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A Recording Water Tube Tiltmeter

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Synopsis

To study the volcanic crustal deformations in the vicinity of Volcano Sakura-jima, one component recording water tube tiltmeter was installed in the underground tunnel at the Hiyamizu observation station which belongs to the Sakura-jima Volcanological Observatory.

The recording methods of the apparatus contain two groups: instruments with visual point readings by micrometers, and the instruments with continuous recordings by the optical method.

A hardend vinyl chloride tube with an inside diameter of 16 mm and 45 meters in length serves as the water tube.

The sensitivity of the water tube tiltmeter on the recording photographic paper can be changed to 0.0069 second/mm, 0.011 second/mm and 0.013 second/mm by the diameter of the pulley.

Good results have been obtained when this water tube tiltmeter was set in the underground tunnel where the effects of the atmospheric pressure gradient and the temperature change were few.

1. Introduction.

In order to record continuous information on crustal deformations, some kind of routine instrumental observation is necessary. For this purpose, tiltmeters and extensometers have been developed. There are two types of tiltmeter: the horizontal pendulum type and the water tube type. Horizontal pendulum type tiltmeters have the advantages of being compact and highly sensitive, but they are not suitable for long term observation of slow secular variations because of the flow of the zero-line. Although requiring much space, water tube type tiltmeters which measure the respective elevations of points of the earth's surface are indispensable for the measurement of secular tilting movements of the earth's crust. Hence, we began to develop the recording water tube tiltmeter with a long distance between two water reservoirs installed at a certain depth under the ground.

2. Dynamic characteristic of the water tube tiltmeter.

2-1 Equation of motion of the recording water tube tiltmeter.

Subsequent to the development and construction of the recording water tube tiltmeter herein described, it was brought to the writer's attention that the basic principle had been used previously by T. Hagiwara[1] and the theoretical problem of the behavior of the water tube tiltmeter had been discussed with J. P. Eaton.[2]

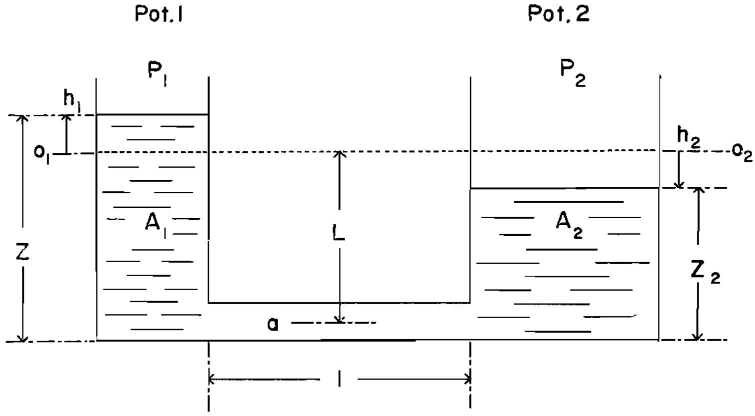

Fig. 1. Diagram of the water tube tiltmeter : movement of water in the system.

We shall now consider the equation of motion of the water surface in the water tube tiltmeter (Fig. 1). In the following discussion it is assumed that the liquid which fills the water tube tiltmeter is considered to be ideal, homogeneous, frictionless, continuous and incompressible.

From Euler's equation, we can write

$$\frac{\partial}{\partial s}\left(\frac{1}{2}\cdot V^2\right)+\frac{1}{\rho}\cdot\frac{\partial P}{\partial s}+g\cdot\frac{\partial z}{\partial s}+\frac{\partial V}{\partial t}=0$$

where bodily force is conservative and

$V(s)$: velocity of the liquid.
ds : element of a streamline.
P : external pressure.
z : height above an arbitrary datum.
g : acceleration of gravity.
ρ : density of the liquid.

