

## Study on the Relation between Local Earthquakes and Minute Ground Deformation

Part 2. An Application of the Digital Filtering to the Tiltgram  
for the Detection of the Minute Anomalous Tilting of  
the Ground.

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### Abstract

Digital band-pass and high-pass filters have been applied to the tiltmetric records observed at Wakayama, in order to detect small anomalous ground deformations. Tiltmetric or extensometric observations of the ground deformation are usually disturbed by meteorological and tidal changes and other unknown factors, so the elimination of these disturbances is necessary for the investigation of the minute deformations relating to earthquake occurrences. It is shown that the band-pass filters designed on the basis of the results of the Fourier analysis can be effectively used for this purpose. Small anomalous changes of the ground tilt, which might be related to the local shocks in the Wakayama region, are seen on some of the filtered results, but more observational data are needed to verify the relation between them.

### Introduction

Records of the ground deformations observed by tiltmeters or extensometers are based on many factors, such as earth tides, loading effects of the oceanic tides, changes caused by meteorological disturbances, deformations peculiar to the observation station and so on. Of course we must use highly sensitive instruments in order to observe such a minute deformation of the ground, but the above-mentioned disturbances are also magnified simultaneously so far as these instruments have flat responses with frequencies.

When an anomalous deformation relating to the occurrences of earthquakes is larger than those disturbances, we can easily detect it, but it is difficult to identify the deformation when it is of the same magnitude as that of the disturbances or smaller, unless we eliminate these disturbances from our observational results in some adequate way.

Two methods are possible for this purpose: One is by deciding quantitatively the amount of the deformation caused in each disturbing factor, eliminating the disturbances after the result, and the other is by extracting the characteristic deformation relating to an earthquake alone by an appropriate filter. The former is preferable, but it is impossible to carry it out accurately and approximation is inevitable. As the first step, some results which had been obtained from an approximate method using simple assumptions, were reported in the

previous paper<sup>1)</sup>. In the present article, the latter method, namely an attempt to use digital filters to detect anomalous changes of the ground deformation is presented.

A sequence of quantitative data of the ground deformation, assigned to specific moments in time - for example, the tilting motion of the ground in some direction - is a time series. When we consider the deformation relating only to the occurrence of an earthquake as a signal, and the disturbing deformations due to the meteorological changes, earth tides and so on as noise, then the detection of anomalous movements of the ground concerning the earthquakes is identical with the procedure of picking out a signal from the time series with noise. In general, it is thought that the disturbances to be considered as noise are periodic and the anomalous deformation as a signal is aperiodic in the present case (unless the occurrences of earthquakes are periodical). If we know its characteristic, namely the spectral structure of the signal, it is possible to take out the signal by using suitable filter, which is able to pass the signal only. But, unfortunately, we have not the knowledge about the characteristic of the signal of the deformation related to small earthquakes and, moreover, it is doubtful, at the present stage, whether such a minute deformation can be observed. Therefore we cannot avoid adopting the procedure whereby, making digital filters of all kinds and applying these filters to the time series, we identify the signal by examining the correlation between the output and the occurrences of earthquakes by try and error. In other words, it means that we must look for a filter that makes S/N of the output larger than the original time series.

Thus we must decide which filter is most effective for this purpose.

Hitherto, such filters of weight functions as the Pertzov's method, 25 hours running mean and so on, have been used for the analysis of tidal phenomena, and these low-pass filters are used for the detection of the secular changes of the crustal movement<sup>2)</sup>.

We try to detect anomalous deformations by using various band-pass and high-pass filters, which are designed according to the Fourier spectrum of the original records in this paper.

### Data

Tiltgrams obtained by the tiltmeters of horizontal pendulum type at the Akibasan Observation Station (135°10'.4 E, 34°11'.8 W) are used for analysis. The sensitivities of the tiltmeters are both about 0.004"/mm<sup>1)</sup>.

