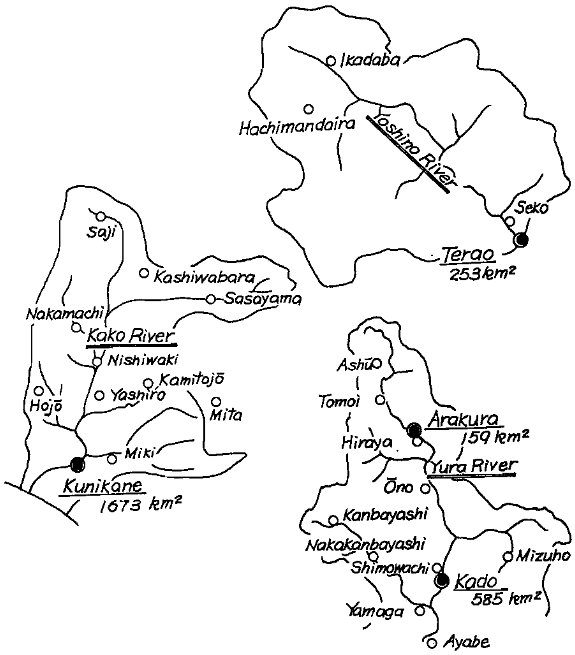


Fig. 2. General remarks of the basin.

γ, γ' : porosities of the strata,
l, L : length of strata,
f : cross sectional area per unit width of the confined stratum,
F : cross sectional area per unit width of the tank at upstream end of the confined model.

and other symbols are shown in the figure. General remarks on the basins and data which are analysed and discussed here, are shown in Fig. 2 and

TABLE 1.
General remarks on the basins and data which are analysed and discussed.

River, Gauging station, Catchment area	Symbol of data	The day, when the discharge is on the peak in daily gauging	The peak discharge in daily gauging m^3/s
Kako River Kunikane 1674 km²	K- 1	Mar. 7, 1950	828.1
	K- 2	July 20, 1954	196.7
	K- 3	July 31, 1954	229.0
	K- 4	Sep. 29, 1954	187.3
	K- 5	July 24, 1956	184.3
	K- 6	Sep. 27, 1956	1722.0
	K- 7	Oct. 31, 1956	370.5
	K- 8	July 28, 1957	304.0
	K- 9	July 4, 1957	218.0
Yoshino River Terao 253 km²	Y- 1	July 6, 1949	68.1
	Y- 2	July 13, 1951	357.3
	Y- 3	Nov. 5, 1952	225.3
	Y- 4	July 24, 1955	128.5
	Y- 5	Aug. 27, 1955	1331.4
	Y- 6	July 23, 1958	145.6
Yura River Arakura 159 km²	A- 1	Aug. 3, 1950	88.6
	A- 2	Oct. 6, 1950	52.8
	A- 3	Jun. 8, 1953	90.7
	A- 4	July 5, 1953	172.1
	A- 5	Sep. 26, 1953	331.8
	A- 6	July 6, 1954	35.8
	A- 7	Jun. 19, 1955	15.5
	A- 8	July 7, 1955	36.6
	A- 9	July 24, 1956	92.3
	A-10	Sep. 24, 1952	42.3
Yura River Kado 585 km²	K′- 1	July 30, 1949	229.0
	K′- 2	Oct. 6, 1949	69.2
	K′- 3	July 11, 1952	386.2
	K′- 4	July 30, 1954	66.6
	K′- 5	July 7, 1955	78.2
	K′- 6	July 24, 1955	13.2
	K′- 7	July 24, 1956	251.8

Table 1. All the drainage basins discussed in this paper are located in Kansai-district in Japan. Data of discharge and rainfall have been obtained by daily gaugings.

3. Characteristics of the Recession of Low Flow

It is thought that the appearances of the recession curve show the characteristics of ground-water runoff or low flow[1~6], which are also considered to depend mainly upon the basin characteristics.

In this section, we discuss the significances of the recession characteristics of low flow, and propose the recession equations.

3-1 Fundamental equations and their solutions

(i) the confined ground-water: In Fig. 1-(b), the equation of continuity is

$$fv=-F\frac{dH_c}{dt}, \tag{1}$$

in which v denotes the mean velocity of water through the stratum. If we consider the stratum and the upstream water tank as a flow pipe, we may use Bernoulli's equations, so that

$$H_c+\frac{1}{2g}\left(\frac{dH_c}{dt}\right)^2=\frac{1}{2g}v^2+h_B, \tag{2}$$

$$\frac{1}{2g}v^2+h_l=\frac{1}{2g}v^2+h_B, \tag{3}$$

where h_B is the head at point B in Fig. 1-(b), h_l is the head loss through the stratum and the pressures at A and C are considered atmospheric. By Darcy's law we may write

$$h_l=l\cdot v/k. \tag{4}$$

From the four equations above mentioned, we obtain the fundamental equation

$$-\frac{f}{F}v=\frac{l}{k'}\frac{dv}{dt}+\frac{v}{g}\left\{1-\frac{f^2}{F^2}\right\}\frac{dv}{dt}. \tag{5}$$

This equation can be easily solved and the solution becomes

$$-\frac{Fl}{fk'}\log_e q_c-\frac{F}{f^2g}\left\{1-\frac{f^2}{F^2}\right\}q_c=t+C, \tag{6}$$

in which $q_c=fv$ and C denotes the integral constant.

(ii) the unconfined ground-water: In this case, it is assumed that the assumption of Dupuit-Forchheimer holds for the water movement. We then have the following Darcy's equation and equation of continuity,

$$v=-\frac{\partial}{\partial x}\{k\cdot H_u(x,t)\}, \tag{7}$$

$$\frac{\partial(\gamma H_u)}{\partial t}=-\frac{\partial}{\partial x}(H_u\cdot v). \tag{8}$$

From these equations, the fundamental equation may be written as follows.

$$\frac{\partial(\gamma H_u)}{\partial t}=kH_u\frac{\partial^2 H_u}{\partial x^2}+k\left(\frac{\partial H_u}{\partial x}\right)^2.$$

If we assume γ is constant and the slope of water surface is very small, we may ignore the 2nd term of the right hand side by comparing with the 1st term. So, the fundamental equation of the recession state becomes

$$\frac{\partial H_u}{\partial t}=\beta H_u\frac{\partial^2 H_u}{\partial x^2}, \tag{9}$$

in which

$$\beta=k/\gamma. \tag{10}$$

There are many difficulties in defining the boundary and initial conditions for a natural river for the following reasons: Stream water is supplied by ground-water runoff, however, its behaviour also controls the ground-water movement as the boundary conditions. But in the low flow regime, we may consider that ground-water runoff mainly controls the channel water than being controled by channel water. So, in this analysis, the following initial and boundary conditions are used.

$$\left.\begin{aligned} &H_u(0,0)=H_0,\\ &H_u(L,0)=h_0,\\ &\left.\frac{\partial H_u}{\partial x}\right|_{x=0}=0. \end{aligned}\right\} \tag{11}$$

Then the solution is easily obtained for the unknown function $H_u(x,t)$ as follows,

$$H_u(x,t)=\frac{1}{\frac{2\beta}{L^2}(H_0-h_0)t+1}\left\{-\frac{H_0-h_0}{L^2}x^2+H_0\right\}. \tag{12}$$

By some calculations, we have obtained the following equation for the recession curve of the unconfined runoff component,

$$q_u=q_{u0}/(at+1)^2, \tag{13}$$

in which

$$a=\frac{2\beta}{L^2}(H_0-h_0). \tag{14}$$

and q_{u0} denotes the initial discharge of the unconfined component.