The line integral of Euler's equation along a streamline from Pot 1 through the water tube to Pot 2 can be written

$$\frac{1}{2}\left(V_2{}^2-V_1{}^2\right)+\frac{1}{\rho}(P_2-P_1)+g(Z_2-Z_1)+\frac{dV_1}{dt}\int_1^2\frac{A_1}{A(s)}\cdot ds=0 \quad\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where the subscripts 1 and 2 indicate the values of the variables at Pot 1 and Pot 2, respectively. Figure 1 shows the diagram of the recording water tube tiltmeter,

where A_1, A_2 : cross-sectional areas of Pot 1 and Pot 2.
a : cross-sectional area of the water tube.
l : length of the water tube.
L : height of the water surfaces in the two pots above the center of the water tube after equilibrium has been established.
Z_1, Z_2 : heights above datum of the water surface in Pot 1 and Pot 2.
h_1, h_2 : heights of the water surfaces in Pot 1 and Pot 2 above its equi-

librium position.

We can calculate the terms in equation (1) as follows,

$$\frac{dV_1}{dt}\int_1^2 \frac{A_1}{A(s)}\cdot ds = \left(L+\frac{A_1}{a}l+\frac{A_1}{A_2}L\right)\frac{dV_1}{dt}$$

$$g\cdot(Z_2-Z_1) \qquad = -g\cdot(h_1+h_2)$$

$$= -g\cdot\left(1+\frac{A_1}{A_2}\right)h_1$$

$$\frac{1}{2}(V_2{}^2-V_1{}^2) \qquad = -\frac{1}{2}\left(1-\frac{A_1{}^2}{A_2{}^2}\right)\left(\frac{dh_1}{dt}\right)^2$$

If we assume that atmospheric pressures on the water surfaces of the two pots are the same,

$$\frac{1}{\rho}(P_2-P_1)=0$$

then, equation (1) becomes

$$\left(L+\frac{A_1}{a}l+\frac{A_1}{A_2}L\right)\frac{dV_1}{dt}-\frac{1}{2}\left(1-\frac{A_1{}^2}{A_2{}^2}\right)\left(\frac{dh_1}{dt}\right)^2-g\left(1+\frac{A_1}{A_2}\right)h_1=0$$

Multiplying this equation by ρ and setting

$$V_1=-\frac{dh_1}{dt}=-\frac{dh}{dt}$$

we have

$$\rho\left(L+\frac{A_1}{a}l+\frac{A_1}{A_2}L\right)\frac{d^2h}{dt^2}+\frac{\rho}{2}\left(1-\frac{A_1{}^2}{A_2{}^2}\right)\left(\frac{dh}{dt}\right)^2+\rho g\left(1+\frac{A_1}{A_2}\right)h=0 \qquad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

This is the equation of motion of a frictionless fluid in the water tube tiltmeter.

But in reality, the liquid which fills the water tube being viscous, we cannot neglect the pressure required to overcome the internal friction in the water moving through the tube.

From the Hagen-Poiseuille's law this pressure p is written

$$p=8\eta l\cdot\frac{u_m}{r^2}$$

where η is the viscosity of water, l is the length of the water tube, r is the radius of the tube and u_m is the average velocity of water in the tube.

Here

$$u_m=\frac{A_1}{a}\cdot\frac{dh_1}{dt}=\frac{A_1}{a}\cdot\frac{dh}{dt}$$

Substituting the value of u_m into the Hagen-Poiseuille's law, we find

$$p=\frac{8\pi\eta l A_1}{a^2}\cdot\frac{dh}{dt}$$

Including this term in equation (2)

$$\rho\left(L+\frac{A_1}{a}l+\frac{A_1}{A_2}L\right)\frac{d^2h}{dt^2}+\frac{\rho}{2}\left(1-\frac{A_1^2}{A_2^2}\right)\left(\frac{dh}{dt}\right)^2+\frac{8\pi\eta lA_1}{a^2}\cdot\frac{dh}{dt}+\rho g\left(1+\frac{A_1}{A_2}\right)h=0 \qquad \cdots\cdots(2')$$

In the recording water tube tiltmeter we are now planning,

$$L\ll l, \quad a\ll A_1,\ A_2 \ \text{ and } \ A_1\ll A_2$$

therefore,

$$L+\frac{A_1}{a}l+\frac{A_1}{A_2}L\doteqdot\frac{A_1}{a}l \qquad \text{approximately}$$

The coefficient of $\left(\frac{dh}{dt}\right)^2$ is relatively small, we neglect this term. Then, the equation (2′) becomes

$$\frac{d^2h}{dt^2}+\frac{8\pi\eta}{\rho a}\cdot\frac{dh}{dt}+\frac{ga}{A_1l}\cdot\frac{(A_1+A_2)}{A_2}h=0 \qquad \cdots\cdots(3)$$

This is the equation of motion of the water surface in the system.