### Fourier spectra of the tiltgrams

First of all, the Fourier spectra of tiltgrams have been calculated in order to investigate the periodical construction of the observed tilting motions of the ground. As is well known, a time function  $f(t)$  is represented by the Fourier integral as follows,

$$f(t) = \frac{1}{\pi} \cdot \int_0^{\infty} F(\omega) \cos(\omega t + \phi(\omega)) d\omega,$$

and then the Fourier amplitude and phase spectra are given as

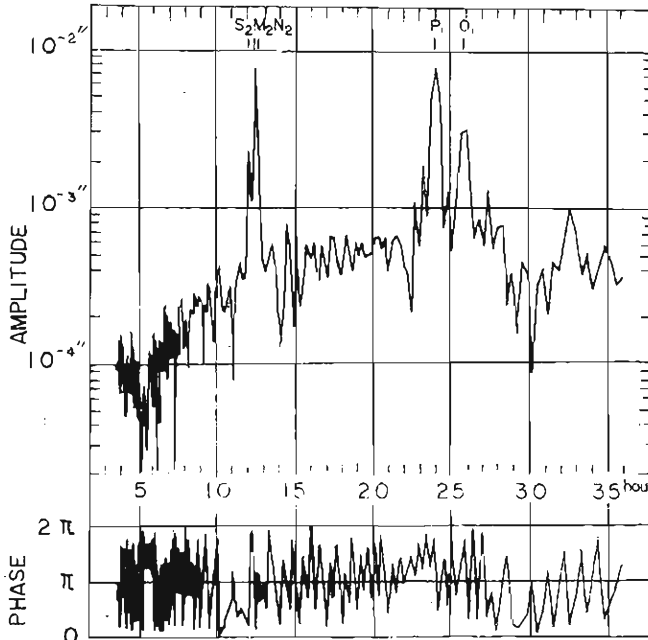


Fig. 1. Amplitude and phase spectra of the N-S component ground-tilt observed at Akibasan in Wakayama City.

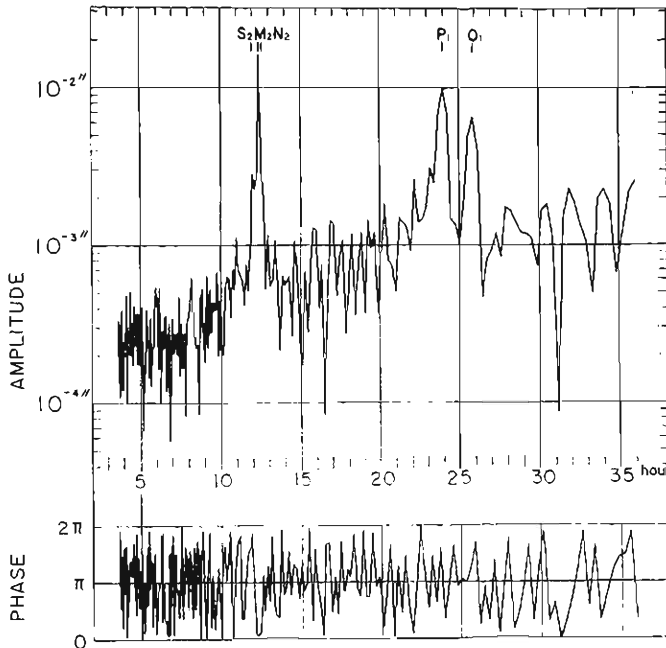


Fig. 2. Amplitude and phase spectra of the E-W component ground-tilt observed at Akibasan.

$$F(\omega) = \sqrt{[a(\omega)]^2 + [b(\omega)]^2}$$

$$\phi(\omega) = \tan^{-1}(-b(\omega)/a(\omega))$$

where

$$a(\omega) = \int_{-\infty}^{\infty} f(t) \cos \omega t \, dt$$

$$b(\omega) = \int_{-\infty}^{\infty} f(t) \sin \omega t \, dt.$$

The amplitude and phase spectra obtained are shown in Fig. 1 and 2. The period of the analysis is fifty days from May 6, 00<sup>h</sup>00<sup>m</sup> to June 24, 23<sup>h</sup>00<sup>m</sup>, 1961. We used the hourly values of the records and limited the analyses within the period of 36 hours this time.