3-2 Equations of the recession curves

As mentioned in the preceding article, the behaviour of the recession of both components is expressed by Eq.'s (6) and (13). Although the equation (6) is very complicated, the 2nd term in its left hand side may be ignored by comparing with the 1st term through simple consideration. So, we may rewrite Eq. (6) as follows:

$$q_c=q_{c0}\exp(-\alpha t), \tag{15}$$

$$\alpha=fk'/Fl, \tag{16}$$

in which q_{c0} denotes the initial discharge of the confined component. Hence the recession equations become Eq.'s (15) and (13).

These equations are for the water movement per unit width of the model shown in Fig. 1-(b). They may be also used for the recession in natural basins after substituting Q into q in Eq.'s (15) and (13), that is

$$Q_c = Q_{c0} \exp(-\alpha t), \tag{15'}$$

$$Q_u = Q_{u0}/(at+1)^2, \tag{13'}$$

in which Q_c and Q_u denote the discharge of both components in the natural stream and subscript 0 shows the value at initial state. In these equations, α and a are also constant as given by Eq.'s (16) and (14) respectively. Then the discharge Q in a river may be written as follows,

$$Q = Q_c + Q_u.$$

Since the relationship

$$\lim_{t\to\infty}\frac{Q_c}{Q_u} = \lim_{t\to\infty}\frac{Q_{c0}}{Q_{u0}}\cdot(at+1)^2\cdot\exp(-\alpha t) = 0$$

shows that the rate of recession of the confined component is faster than that of the unconfined one, low flow in a long period is mainly supplied by the unconfined runoff component. By means of this characteristic we can separate the unconfined component from the confined one.

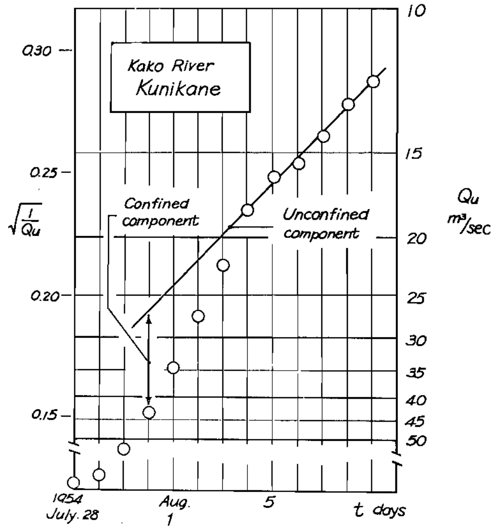


Fig. 3. An example of separation of runoff components.

Fig. 3 shows an example for the separation of runoff components. In this figure, the straight line fitted to the plots shows the unconfined recession curve, Eq. (13′). The confined components are obtained by the difference between this straight line and the plots. If one plots these confined components obtained for each day on semi-logarithmic paper and fits a straight line to new plotting, then this line will show the recession curve for the confined component.

In the analysis, the starting day of the recession should be taken at the time of the ground-water runoff peak. There are many difficult problems yet unsolved in choosing this time. However, in considering the drainage area and some investigations of direct runoff[7)8)], in this paper, we understand that the ground water runoff will be at its peak and the direct runoff will cease on the following day.

for Kunikane gauging station	: 3 days after the day when the runoff discharge is at its peak.
for Kado gauging station	: 2 days, 〃
for Arakura and Terao gauging station	: 1 day, 〃

TABLE 2.
Characteristic values obtained by the analyses.

	Data	Q_{u0} m^3/sec	a 1/day	Q_{c0} m^3/sec	α 1/day
Kako River Kunikane	K- 1	82.9	0.089	53.3	—
	K- 2	28.2	0.028	6.9	—
	K- 3	19.5	0.024	23.5	0.387
	K- 4	38.4	0.024	16.6	0.363
	K- 5	15.9	0.061	16.4	0.421
	K- 6	77.2	0.079	26.8	0.431
	K- 7	47.2	0.061	21.3	—
	K- 8	41.6	0.106	27.4	0.504
	K- 9	22.3	0.024	28.7	0.554
Yoshino River Terao	Y- 1	18.6	0.019	23.9	0.569
	Y- 2	46.0	0.084	109.7	0.618
	Y- 3	15.7	0.037	37.0	0.628
	Y- 4	13.8	0.023	64.0	0.626
	Y- 5	28.6	0.055	109.4	0.719
	Y- 6	17.8	0.043	61.8	0.642
Yura River Arakura	A- 1	4.7	0.027	24.0	0.774
	A- 2	8.0	0.037	17.1	0.462
	A- 3	12.2	0.055	40.0	0.770
	A- 4	22.2	0.079	92.2	0.618
	A- 5	16.0	0.054	18.4	0.685
	A- 6	18.2	0.066	1.3	—
	A- 7	4.9	0.027	14.5	0.613
	A- 8	7.7	0.048	14.4	0.620
	A- 9	7.7	0.032	30.6	0.648
	A-10	9.4	0.077	13.3	—
Yura River Kado	K′- 1	23.2	0.017	26.0	0.300
	K′- 2	27.0	0.027	6.1	0.366
	K′- 3	66.0	0.136	1.4	—
	K′- 4	16.4	0.069	10.9	0.790
	K′- 5	14.6	0.042	22.0	0.524
	K′- 6	5.3	0.057	1.5	0.742
	K′- 7	29.3	0.067	16.1	0.847

The characteristic values obtained by the analyses in the natural rivers are shown in Table 2.

The latter discussion in this section will be observations upon the analysed data from rivers togather with the consideration of theoretical research.

3-3 Characteristics of the confined runoff component

The equation of the recession curve is given by Eq.'s (15′) and (16). The appearances of the recession characteristics of the confined runoff component are expressed synthetically by the recession coefficient α.

According to Eq. (16), that is $\alpha = fk'/Fl$, α consists of topological and geological factors f, F, k' and l, all of which may be considered constant in a river basin, so α will be constant for a basin and indicate the basin characteristics for the confined runoff component. In Fig. 4, we can see the relationship between the initial discharge Q_{c0} and the values of α obtained in the basin. This figure shows that α may be considered to be constant for any initial discharge, and α_0 in the figure denotes the mean value of α.