The displacement of the water surface which is given by the tilting motion of earthquakes and crustal deformations must be damped as rapidly as possible.

Setting

$$\frac{8\pi\eta}{\rho a}=2\beta\omega_0 \quad \text{and} \quad \frac{ga}{A_1l}\cdot\frac{(A_1+A_2)}{A_2}=\omega_0^2$$

then, the equation of motion (3) may be written

$$\frac{d^2h}{dt^2}+2\beta\omega_0\frac{dh}{dt}+\omega_0^2h=0 \qquad \cdots\cdots(3')$$

As the initial conditions, we give

$$\frac{dh}{dt}=0, \quad h=h_0 \quad \text{at time} \quad t=0$$

For the recording water tube tiltmeter with critical damping $\beta=1$, the solution of equation (3′) takes the form

$$h(t)=h_0\omega_0te^{-\omega_0t} \qquad \cdots\cdots(4)$$

This is the equation of the critical damping response of the water surface in the system.

If we set $\beta=1$, we can solve the equations for β and ω_0 for a_c, the tube cross-sectional area required to achieve critical damping and d_c, the diameter of the tube required to achieve critical damping. Hence

$$a_c^3=\frac{16\cdot\pi^2\cdot\eta^2\cdot A_1\cdot A_2\cdot l}{\rho^2\cdot g\cdot(A_1+A_2)}$$

or

$$d_c=2\cdot\left(\frac{\eta}{\rho}\right)^{1/3}\cdot\left\{\frac{4lD_1^2D_2^2}{g(D_1^2+D_2^2)}\right\}^{1/6} \qquad \cdots\cdots(5)$$

where D_1 and D_2 are the diameters of Pot 1 and Pot 2, respectively.

For the case of the recording water tube tiltmeter set at the Hiyamizu station, $l=45$ meters, $D_1=10.4$ cm and $D_2=50.0$ cm. Substituting $\rho=1$ gr/cm^3, g= 980 cm/sec^2, $\eta=0.0130$ poise (water at 10°C) and $\eta=0.0114$ poise (water at 15°C), equation (5) becomes

$$\begin{cases} 10°\text{C} : & d_c=16.57 \text{ mm.} \\ 15°\text{C} : & d_c=15.87 \text{ mm.} \end{cases}$$

As the temperature in the underground tunnel at the Hiyamizu station is about 14°C～15°C throughout the year, the water tube actually used at Hiyamizu has just 16 mm in diameter.

To know the time for the system to reach equilibrium after the displacement is given at the water surface, $h(t)/h_0$ calculated from equation (4) where $\eta=0.0114$ poise, is compared with the experimental data in Table 1. 140 seconds are sufficient time for the water surface of the recording water tube tiltmeter to attain equilibrium.

TABLE 1.
Comparison of h/h_0 vs. time calculated from equation (4) with h/h_0 determined experimentally by the calibration test.

Time	h/h_0	
	Calculated	Experimental
0 second	1.00	1.00
20	0.30	0.47
40	0.10	0.05
60	0.025	−0.04
80	0.0056	−0.03
100	0.0012	−0.01
120	0.00023	−0.004
140	0.000045	0.0
160	8.6×10^{-6}	0.0
180	1.6×10^{-6}	0.0

2-2 Tilting and the displacement of the water level in the water tube tiltmeter.

We consider the case when we have a recording pot of cross-sectional area A_1 and a compensating reservoir of cross-sectional area A_2 on both sides of the water tube, where $A_1 \ll A_2$ (Figure 2).

We assume that the relative vertical displacement Δh is given to the compensating reservoir relative to the recording pot. The level of liquid in Pot 1 and Pot 2 changes by the values h_1 and h_2, respectively, to become a new equilibrium level O_1 O_2.