As seen in the Figures, not only the amplitudes of tidal changes of  $M_2$ ,  $O_1$ ,  $N_2$  and  $S_2$ , but those of the daily changes in the period of 24 hours are predominant. The amplitudes due to tidal terms of the E-W component are larger than those of the N-S, while  $S_2$  and the diurnal changes of the period of 24.0 hours, which might be considered mainly with the meteorological origins, are nearly equal for both components.

The noise level of the E-W component is higher than that of the N-S. This may be explained by the influence of the meteorological changes and oceanic tidal changes which appear more noticeably in the E-W component than in the N-S<sup>1)</sup>.

The spectral structure will be discussed in another paper in detail.

### Digital Filtering

On the basis of the Fourier spectra, we designed some types of the filters, and applied the digital filters to the tiltgrams. Now we show the center frequency and the band width of a rectangular filter as  $\omega_0$  and  $2\omega_c$  respectively, and we may describe its amplitude characteristic  $A(\omega)$  as

$$A(\omega) = \begin{cases} 0, & (\omega < \omega_0 - \omega_c) \\ A(\omega), & (\omega_0 - \omega_c \leq \omega \leq \omega_0 + \omega_c) \\ 0, & (\omega_0 + \omega_c < \omega) \end{cases},$$

and the impulse response of this filter  $h(t)$  is given as

$$h(t) = \{2A(\omega_0)/\pi\} \cdot \{\sin[\omega_c(t-t_0)]/(t-t_0)\} \cos \omega_0 t.$$

We can get the output  $g(\tau)$  for the band-pass filtering of  $f(t)$  by taking the cross-correlation between  $h(t)$  and  $f(t)$ , namely

$$g(\tau) = \int h(t) \cdot f(t+\tau) dt.$$

In order to calculate the impulse response of an arbitrary filter, we divide it into  $n$  rectangular sections. Then the impulse response of the filter is given as the summation;

$$h(t) = \sum_j^n h_j(t), \quad (\omega_c \rightarrow \Delta\omega_j, \quad \omega_0 \rightarrow \omega_j, \quad \omega_{j+1} - \omega_j = \Delta\omega_{j+1} + \Delta\omega_j)$$

where  $h_j(t)$  is the impulse response of each small rectangular filter,  $\omega_j$  is in the band of the filter and  $\Delta\omega_j$  is the width of it<sup>3)</sup>.

Examples of comparison of the digital filters used with those which are designed, are shown in Fig. 3, 4 and 5, where the former and the latter are illustrated by solid and dashed lines respectively. The differences of their shapes are probably due mainly to the errors in the integration caused by the finite time length of  $h(t)$ . We limited  $h(t)$  within 360 hours and there is an indication that the more complex the shape of the filter is, the more the response is distorted. However, these digital filters are satisfactory for our present purpose.

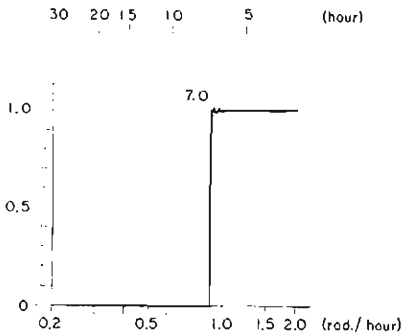


Fig. 3. Responses of the designed (solid line) and digital (dashed line) filters.

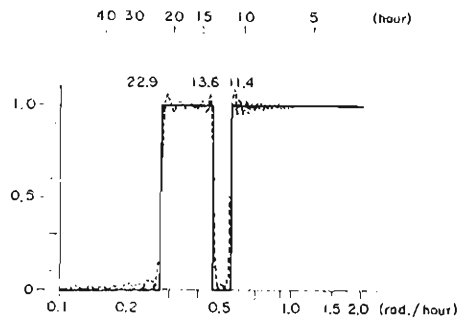


Fig. 4. Responses of the designed (solid line) and digital (dashed line) filters.