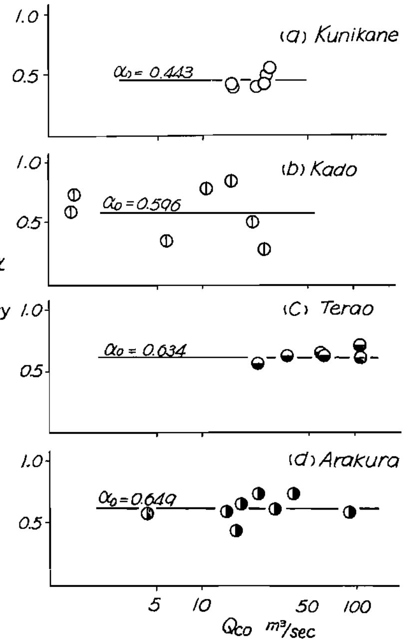
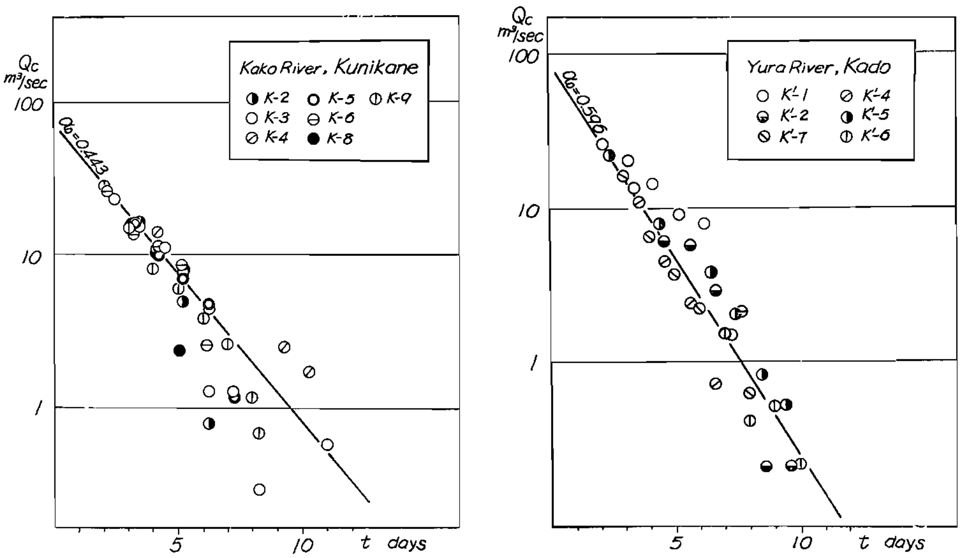
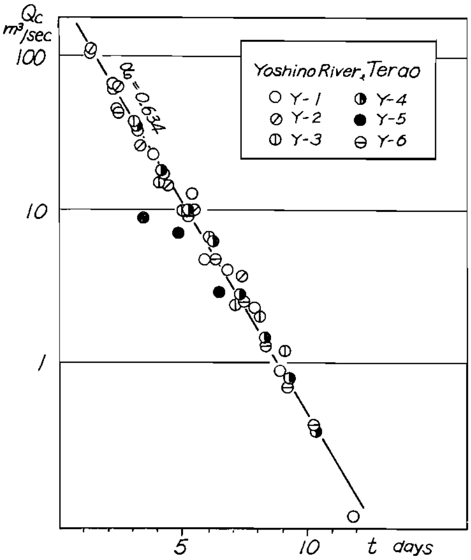


Fig. 4. The relationship between α and Q_{c0}.

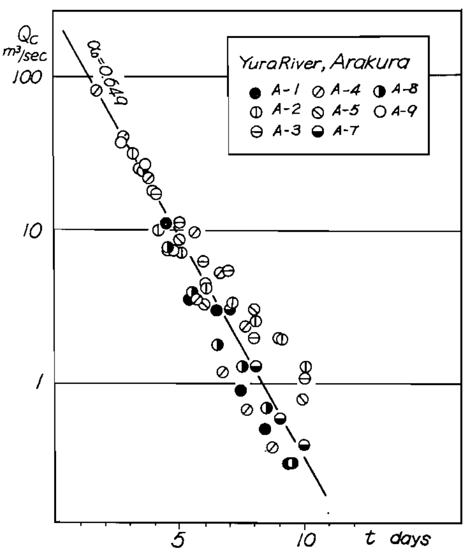
In a case where α is constant, the discharge due to the runoff of the confined component will decrease only along one curve in every case. This fact can be seen in Fig. 5, which shows the analysed discharge of confined components for several water basins, and the recession curve by the Eq. (15') for mean value α_0.

As mentioned in the preceding article, this component will decrease and vanish in a few days, but some times it will supply a considerable quantity of discharge in the early stage of re-





(c)



(d)

Fig. 5. The recession curve of the confined runoff component.

cession as seen Fig. 5 and Table 2.

3-4 Characteristics of the unconfined runoff component

The recession equation for the unconfined component is given by Eq.'s (13′) and (14), in which case, the parameter a expresses the recession characteristics. According to the Eq.'s (10) and (14), a depends on both of the geological factors γ, k, L and the initial state H_0 and h_0. Since the geological factors are constant for a river basin, we can consider a as a quantity defined by the initial condition. So there may exist several appearances of recession curves with respect to the difference of the initial states. On the other hand, the initial discharge Q_{u0} is also the function of H_0 and h_0. Therefore, it will be seen that there are some functional relationship between a and Q_{u0} by means of the parameter H_0 and h_0.

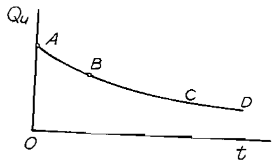


Fig. 6.

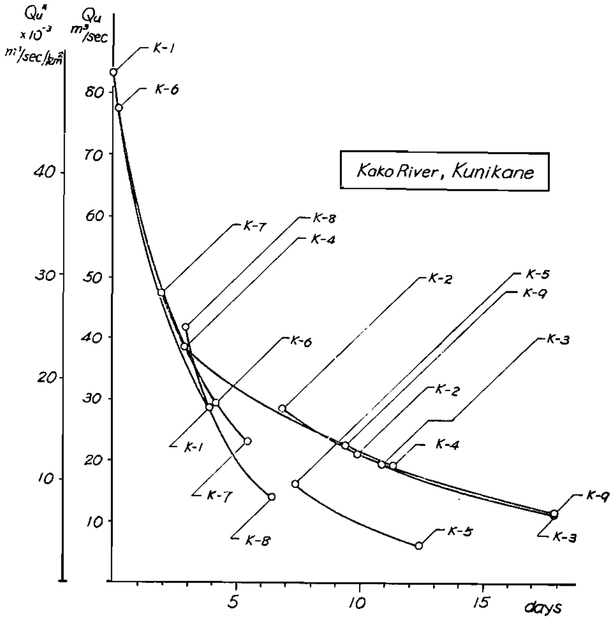
Now we assume that the discharge of the unconfined component decreases only along one curve for every initial state. That is to say, in Fig. 6 the discharge of which initial discharge is at point A, will decrease along the curve ABCD, and the other recession which starts from the point B is assumed to decrease along the curve BCD, and any recession will follow the same curve. Then we may deduce theoretically the relationship between a and Q_{u0} by Eq.'s (12), (13′) and (14) as follows

$$a=K\sqrt{Q_{u0}}, \tag{17}$$

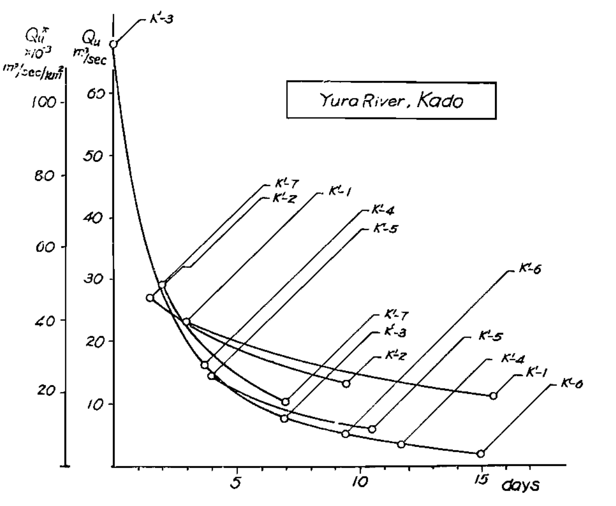
in which K is the constant for a river basin.