For a incompressible fluid of density ρ in the system,

$$\begin{cases} \rho\cdot h_1 A_1 = \rho\cdot h_2 A_2 \\ \Delta h \quad = h_1 + h_2 \end{cases}$$

Consequently, we obtain

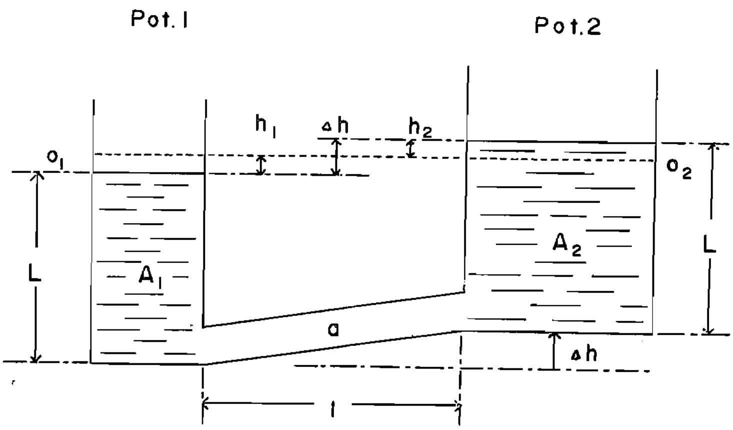

Fig. 2. Movement of water in the system when vertical displacement Δh is given at the pot 2.

$$\begin{cases} h_1 = \dfrac{A_2}{A_1 + A_2} \cdot \Delta h \\ h_2 = \dfrac{A_1}{A_1 + A_2} \cdot \Delta h \end{cases} \qquad \cdots\cdots(6)$$

Determining as $A_1 \ll A_2$, the change of level of the water surfaces by the relative vertical displacement Δh appears as a greater part in the recording pot than in the compensating reservoir.

The angle ΔT corresponding to the relative vertical displacement Δh is given,[3]

$$\Delta T(\text{second}) = \frac{360 \cdot 60 \cdot 60}{2\pi} \cdot \frac{\Delta h}{l}$$

$$= 20.626 \frac{\Delta h(\mu)}{l(\text{cm})}$$

$$= \frac{20.626}{l(\text{cm})} \cdot \frac{A_1 + A_2}{A_1} \cdot h_1(\mu) \qquad \cdots\cdots(7)$$

When $h_1 = 1000$ micron in the water tube tiltmeter at the Hiyamizu station, corresponding ΔT is calculated from equation (7),

$$\Delta T = 4.785 \text{ second in angle}$$

where $A_1 = 85.6\,\text{cm}^2$, $A_2 = 1962.5\,\text{cm}^2$ and $l = 4.5 \times 10^3$ cm.

When the change of level in the recording pot h_1 is magnified by the optical method, the sensitivity of the water tube tiltmeter on the recording photographic paper is given by,

$$S(\text{second/mm}) = \frac{\Delta T^{('')} \cdot 10^3}{h_1(\mu) \cdot Q}$$

$$= \frac{20.626(A_1 + A_2) \cdot 10^3}{l \cdot A_2 \cdot Q} \qquad \cdots\cdots(8)$$

where Q is the optical magnification.

It is possible to control the sensitivity S by changing Q in various methods.

2-3 Calibration of the recording water tube tiltmeter.

Once having installed and fixed the recording water tube tiltmeter in the observing tunnel, we cannot give the vertical displacement to Pot 1 or Pot 2 themselves. So, we drop some material in the compensating reservoir to give the change of level on the water surfaces of the system. This is a good method to determine experimentally the sensitivity of the recording water tube tiltmeter.

When we drop some level-changing mass in the liquid of the compensating reservoir, the changes of level h_1 and h_2 are introduced to be equilibrium level to the water surfaces of Pot 1 and Pot 2, respectively. The changes of level h_1 and h_2 by the level-changing mass V are written,

$$\begin{cases} h_1 = \dfrac{A_2}{A_1+A_2}\Delta h = \dfrac{V}{A_1+A_2} \\ h_2 = \dfrac{A_1}{A_1+A_2}\Delta h = \dfrac{A_1 V}{A_2(A_1+A_2)} \end{cases}$$

where $\Delta h = V/A_2$: change of level in the Pot 2 in the instant of dropping a level-changing mass.