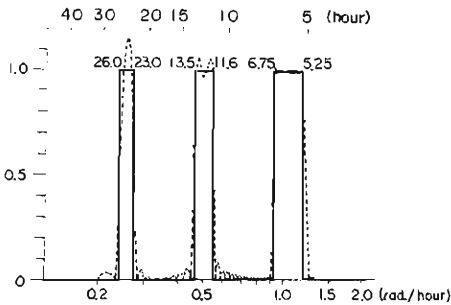


Fig. 5. Responses of the designed (solid line) and digital (dashed line) filters.

filters which have such amplitude characteristics as 1 in the periodic ranges from 2 to 5, 5 to 7, 7 to 11, 11 to 13, 13 to 23 and 23 to 25 hours and 0 outside the intervals, respectively. Upward changes in the Figures mean a tilting motion in the direction of the south. The amplitudes are shown in different scales.

Then, with regard to the periodic changes of the diurnal, semi-diurnal and quarterly-diurnal terms we made a filter, the amplitude characteristic of which is 1 in the intervals from 23.0 to 26.0, 11.5 to 13.5 and 5.25 to 6.75 hours, as seen in Fig. 5, took out the major periodic change only, and then subtracted it

The results of the practical filtering are now described. According to the Fourier spectra, in which the amplitudes near diurnal and semi-diurnal terms are predominant, we divide the periodic range into six parts, and filtered the tiltgram from Oct. 1st to Dec. 31, 1960, by the six filters. The outputs are shown in Fig. 6 and 7, together with the original record 0. The curves 1, 2, 3, 4, 5 and 6 in the Figures are the outputs by the digital rectangular