Fig. 7 shows the several recession curves analysed in river basins, and Fig. 8 shows the relationship between a and the initial discharge Q_{u0}.

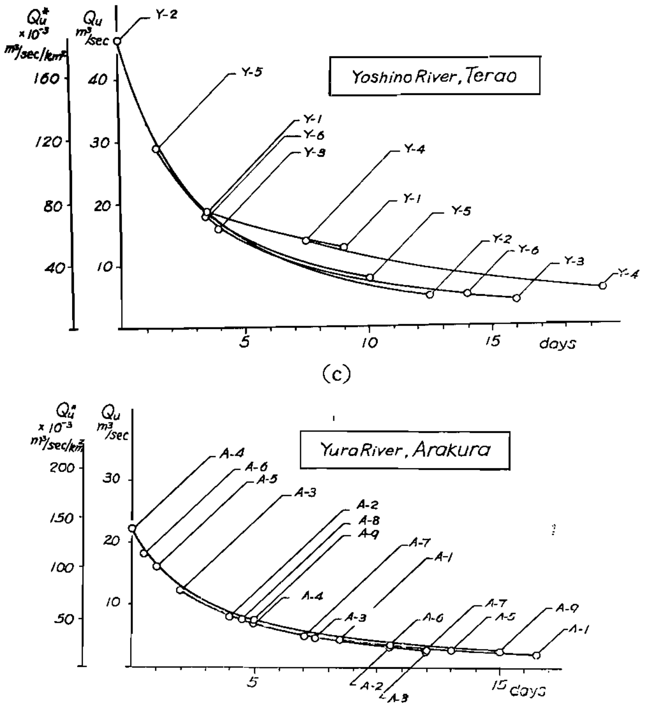
Since according to Fig. 7-(d) it seems that the recession curve may be expressed by only one curve for any initial state for the recession at Arakura gauging station where the drainage basin is mountaneous and its area is very small (159 km^2), we have the relationship of Eq. (17). This relationship is shown in Fig. 8. From Fig. 8, we may guess that the same tendencies occur in other basins too, however, the plots scatter and we cannot define the relationships in detail, especially if the area of the water basin is large whereupon the data be-



(a)



(b)



(d)
Fig. 7. The recession curve of the unconfined component.

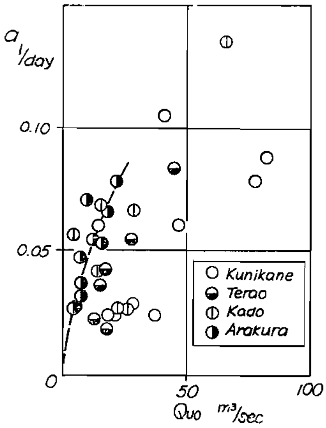


Fig. 8. The relationship between a and Q_{u0}.

comes much more scattered.

3-5 Some factors effecting the recession

From the discussion in the preceding articles, we find clearcut recession curves for both components in the data for small mountaneous basin, like Arakura. But in Fig. 4, 5, 7 and 8, it seems that there is much scattering of the plots for a large river basin.

Of course, the recession characteristics of low flow or ground-water runoff is mainly influenced by the physical characteristics of a basin. While differences in the regional distribution of rainfall changes the distribution of ground-water in a basin, so that rainfall distribution will also influence the recession characteristics.

In a large basin, for the sake of variety in the distribution of factors effecting the recession, there will appear various states or characteristics on the curve. But the actual conditions of these factors are very complicated, so it is very difficult to discuss in general the recession with due consideration to these factors.

In this article, we shall discuss some factors effecting the recession characteristics by making observations of the analysed data.

(i) the confined component : The result of analyses applied to rivers, shows that the mean value of α ; α_0 decreases with an increase of basin area as seen in Fig. 4. α is made up of F, f, k' and l. F and f are the assumed factors in the model to express the effect of pressure, and we cannot compare the data in several basins by means of these factors, nor are there any big differences in k' in these basins. Therefore, it may be assumed that the value of l will cause the differences among various values of α_0 in several basins.

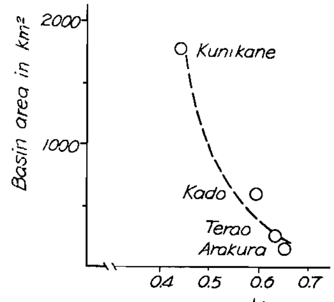


Fig. 9. The relationship between α_0 and the basin area.

We may see the tendency of α in relation to the basin area in Fig. 9. Now, assume the value l is proportional to the basin area, then the value α is inversely proportional to the value of l by means of Eq. (16) $\alpha=fk'/Fl$. We may explain the tendency in Fig. 9 in some extent by this consideration.

(ii) the unconfined component : The effects of rainfall on the recession of the unconfined component, may be seen in the analysed data at Kado and Terao gauging stations. In the case of data K′-3, 4, 5, 6 and 7, the daily rainfall distributes homogeneousely from the downstream region to the upstream region. On the other hand, the daily rainfall for the datum K′-1, concentrates at the mountaneous region upstream in the basin. While the recession for the datum K′-1 decreases much more gently by comparison with those of the others as seen in Fig. 7-(b).

We may see such a tendency at Terao in Yoshino river basin, that the recession rate becomes much more gentle when the rainfall concentrates at the upstream region. By comparison of the data Y-4 and Y-5, Fig. 7 shows that the recession rate for Y-5 is steeper than that of Y-4. The rainfall for Y-5 distributes homogeneousely all over the basin, but for Y-4 it concentrates in the upstream region. From these tendencies, therefore, we may conclude that the recession rate of runoff from the upstream region is gently and that from the downstream region it is steep. It regrets that the data of rainfall for K′-2 and Y-1 lack, of which recessions are as gentle as those of K′-1 and Y-4.

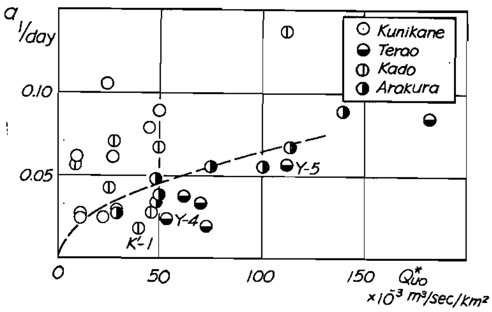


Fig. 10. The relationship between a and the initial discharge per unit area.

In Arakura and Terao basins, which are also mountaneous with basin areas nearly equal to each other, it seems that the appearances and characteristics of the runoff are similar to each other. We can see this similarity in Fig. 7 and Fig. 10, in which Q^*_u denotes the discharge of the unconfined component per unit area in the basin, and subscript 0 denotes the initial state.