$V\text{cm}^3$: volume dropped in the liquid of Pot 2.

Registering the change of level h_1 on the recording paper by the optical magnification Q,

$$Q \cdot h_1(\text{mm}) = \frac{10 \cdot V \cdot Q}{A_1+A_2}$$

To give at first the change of level Δh to Pot 2 corresponds to giving the relative vertical displacement Δh to the compensating reservoir. From equation (7), the angle correspond to Δh is calculated

$$\Delta T^{('')} = \frac{20.626 \cdot \Delta h(\mu)}{l(\text{cm})} = \frac{20.626 \cdot 10^4 \cdot V}{l(\text{cm}) \cdot A_2}$$

The sensitivity S on the recording photographic paper can be written

$$S(\text{second/mm}) = \frac{\Delta T^{('')}}{Q \cdot h_1(\text{mm})} = \frac{20.626}{l} \cdot \frac{10^4 V}{A_2} \cdot \frac{(A_1+A_2)}{10 \cdot V \cdot Q} = \frac{20.626 \cdot (A_1+A_2) \cdot 10^3}{l \cdot A_2 \cdot Q}$$

This equation is the same as equation (8).

In a routine observation, the optical magnification Q becomes smaller than calculated values by the friction of the pulley axis or some other reasons. If displacements f mm are recorded on the recording paper by dropping the level-changing volume V in the compensating reservoir,

$$h_1 \cdot Q' = f$$

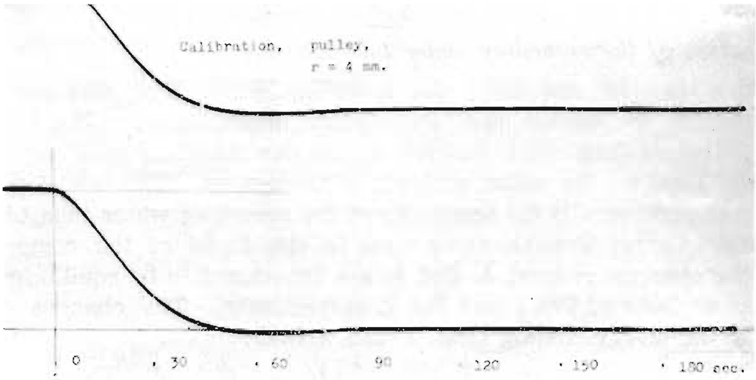

Photo. 1. A record of the calibration, using a pulley of 8mm in diameter.

TABLE 2.
Comparison of the planned sensitivity on the recording paper with the calibrated sensitivity of the recording water tube tiltmeter.
Optical length : 200 cm. Optical magnification : Q

Diameter of the pulley	Planned		Calibrated		
	Q	S second/mm	f mm	Q'	S' second/mm
8 mm	1000	0.0048	40.5	692	0.0069
16 mm	500	0.0096	25.0	427	0.011
22 mm	364	0.0132	21.0	359	0.013

$$\therefore \quad Q' = \frac{f}{h_1} = \frac{A_1 + A_2}{10V} \cdot f \qquad \cdots\cdots(9)$$

where Q' is the optical magnification in the routine observation.

The sensitivity on the recording paper in the routine observation is given by,

$$S'(\text{second/mm}) = \frac{\Delta T^{('')}}{f(\text{mm})}$$

$$= \frac{20.626 \cdot V \cdot 10^4}{l \cdot A_2 \cdot f} \qquad \cdots\cdots(10)$$

If V cm^3 and f mm are measured experimentally, we can calculate the sensitivity S' from equation (10).

Photo. 1 shows a record of the period response characteristic of the recording water tube tiltmeter to the influence of a level-changing mass $V = 12$ cm^3 using a pulley 8 mm in diameter.

Comparisons of the sensitivity of the system between the planned and the experimental are summarized in Table 2.