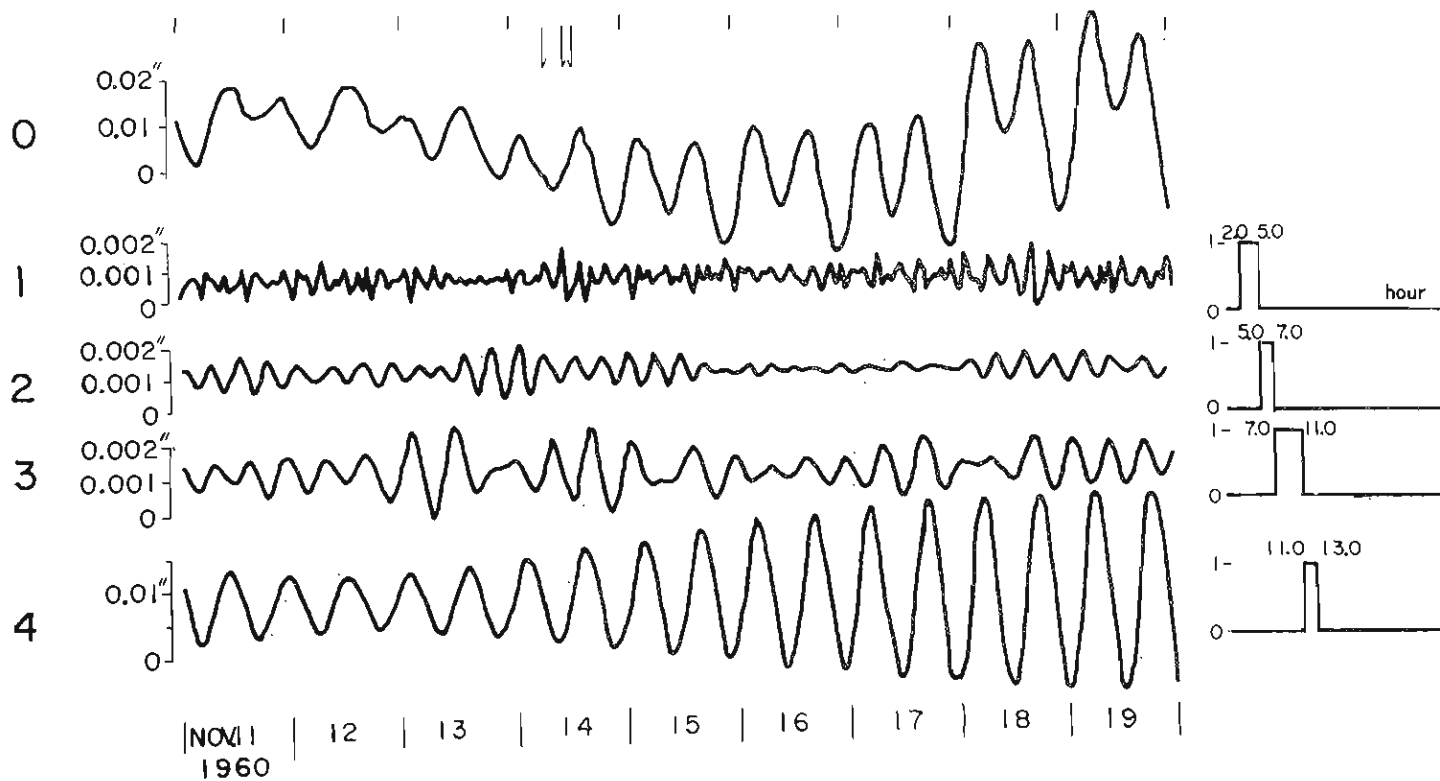


Fig. 6. The original record of the N-S component tilt (uppermost curve), filtered results and amplitude characteristics of the applied filters. The arrows show the occurrences of the felt local shocks.

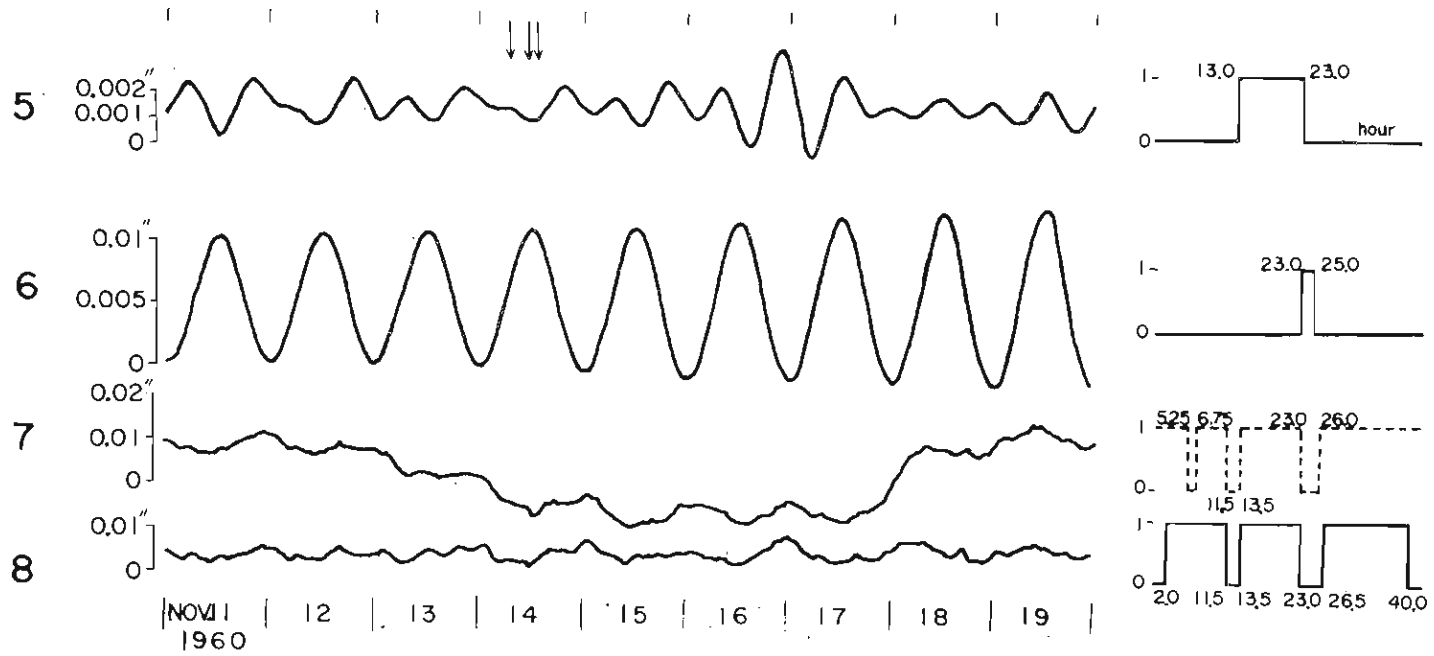


Fig. 7. The filtered results of the N-S component tilt.

from the original record. The result is the curve 7 in Fig. 7. The curve 8 is the output from the digital filtering which reduces the periodic changes of the diurnal and semi-diurnal terms and long period fluctuations.