Moreover, Fig. 10 suggests, if the basin area is small, a is small for the same discharge per unit area of basin, that is, the recession rate is gentle. Although we may guess these tendencies depend on the differences between the runoff from the upstream mountaneous region and that from the plains down-stream, we cannot clarify the causes of the tendencies in this stage of investigation.

4. Characteristics of Variation of Low Flow due to Rainfall

The process of seepage of rainfall through strata from the ground surface to the ground water table is an unsaturated process and is very complicated. There are many problems unsolved, for instance the variation of moisture content, exchange between liquid water and void air etc, and today we cannot describe the phenomena in detail.

In the last section, we clarify the several characteristics of the confined and unconfined runoff components during no-rainfall periods. It may be considered that the characteristics of these components will reflect the behaviours of the increments or variations of these components due to rainfall.

4-1 Recharge regimes

By application of the recession curve, we can see the macroscopic features of the increment of runoff discharge and ground-water storage due to rainfall.

(i) increment of the runoff discharge : Fig. 11 shows schematically the variation of hydrograph due to rainfall. From rainfall ground-water runoff will rise along the broken line B′-C in this figure, and the precedent recession curve I will be replaced by the recession curve II. Now, let the recession characteristics of these two curves be α, Q_{c0}, a, Q_{u0}, obtained at $t=t_c$ as the origin of time for curve II, and α', Q'_{c0}, a', Q'_{u0} at $t=0$. The prime denotes the value for the precedent curve I.

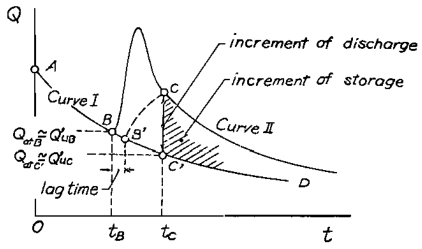


Fig. 11.

If there is no rainfall, the recession must decrease along the curve $ABC'D$. Denote the confined and unconfined runoff discharge for the point C' by Q'_{cO} and Q'_{uO}, then the increment of the discharge ΔQ_u, ΔQ_c for these two components at the time $t=t_c$, may be easily calculated by the following equations.

For the unconfined component :

$$\Delta Q_u = Q_{u0} - Q'_{uO} \tag{18}$$

For the confined component :

$$\Delta Q_c = Q_{c0} - Q'_{cO}. \tag{19}$$

If the recession period for curve I is sufficiently long, $Q'_{cO} \cong 0$ and we have

$$\Delta Q_c \cong Q_{c0} \tag{19'}$$

(ii) increment of ground-water storage : For both components, the total amount of runoff which will flow out to the stream in future, may be given by the integrals of the recession curve from $t=t$ to $t=\infty$. The total amount may be considered as the ground-water storage in the basin at that time $t=t_c$. Therefore, the increment of the storage due to rainfall is approximately given by

$$\Delta S_u=\int_0^\infty \frac{Q_{u0}}{(at+1)^2}dt-\int_0^\infty \frac{Q'_{u0}}{(a't+1)^2}dt=\frac{Q_{u0}}{a^2}-\frac{Q'_{u0}}{a'^2}, \tag{20}$$

for the unconfined component, and

$$\Delta S_c=\int_0^\infty Q_{c0}e^{-\alpha t}dt-\int_0^\infty Q'_{c0}e^{-\alpha' t}dt=\frac{Q_{c0}}{\alpha}-\frac{Q'_{c0}}{\alpha}, \quad (\because\ \alpha=\alpha') \tag{21}$$

for the confined component. In general, ΔS_c is negligible small in comparison with ΔS_u, and we may express the increment of storage approximately by ΔS_u.

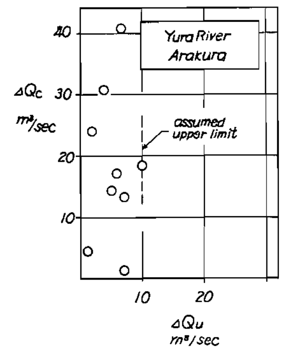


Fig. 12. The relationship between ΔQ_c and ΔQ_u.

In order to calculate the increment of discharge and storage by the above equations, it should be emphasized that the recession analysis must be clarified definitely, because a small error in the recession analysis may cause a great error in estimation of the increment. Therefore, in this section, we will treat only the data at Arakura in Yura river basin, of which recession characteristics are clarified in the last section.

(iii) recharge regimes: The results of analysis applied to the rivers are summarized in Table 3. Fig. 12 shows the relationship between ΔQ_c and ΔQ_u. This figure suggests that there is an upper limit for ΔQ_u, but on the contrary, ΔQ_c may reach considerably large amount.

TABLE 3.
The results of analyses applied to the rivers.

Data	Increment of discharge		Increment of storage ΔS_u $\times 10^4$m^3	Rainfall		Discharge of the precedent recession curve, at the point B in Fig. 11 Q'_{uB}, m^3/sec
	ΔQ_c m^3/sec	ΔQ_u m^3/sec		Duration time T days	Mean daily rainfall R_m mm/day	
A- 1	24.0	1.8	253	1	26.0	3.3
A- 2	17.1	5.8	405	4	30.2	2.6
A- 3	40.6	6.6	608	5	40.8	7.8
A- 4	92.2	—	—	—	—	—
A- 5	18.4	10.0	—	—	—	7.8
A- 6	1.3	7.1	531	5	14.3	15.5
A- 7	4.5	1.1	—	—	—	4.6
A- 8	14.4	4.9	607	4	28.3	3.7
A- 9	30.6	3.8	152	2	73.0	4.4
A-10	13.3	6.9	810	9	23.2	3.0

In general, we may consider that the regime of recharge to the groundwater table may be classified into the following categories.

(a) the recharge by infiltration or seepage through the ground,

(b) the recharge by the waterflow through the cracks, chinks and orifices in the ground, which will be somewhat similar to that of open- and closed-channel flows.

Since the unconfined ground-water seems to distribute homogeneously all over the basin, the recharge to it depends mainly on the infiltration of rainfall through the ground. This will be suggested by the upper boundary for the unconfined component as seen in Fig. 12. On the other hand, a confined aquifer may be considered to have the catchment area for itself, and a certain part of the rainfall reaching this area supplies the confined ground water. But we cannot see clearly the recharge regime itself.

The following problems should be considered as most important, in regard to the basinwide infiltration of rainfall or soil water in the ground.

(a) the intensity of infiltration through the ground, and that of recharge to the ground-water table.

(b) the time lag expected for water movement from the ground surface to the ground-water table.

(c) the ability of the soil or layer to hold the water.

On these problems, we have several suggestions from some other studies. Making measurement of the variation of soil moisture in the field, Dreibelbis has clarified that the layer of 10–20 inches deep beneath the ground surface plays an important role in the runoff[9]. His studies suggest indirectly that the lag-time is not so long and the recharge intensity to the ground water table may be considered constant. Recently Prof. Takasao[10] and the others[11] have given some suggestions on infiltrations. From some results made by them[11], we can obtain the time expected for the infiltration intensity to reach nearly the final infiltration intensity at Yura river basin as follows.

TABLE 4.

The precedent norainfall days.	The time expected for the infiltration intensity to reach nearly the final infiltration intensity at Yura river basin
4 days	0.89 day
2–4	0.84
1–2	0.78

Therefore, we may use 1 day for the lag time and that the infiltration intensity is almost constant in the discussion of low flow.