3. Description and observation of the recording water tube tiltmeter.

A one component recording water tube tiltmeter was installed in the under-

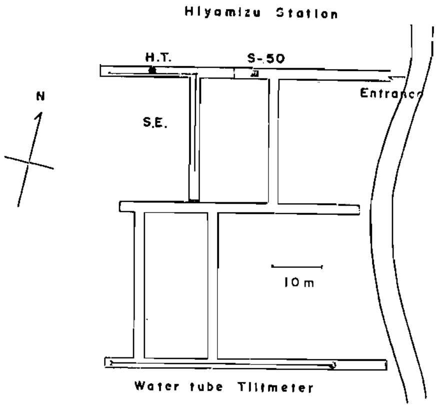

Fig. 3. Hiyamizu observation station.
S-50 : S-50 seismographs (NS, EW).
S. E. : Sassa type extensometers (NS, EW).
H. T.: Horizontal pendulum type tiltmeters (NS, EW).

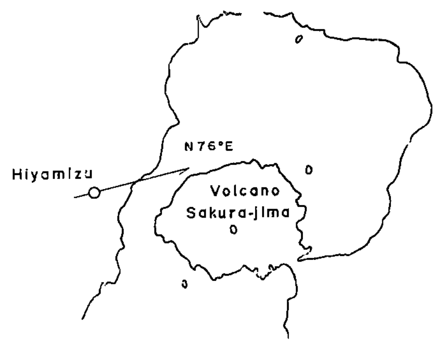

Fig. 4. Position of the Hiyamizu station and the direction of the recording water tube tiltmeter.

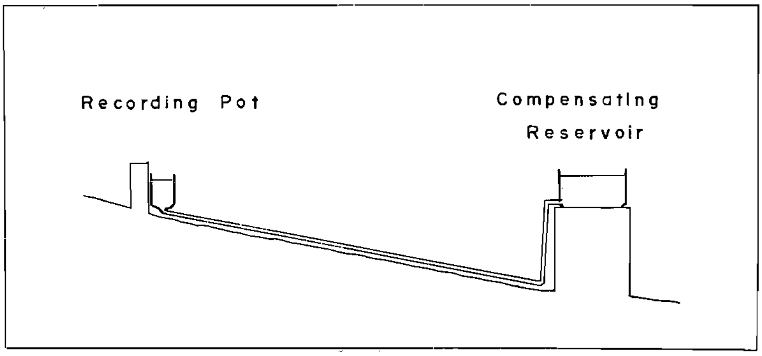

Fig. 5. Arrangement of the water tube.

ground tunnel at the Hiyamizu station in March, 1964. The position of the station and the direction of the recording water tube tiltmeter are shown in Figure 3 and 4.

3-1 Water tube.

A hardend vinyl chloride tube with an inside diameter of 16 mm and 45 meters in length serves as the water tube at the Hiyamizu water tube tiltmeter. The critical diameter of the water tube determined from equation (5) is $d_c=15.87$ mm (water at 15°C), so the water tube actually used has almost exactly this diameter.

3-2 Compensating reservoir (Pot 2).

The conpensating reservoir is a stainless steel tub which is 50 cm in diameter and 25 cm in depth. To prevent evaporation, the water surface of the compensating reservoir is covered with a spindle oil layer. For the visual reading

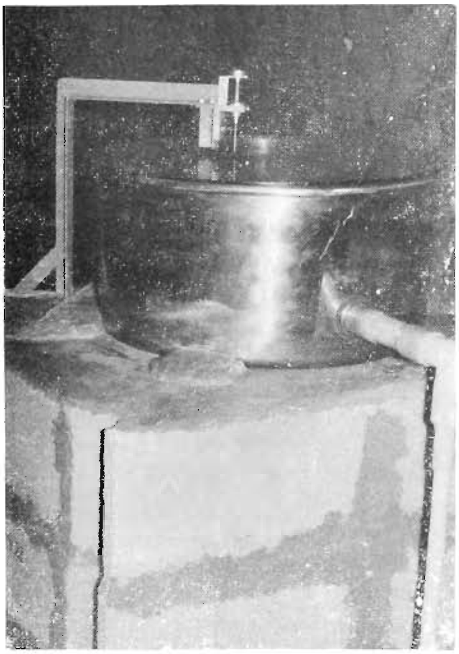

Photo. 2. Compensating Reservoir (Pot 2).