Similarly the results are shown in Fig. 8 as to the component of the E-W tilt. The curve 0 is the original record. The curves 1 and 2 are the outputs by the filter, having the amplitude that is 1 for the period of 22.9 to 13.6 and 11.4 to 2.0 hours, and by the high-pass filter which passes the changes of the shorter periods than 7.00 hours, respectively. The upward changes in the Figure show the westward decline.

As for the weather conditions of the days shown in the Figures from Nov. 11 th to 19 th, 1960, we had precipitation of about 14 mm and 26 mm in the afternoons on 12 th and 17 th, and the other days were fine.

We made the exchanges of the recording paper between 14 hrs and 16 hrs on 13 th and at about 16 hrs on 16 th.

Since the record was disturbed after the resetting of the instrument on 13 th, we removed the artificial disturbances from the record.

On Nov. 14 th, three noticeable local earthquakes occurred in the Wakayama region. They were the same earthquakes that we examined the relation to the ground deformation in the previous paper<sup>1)</sup> and the times of their occurrences are shown by the arrow lines in the Figures.

We shall now discuss the filtered results in some detail.

The amplitude of the output by the filtering in the period from 2.0 to 5.0 hours is predominant near the time of the occurrences of the earthquakes on 14 th and from 18 th to 19 th (Curve 1 in Fig. 6).

It may safely be said that the unusual change on 14 th arose from those local earthquakes, although we cannot judge whether it was the anomalous tilting motion of the ground relating to the earthquakes or the instrumental effect from the shocks of them. The amplitude from 18 th to 19 th is considered to be the effect of the rainfall.

Then the fluctuation in the period from 5.0 to 7.0 hours (Curve 2) is large before and after 14 th and from 18 th to 19 th. It is reasonable to attribute the change in the former to the influence of the work of the exchange of the recording paper and the latter to rainfall.

Increments of amplitude on 13 th and 14 th are seen in the curve 3, which is the change in the period from 7 to 11 hours. The large fluctuation on 13 th and 14 th may be explained as the effects of the rainfall in the afternoon of 12 th and the occurrences of the earthquakes respectively.

The curve 4 in the period from 13 to 23 hours consists mainly of the tilting motion of the ground caused to the  $M_2$ - and  $S_2$ - constituents of the earth tides.

The fluctuation of the curve 5 is noticeable from 16 th to 17 th and might be the effect of the artificial disturbance at the time of the exchange of the recording paper.

The filtered result in the period from 23 to 25 hours (Curve 6) consists of the diurnal terms of the earth tides such as  $O_1$  and  $P_1$ , and those caused to the daily meteorological variation such as the atmospheric pressure and temperature.

The curve 7, in which there is the change of the long period left alone, turns upwards on 17 th, and this means the southward tilting motion of the ground