On the other hand, it is very difficult to define the behaviour of the recharge through cracks, chinks, and orifices in the ground because there are but few studies on these mechanisms. However, since phenomenological consideration suggests that the lag time is shorter than that of infiltration, we consider that the lag time is 1 day in this case too and assume that the recharge intensity to the confined stratum is also constant. This will be discussed later in this section.

When we assume the recharge regime as mentioned above, the increment of discharge and the storage due to rainfall will be influenced mainly by

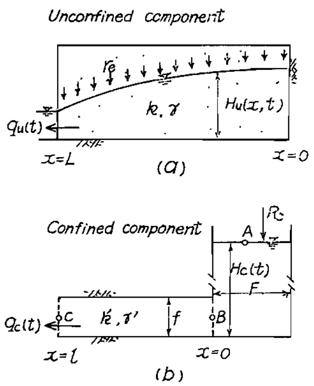


Fig. 13. Runoff models.

the duration time of recharge. It does not always follow that the duration time of recharge corresponds with that of rainfall or precipitation. But there must be a close relationship between them, and we can guess that the intensity of infiltration will become very small after the ceasing of rainfall. Therefore, in the discussion we adopt the duration days of rainfall for those of recharge.

4-2 Fundamental equations and their solutions

(i) the confined component: Denote the constant recharge intensity by R_c in Fig. 13-(b), then we have the following equations for the water movement in the confined aquifer.

$$v=k'h/l, \tag{22}$$

$$H_c+\frac{1}{2g}\left(\frac{dH_c}{dt}\right)^2=h+\frac{v^2}{2g}, \tag{23}$$

$$F\frac{dH_c}{dt}=R_c-fv. \tag{24}$$

In Eq. (23) we may ignore the term $(dH_c/dt)^2$ in comparison with the others similar to the case of section 3. Therefore, from these three equations we have the equation

$$F\left\{\frac{l}{k'}+\frac{v}{g}\right\}\frac{dv}{dt}=R_c-fv. \tag{25}$$

And we obtain the solution for the discharge $q_c=fv$ through the confined aquifer as follows

$$-\frac{F}{f}\left(\frac{l}{k'}+\frac{R_c}{fg}\right)\log_e(R_c-q_c)+F\frac{R_c-q_c}{f^2g}=t+C_1, \quad \text{for} \quad fv_0<R_c, \tag{26}$$

$$-\frac{F}{f}\left(\frac{l}{k'}+\frac{R_c}{fg}\right)\log_e(q_c-R_c)+F\frac{q_c-R_c}{f^2g}=t+C_2, \quad \text{for} \quad fv_0>R_c, \tag{27}$$

in which C_1 and C_2 are the integral constants given by the initial condition, and v_0 is the initial velocity through the aquifer. In these equations, we can ignore the 2nd term by comparing it with the 1st term in the left hand side. So, we may rewrite the Eq.'s (26) and (27) into the following one.

$$-\frac{F}{f}\left\{\frac{l}{k'}+\frac{R_c}{fg}\right\}\log_e\left|\frac{R_c-q_c}{R_c-q_{c0}}\right|=t, \tag{28}$$

that is,

$$q_c=R_c\{1-e^{-\delta t}\}+q_{c0}\cdot e^{-\delta t}, \tag{29}$$

in which

$$\delta=\frac{f}{F}\frac{1}{\dfrac{l}{k'}+\dfrac{R_c}{fg}}, \tag{30}$$

and 0 denotes the initial condition for the rising limb of discharge due to ground-water runoff, i.e. for point B' in Fig. 11.

(ii) the unconfined component: In this case, at first, we assume that the recharge rate is given by $r_e(t)$ and the inclination of water surface is very small, i.e. $(\partial H_u/\partial x)^2\cong 0$. Then the fundamental equation becomes

$$\frac{\partial H_u}{\partial t}=\beta H_u\frac{\partial^2 H_u}{\partial x^2}+\alpha r_e(t), \tag{31}$$

where

$$\alpha=\frac{1}{\gamma},\quad \beta=\frac{k}{\gamma}. \tag{32}$$

For this equation, the authors use the following conditions.

$$H_u(x, 0)=f(x), \tag{33}$$

$$\left.\frac{\partial H_u}{\partial x}\right|_{x=0}=0, \tag{34}$$

$$H_u(L, t)=h_0'. \tag{35}$$

In actual conditions, the water stage in the river or stream may vary rapidly, so it is not constant. However, since violent variation of the stream water stage is limited to a short period, it is assumed here that h_0' expresses synthetically the mean water stage during the rising state in the stream.

Now we write

$$H_u(x, t)=H_0'+h(x, t), \tag{36}$$

in which

$$H_u(0, 0)\equiv H_0'. \tag{37}$$

If we assume

$$H_0'\gg h(x, t), \tag{38}$$

then the fundamental equation becomes

$$\frac{\partial h}{\partial t}=\beta H_0'\frac{\partial^2 h}{\partial x^2}+\alpha r_e. \tag{39}$$

And the conditions may be rewritten as follows,

$$h(x, 0)=f(x)-H_0'=F(x), \tag{40}$$

$$h(L, t)=h_0'-H_0', \tag{41}$$

$$\left.\frac{\partial h}{\partial x}\right|_{x=0}=0. \tag{42}$$

Here we may separate the unknown function $h(x, t)$ into two parts.

$$h(x, t)=h_1(x, t)+h_2(x, t), \tag{43}$$

in which h_1 and h_2 satisfy the following equations respectively,

$$\left.\begin{aligned} &\frac{\partial h_1}{\partial t}=\kappa^2\frac{\partial^2 h_1}{\partial x^2}, \\ &h_1(x,0)=F(x), \\ &\frac{\partial h_1}{\partial x}\Big|_{x=0}=0, \\ &h_1(L,t)=h_0'-H_0'. \end{aligned}\right\} \quad \text{for } h_1, \tag{44}$$

$$\left.\begin{aligned} &\frac{\partial h_2}{\partial t}=\kappa^2\frac{\partial^2 h_2}{\partial x^2}+R(t), \\ &h_2(x,0)=0, \\ &\frac{\partial h_2}{\partial x}\Big|_{x=0}=0, \\ &h_2(L,t)=0. \end{aligned}\right\} \quad \text{for } h_2, \tag{45}$$

where

$$\kappa^2=\beta H_0', \quad \text{and} \quad R(t)=\frac{1}{\gamma}r_e=\alpha r_e.$$

The solutions of these equations become

$$\begin{aligned} h_1(x,t)=h_0'-H_0'+\frac{2}{L}\sum_{s=0}^{\infty}\Big[&\exp\Big\{-\frac{\kappa^2(2s+1)^2\pi^2}{4L^2}t\Big\}\cos\Big\{\frac{(2s+1)\pi x}{2L}\Big\} \\ &\times\Big\{\int_0^L F(\lambda)\cos\frac{(2s+1)\pi\lambda}{2L}d\lambda-(-1)^s\frac{2L(h_0'-H_0')}{(2s+1)\pi}\Big\}\Big], \end{aligned} \tag{46}$$

$$\begin{aligned} h_2(x,t)=\frac{2}{L}\sum_{s=0}^{\infty}\Big[&\cos\frac{(2s+1)\pi x}{2L}\int_0^L\exp\Big\{-\frac{\kappa^2(2s+1)^2}{4L^2}t\Big\} \\ &\times\Big\{C+\int R(t)\exp\Big(\int\kappa^2\frac{(2s+1)^2}{4L^2}dt\Big)dt\Big\}\cos\frac{(2s+1)\pi\lambda}{2L}d\lambda\Big]. \end{aligned} \tag{47}$$