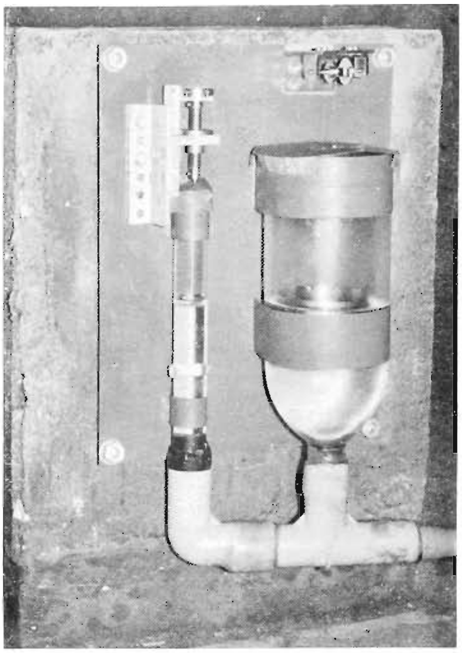

Photo. 3. Recording pot and its Apparatus.

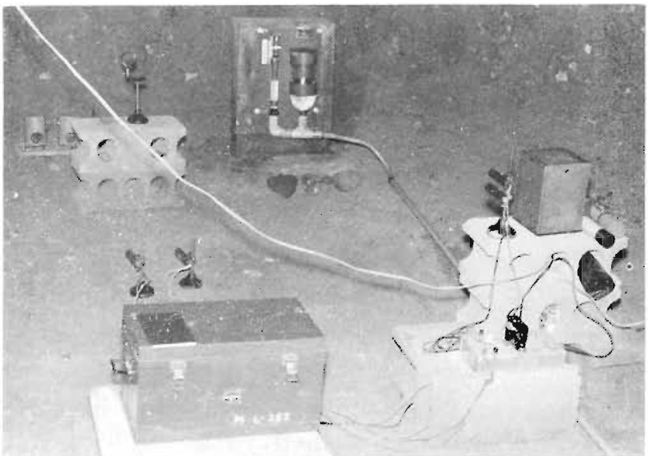

Photo. 4. Recording Apparatus of the Water tube Tiltmeter.

of the water surface, a micrometer is attached (Photo. 2).

3-3 Recording pot (Pot 1) and its optical recording apparatus.

The measuring container is a glass pot which is 12 cm in diameter. On the water surface of the recording pot, a glass float, 8 cm in diameter, is suspended by a counter-balance with super invar wire of 50 micron in diameter. This wire is hung over a pulley.

Continuous optical recordings of the change of level in the water surface are given by a recording mirror which is mounted on the pulley axis. For the visual reading of the water level of the recording pot, a micrometer is attached to a glass tube of 3 cm in diameter (Photo. 3). The speed of the recording photographic paper can be changed to 6 cm/day and 42 cm/day.

The water surface of the recording pot is covered with a spindle oil layer to prevent evaporation.

3-4 Micrometer reading.

The recording methods of the apparatus are made up of two groups : instruments with visual point readings by micrometers, and instruments with continuous recording by the optical method. So, this water tube tiltmeter can be used simultaneously both for continuous recording and also for visual readings.

A turn of the micrometer screw can be determined with an accuracy of 0.1 of a unit of the scale, which is equivalent to a displacement of the point of the screw by 0.5 micron.

Reading needles of micrometers are made of stain-less steel, and micrometer readings are measured from the top surface of the liquid.

3-5 Air-tube.

In equation (1), atmospheric pressures on the water surfaces of both pots are assumed to be equal. The atmospheric pressure gradient on both sides of the water tube is considered to be small, if the water tube tiltmeter is installed in an underground tunnel. So, the air tube is not attached to the Hiyamizu water tube tiltmeter. When the water tube tiltmeter is set in the field, we recommend the use of the air-tube for the equalization of atmospheric pressure in the whole system.

4. Some examples of the observations at the Hiyamizu station.

We use a pulley of 16 mm in diameter in the continuous routine observation of the recording water tube tiltmeter at the Hiyamizu station.

The recording paper is to be changed once a week, and at that time micrometer readings of the water level on both pots are conducted.