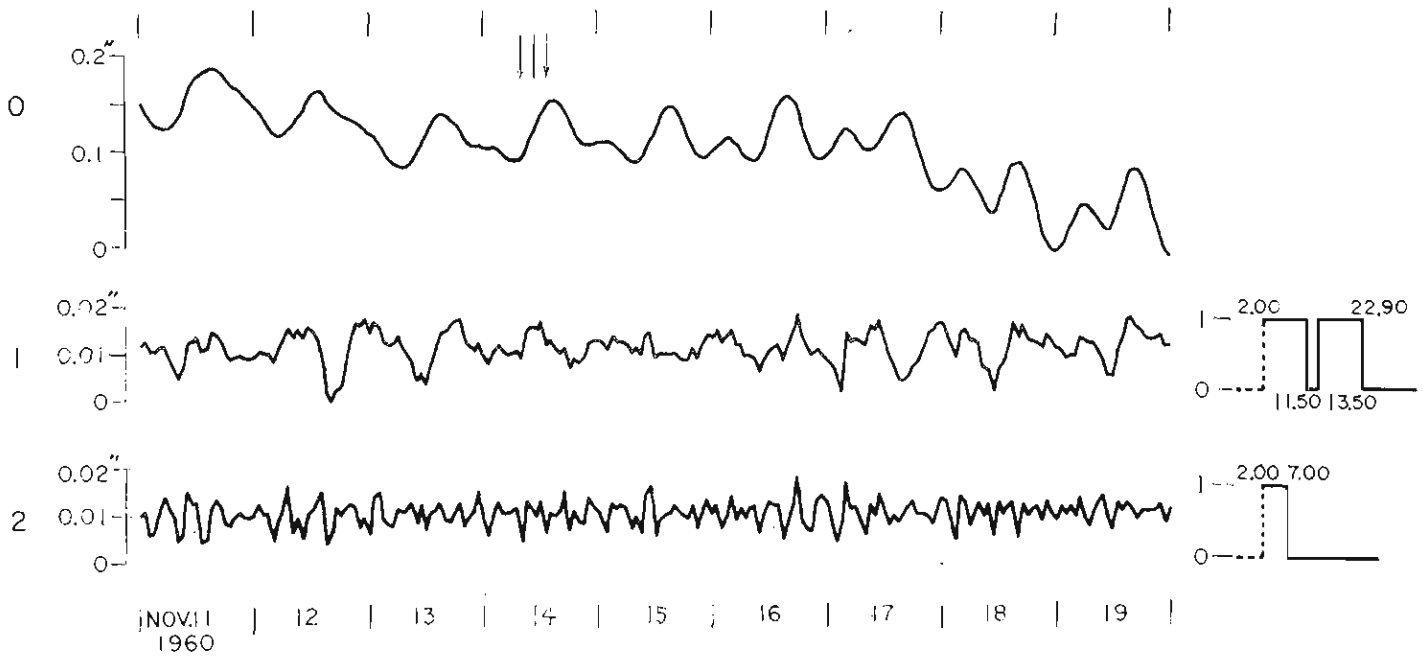


Fig. 8. The original record of the E-W component tilt (uppermost curve), filtered results and amplitude characteristics of the applied filters. The arrows show the occurrences of the felt local shocks.

by the rainfall on 17th. Besides the periodic fluctuation in the period longer than 24 hours is seen in the curve 7. It seems to be attributable to the fact that, although the characteristic of the filter for the period longer than 26 hours is zero, we cannot completely eliminate the  $O_1$ -constituent, the period of which is 25.82 hours.

Therefore, the upper limit of the filter for the diurnal terms is extended to the period of 26.5 hours on the curve 8. Periodic changes of about 26 hours are not yet completely reduced in the latter part.

In these cases, as seen in the Figure, it is somewhat difficult to distinguish the characteristic changes from the noise, in comparison with the case of the filtering by the simple filters.

On the other hand we applied the two kinds of filter to the E-W component of the ground tilt.

The curve 1 in Fig. 8, on which the oscillations in the period from 11.5 to 13.5 hours and the periods longer than 23 hours were already eliminated, shows large fluctuations except 11th and 15th.

The amplitudes of the tilting motion of the period shorter than 7 hours are almost constant in this interval. We cannot recognize the anomalous change of the ground tilt relating to the local earthquakes in both curves. Since this component of the ground tilt at Akibasan is very much disturbed by the changes of the atmospheric pressure and temperature and the earth tides,<sup>1)</sup> it requires further examination, making comparisons with these disturbing effects.

### Concluding Remarks

In the preceding paragraph we have shown some examples of the digital filtering as an attempt to detect minute ground deformations, hidden behind such disturbances as earth tidal and meteorological changes.

It is concluded that the band-pass filters suitably designed, serve the purpose, and those of the narrow frequency responses are more effective.

A peculiar tilting motion of the ground of the amount of about  $0.002''$  is seen on the tiltgram of the N-S component in connection with the occurrences of the local earthquakes.

We are preparing the digital filterings of the atmospheric pressure, temperature and so on, for comparison with the output of the tiltgrams and similar attempts to the change of the longer period by the low-pass filters.

We will discuss these problems in the near future.

### Acknowledgement

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### References

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2) For example

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