From these solutions, we may calculate the runoff discharge $q_u(t)$ by the expression

$$q_c=k\cdot h_0'\Big(\frac{\partial H_u(x,t)}{\partial x}\Big)_{x=L}. \tag{48}$$

Assume r_e is constant, then we have

$$\begin{aligned} q_u(t)=\frac{2k}{L}h_0'\sum_{s=0}^{\infty}\Big[&\exp\Big(\frac{-\kappa^2(2s+1)^2\pi^2}{4L^2}t\Big)\frac{(2s+1)\pi}{2L}(-1)^s \\ &\times\Big\{\int_0^L f(\lambda)\cos\frac{(2s+1)\pi\lambda}{2L}d\lambda-(-1)^s\frac{2L(h_0'-H_0')}{(2s+1)\pi}\Big\}\Big] \\ +\frac{2k}{L}h_0'\sum_{s=0}^{\infty}\Big[&\exp\Big\{-\frac{\kappa^2(2s+1)^2\pi^2}{4L^2}t\Big\}\frac{4L^2}{\kappa^2(2s+1)^2\pi^2}(-1)^s \\ &\times R\Big\{\exp\Big(\frac{\kappa^2(2s+1)^2\pi^2}{4L^2}t\Big)-1\Big\}\Big]. \end{aligned} \tag{49}$$

As seen by the Eq. (44), $h_1(x, t)$ describes the variation of water depth in the model during no-rainfall period, namely it corresponds to that of the recession state. And the 1st term in the right hand side in Eq. (49) is introduced by means of $h_1(x, t)$. Therefore the 1st term expresses the discharge for the recession state and the increment of discharge due to constant recharge is given by the 2nd term, i.e.

$$\Delta q_u = \frac{2k}{L} h_0' \sum_{s=0}^{\infty} \frac{4L^2}{\kappa^2(2s+1)^2\pi^2} R\left\{1-\exp\left(-\frac{\kappa^2(2s+1)^2\pi^2}{4L^2}t\right)\right\}, \tag{50}$$

that is

$$\Delta q_u = \frac{8}{\pi}\frac{h_0'}{H_0'} r_e L \sum_{s=0}^{\infty} \frac{1}{(2s+1)^2}\left\{1-\exp\left(-\frac{kH_0'(2s+1)^2\pi^2}{4\gamma L^2}t\right)\right\}. \tag{51}$$

In this series, the 1st term is dominant, and the other terms are 1/10 or much less in order of the 1st term. So, the increment of discharge may be approximately written as follows.

$$\Delta q_u = \frac{8}{\pi}\frac{h_0'}{H_0'} r_e \cdot L\left\{1-\exp\left(-\frac{H_0' k\pi^2}{4L^2\gamma}t\right)\right\}. \tag{52}$$

4-3 Approximate expressions

In the discussion of variations due to rainfall, the initial state is taken at the day when the recharge begins. It should be noted that the state at point B' in Fig. 11 is taken for the initial state in this section.

(i) the confined component: As known by Eq. (29), if the aquifer is supplied by constant recharge, we have

$$\left.\begin{aligned} q>q_0 &\quad \text{for} \quad R_c>q_0, \\ q<q_0 &\quad \text{for} \quad R_c<q_0, \end{aligned}\right\} \tag{53}$$

for any time as far as recharge continues.

The discharge will continue to increase or decrease unceasingly and reach a steady state if the recharging continues. The variation rate of discharge is expressed by the parameter δ. F, f, l and k' may be considered constant in a river basin, it seems, therefore, that δ is a function of R_c only according to Eq. (30). But k' is of the order $10^{-6}\sim10^{-7}$ (km/day), g: 10^8 (km/day^2) and R_c: 1 (km^3/day/km), then we can ignore R_c/fg for l/k', and we may exchange α into δ in the approximation, that is

$$\delta \simeq fk'/Fl \equiv \alpha.$$

This quantity α corresponds to the value for $R_c=0$, namely the recession rate.

Therefore, the Eq. (29) becomes approximately

$$q_c = R_c(1-e^{-\alpha t}) + q_{c0}\cdot e^{-\alpha t},$$

Considering the effect of the basin area B, we have

$$Q_c = R_c B(1-e^{-\alpha t}) + Q_{c0}e^{-\alpha t},$$

in which Q_{c0} denotes the initial discharge at the time when the recharge begins. In this equation the 2nd term in the right hand side shows the recession curve, so that the increment of discharge due to rainfall may be written

$$\Delta Q_c = R_c B(1 - e^{-\alpha t}). \tag{55}$$

(ii) the unconfined component:
Since

$$\Delta q_u \rightarrow \frac{8}{\pi} \frac{h_0'}{H_0'} r_e L \qquad \text{when } t \rightarrow \infty,$$

Eq. (52) does not satisfy the equation of continuity, and this error may be considered to be caused by several assumptions. So correcting the coefficient, Eq. (52) will be rewritten as follows,

$$\Delta q_u = r_e L\left[1 - \exp\left(-\frac{k}{\gamma} \frac{\pi^2 H_0'}{4L^2}\right)\right], \tag{56}$$

which satisfies the equation of continuity. Moreover in introducing the factor B' expressing the basin area, we have

$$\Delta Q_u = r_e B' L[1 - \exp(-\varepsilon t)], \tag{57}$$

where

$$\varepsilon = \frac{k}{\gamma} \frac{\pi^2 H_0'}{4L^2}. \tag{58}$$

4-4 Variation of the confined runoff component due to rainfall

In the case of confined ground water, we may propose that the recharge is not only due to the infitration but also the water flow through cracks, chinks and orifices in the ground. So, the recharge intensity may vary with the intensity of rainfall to some extent.

If the recharge intensity is constant during the recharge, the increment of discharge may be given by Eq. (55). We can calculate the intensity of recharge $R_c B$ by using the duration time of rainfall T, Eq. (55) and the values of α and Q_c defined by the recession analysis. Fig. 14 shows the relation between the value of $R_c B$ and the mean daily rainfall during the

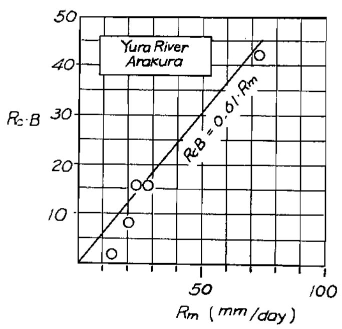


Fig 14. The relationship between the calculated value of $R_c B$ and the mean daily rainfall R_m.

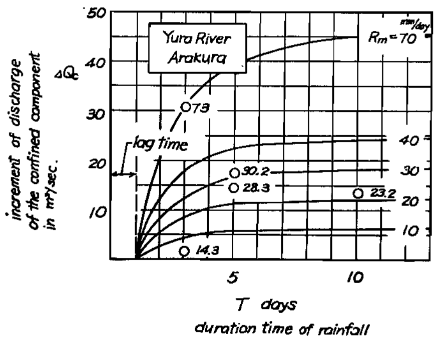


Fig. 15. The relationship between the increment of discharge of the confined component and the duration time of rainfall T. The numbers in the figure denote the mean daily rainfall R_m.

rainy period. It seems that there is a proportional relationship between these two factors, namely that the recharge intensity will vary in proportion to the rainfall intensity R_m.