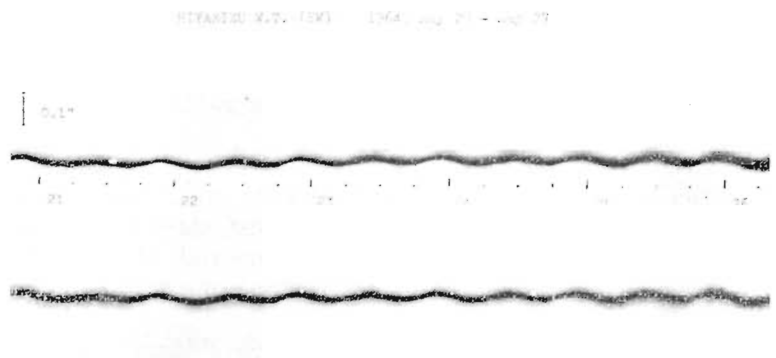

Photo. 5. A tiltgram of the Water tube Tiltmeter, in May, 1964.

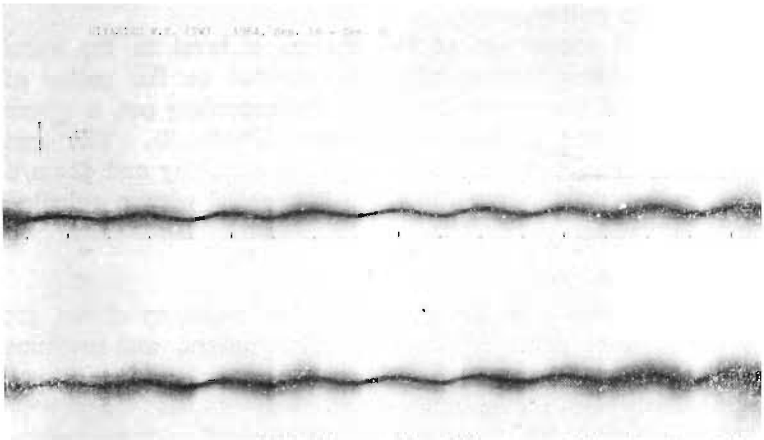

Photo. 6. A tiltgram of the Water tube Tiltmeter, in September, 1964.

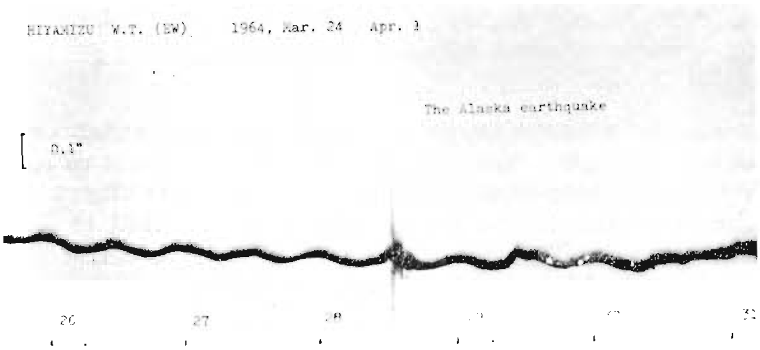

Photo. 7. The Alaskan earthquake (March, 28, 1964) observed by the Recording Water tube Tiltmeter at the Hiyamizu Station.

To study the effects of atmospheric pressure and temperature on the recording water tube tiltmeter, a thermometer and a micro-barograph are also set near the tiltmeter (Photo. 4).

Some examples of the recording water tube tiltgrams are shown in Photo. 5, 6 and 7. The Alaskan earthquake of $03^h36^m13^s$ U. T., March 28, 1964 recorded by the Hiyamizu water tube tiltmeter was shown in Photo. 7. This recording water tube tiltmeter may be useful as a tilt seismograph.[4]

At the conclusion of this report, the writer wishes to express his deep gratitude to Dr. K. Sassa and Prof. I. Ozawa for their kind suggestions and directions. His cordial thanks are also due to the staffs of the Sakura-jima Volcanological Observatory, Disaster Prevention Research Institute, Kyoto University, for their helpful assistance in constructing the recording water tube tiltmeter.

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