If we describe the tendency between them by the straight line in Fig. 14, we have

$$R_c B = 0.61 R_m. \tag{59}$$

And we may write Eq. (55) as follows

$$\Delta Q_c = 0.61 R_m (1 - e^{-at}). \tag{55'}$$

From this equation we may estimate the increment of discharge of the confined component due to rainfall. The family of curves in Fig. 15 shows the increment of discharge by the calculation above mentioned for the parameter R_m. And the dotted plots show the increment for the data at Arakura.

The number of data in Fig. 14 and Fig. 15 is so small that we can not clarify definitely the relationship between the increment of discharge, the recharge intensity, the rainfall intensity and the duration time of rainfall. But comparing it with the characteristics of the unconfined one discussed in the next article, it is very interesting that the recharge intensity has a clear relation to the rainfall intensity, and we may calculate the increment of discharge to some extent by means of the rainfall intensity and the duration time of rainfall by defining the proportionality as seen in Fig. 14.

4-5 Variation of the unconfined runoff component due to rainfall

In this case, Fig. 16 shows the relationship between the duration time of rainfall and the increment of storage for the unconfined component given by Eq. (20). The plots scatter to some extent but we can discern the tendency for the increment of storage to increase in proportion to the duration time of rainfall. That is to say the recharge intensity will be constant for any rainfall condition in a river basin, as mentioned in the article 4-1. The increment of discharge of the unconfined component, therefore, may be expressed by Eq. (57), and we see that the main factor governing the increment is the duration time of rainfall.

The value ε in Eq. (57) consists of the geological factors k, γ, L and the initial condition H_0', as seen in Eq. (58). But if the unconfined

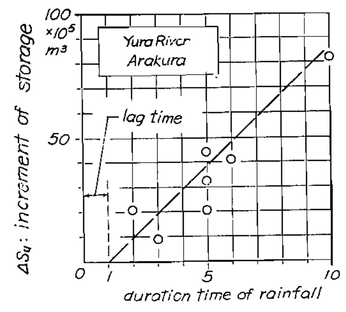


Fig. 16. The relationship between the increment of discharge and the duration time of rainfall.

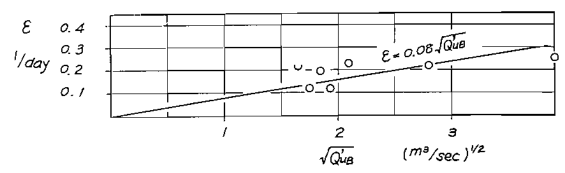


Fig. 17. The relationship between the value of ε and the square root of Q'_{uB}.

component decreases along one curve for any rainfall conditions as seen at Arakura, by Eq'.s (12), (13) and (14) on the recession curve I in Fig. 11, we will have the relationship between Q'_{uB} and H_0' as follows.

$$\sqrt{Q'_{uB}} \propto H_0'$$

So, the value of ε, should be proportional to the square root of Q'_{uB}. Now, if we assume the upper limit of the increment of discharge at Arakura is 10 m^3/sec. by Fig. 12, and we have

$$\Delta Q_u = 10(1 - e^{-n\sqrt{Q'_{uB}}\cdot t}).$$

Then we may calculate the values ε for the data by means of the duration time and the increment of Q_u. The results of calculation is seen in Fig. 17. In this figure we see that the value ε is proportional to $\sqrt{Q'_{uB}}$, and assuming this relation from the straight line in the figure

$$\varepsilon = 0.08\sqrt{Q'_{uB}},$$

then Eq. (60) becomes

$$\Delta Q_u = 10\{1 - \exp(-0.08\sqrt{Q'_{uB}}\cdot T)\}.$$

Fig. 18 shows the data analysed at Arakura and the increment of discharge obtained by Eq. (60) for several initial discharges. Though the plotted data are not sufficient, the result suggests that we may deduce the tendency of increment of discharge to some extent by the initial condition say Q'_{uB} and the duration time of rainfall T.

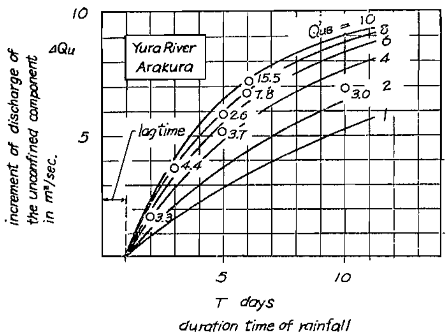


Fig. 18. The relationship between the increment of discharge of the unconfined component and the duration time of rainfall T. The number in the figure denotes the value of Q'_{uB}.

5. Conclusion

In this paper, we discuss the various characteristics of low flow from the view of kinematical mechanism and it is clarified that the low flow may be characterized by the runoff components which are due to seepage through confined strata and the leakage through unconfined strata. We have derived theoretically the expressions for variation of low flow, as expressed in the recession equations and the equations for increment due to rainfall. The results obtained through this research are as follows.

(A) On the recession characteristics:

(i) The recession curve of confined component may be expressed by the exponential function, the recession coefficient α is costant for a river basin and it expresses synthetically the geological characteristics of that basin. On the other hand, the recession curve of the unconfined component is expressed by the reciprocal function, and the recession rate is much more gentle than that of the confined one. The low flow during an extended period, therefore, is mainly supplied by the unconfined runoff component.

(ii) Introducing the unconfined component of which the mechanism is the same as diffusion, we may express in accurately the behaviour of low flow during extended periods.

(iii) In a small river basin, the behaviour of the recession of both components does not scatter, and in such small basins, we introduce theoretically the parabolic relationships between the recession coefficient a and the initial discharge Q_{u0}.

(iv) Although, through this analysis, we can ascertain some relationships between α, a and the basin area, these relationships are vague in regared to the data of large basin.

(B) On the variation due to rainfall:

(i) From the results of theoretical analysis, we introduce the equation for the increment of discharge due to the recharge with constant intensity.

(ii) By means of recession analysis, we calculate the increments of discharge and storage. In the case of confined strata, the intensity of recharge varies with that of rainfall and we clarify the proportional relationships between them by the application of theoretical research, and discuss the increment of discharge by means of the equation obtained. The theoretical research agrees with the observed data fairly well.

On the contrary, for the unconfined component, the increment of storage shows that the recharge intensity is always constant for any rainfall condition. The analysed result suggests that the behaviour of discharge may be expressed by theoretical equation.

For both components, the duration time of rainfall plays an important role in the variation of discharge and storage.

In this research, the runoff model itself and discussion exclude mathematical complications, the complexity of many factors effecting ground-water runoff and the lack of natural data. Hence, of course, it is not sufficient to clarify the natural behaviour of low flow in detail from this analysis. But the clear result as seen in a small basin suggests that for low flow we may divide a basin into small unit basins and discuss the phenomena in a large basin as a combination of those in the unit basins.

Moreover there are many problems yet unsolved, and we have a research programme to study synthetically low flow in near future.

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