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# Vibrational Characteristics of the Ground Investigated by Several Methods

by Soji YOSHIKAWA, Michiyasu SHIMA, and Kojiro IRIKURA

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**Abstract**

The vibrational characteristics of the ground were investgated through the observation of natural earthquakes and blasts and by means of an oscillator and seismographs being set on the ground surface and in the drift. Fourier components were obtained in the case of seismograms of natural earthquakes and blasts, and the amplitude-frequency relations for a constant vibrational force in the case of vibrational test by the oscillator. The ground structure was rather complicated, however, the amplitude and phase distributions at each measuring point (on the ground surface and in the dirft) coincide well with that of theoretical calculations in regards to the ground structure and the spectrum of the place.

## 1. Introduction

It is generally well known that the seismograms of an earthquake at many observation points differ according to the geological structure of each position[1~3]. Many seismologists have attributed this to the fact that the ground has predominant period corresponding to its geological structure and theoretically this is calculated as the multiple reflections of infinite harmonic S waves coming upwards perpendicularly. From the analyses of micro-tremors and natural earthquakes, it has been proved that the ground has its own spectrum which is related to the sub-layers and their physical properties (propagating wave velocity, density, rigidity, and viscosity)[4~6]. The vibrational characteristics of the ground during earthquakes are closely related to the damage caused by destructive earthquakes and it has been established that the presentation of the spectrum corresponding to the dynamic characteristics of the ground is important from the earthquake engineering point of view. Investigations on the vibrational characteristics of the ground chosen as construction sites have been carried out through the observation of earthquakes on the ground surface. The vibrational characteristics can be correlated with the geological structure from the seismogram on the ground surface when it is simple enough, for instance when one or two layers are parallel to the ground surface. However, when the geological structure is rather complicated, that is when the ground is composed of many layers or the boundary is irregular and indefinite, the relationship between the surface seismogram and the structure becomes very complicated and hence seismograms of places deep below ground have to be obtained.

When the foundation of the structure is set deep below ground, the amplitudes and the phase difference along the vertical line from the surface will be required and hence it is necessary to investigate the vibrational characteristics underground. In view of this fact, seismograms from the ground surface to a depth of 40 m were obtained and analyzed. In the case of natural earthquakes we can only get the vibration characteristics of the ground by the earthquakes. When the frequency characteristics of the earthquake are $U(\omega)$, and $O(\omega)$, $Q(\omega)$ and $G(\omega)$ are the frequency characteristics of the origin, the path through the crust and the geological structure of the observing point, respectively, we get

$$U(\omega)=O(\omega)\cdot Q(\omega)\cdot G(\omega)$$

To obtain $G(\omega)$, the forms of $O(\omega)\cdot Q(\omega)$, which are irrespective of the sub-surface geological structure of the observation point have to be clarified. For this purpose observation at a point more than 100 m below ground is required, but in practice this is difficult. It was reported that at the Hitachi mine several hundred meters below ground the velocity spectrum for observing earthquakes was constant[7]. However, the frequency range in this case was limited and hence the spectrum of the earthquake at base rock level is still unclear, because it will be affected by the origin and the path of the seismic ray. Therefore, the spectrum of the ground itself will be unobtainable from the analysis of the seismogram at the ground surface. since it is very difficult to get the one at base rock level. At persent the spectra of the displacement, velocity and acceleration are first obtained and from the spectral form, i. e., the relative positions of peak and dip, the predominant period of the ground is generally presumed.

When an oscillator is used as the disturbance origin, the frequency characteristics of the source are relatively controlled and hence the predominant period of the ground can be conveniently obtained. However, the wave in this case is spherical and not a plane wave as is the case with an earthquake wave. It can be presumed that the vibrational characteristics will be clarified when the origin is designed so as to emit SH type waves, even though the method will be indirect. In view of the above, the vibrational characteristics of the ground were examined experimentally by means of natural earthquakes, blasts and an oscillator.

The experiments were carried out at Fukui Pref. where the seismographs were set at the ground surface, and the pit and drift were 10-40 m below ground. From the seismograms, Fourier components were obtained by IBM 7090 and the spectra were compared with those of the oscillator and blasts, and the theoretical results were obtained by the calculation of multiple reflections of the waves corresponding to the geological structure obtained by the seismic prospecting and boring data.

## 2. Experimental arrangement

The natural period of the seismographs used for the observation of natural earthquakes was one second and the seismograph was connected to the amplifier so as to give a magnification of 1000-9000 and the paper speed of the oscillogram is 2.5 cm/sec. The frequency characteristics are constant in the range 2-30 cps as to the displacement amplitude and over this range the amplitude was corrected in the data analysis. The instrumental layout is shown in Fig. 1.

As shown in Fig. 1, S-1 is set on the ground surface, S-2 in the oblique drift 10 m in depth and S-3 and S-4 in the drift 40 m in depth. At S-3 three compo-

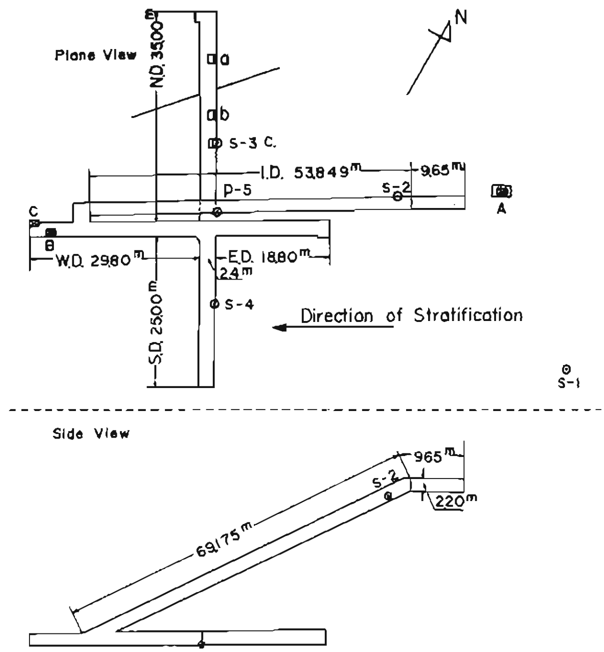



Fig. 1 Plane and side view of drift and the locality of seismographs.

nent observations, two in the horizontal direction and one in the vertical were made and at the other points they were made only in the horizontal direction.

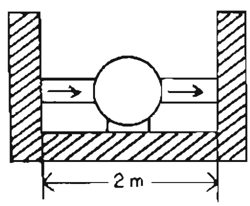



Fig. 2 Oscillator

In the cases of the blasts and the oscillator, three points (a, b and c shown in Fig. 1) were added besides the above observation points and connected to the data-recorder whose frequency characteristics were constant in the range 2-100 cps.

The oscillator was set as shown in Fig. 2 and the force in this case is $F=286f^2$, where f is frequency in cps and F in kg. F is about 1-14 tons in the range 2-7 cps.

## 3. Geological structure

The geological structure of the site is shown in Fig. 3. The velocity distributions (Vp and Vs : propagating velocities of P waves and S waves, respectively) are shown in the following table :

| | | Vp (km/sec) | Vs (km/sec) |
|---|---|---|---|
| First layer | Surface layer | 0.3-0.8 | 0.1-0.2 |
| Second layer | Granitic weathering soil | 1.7 | 0.5 |
| Third layer | Weathered granite | 3.0-3.3 | 1.0-1.1 |
| Fourth layer | Granite | 3.8-4.2 | 1.3-1.6 |

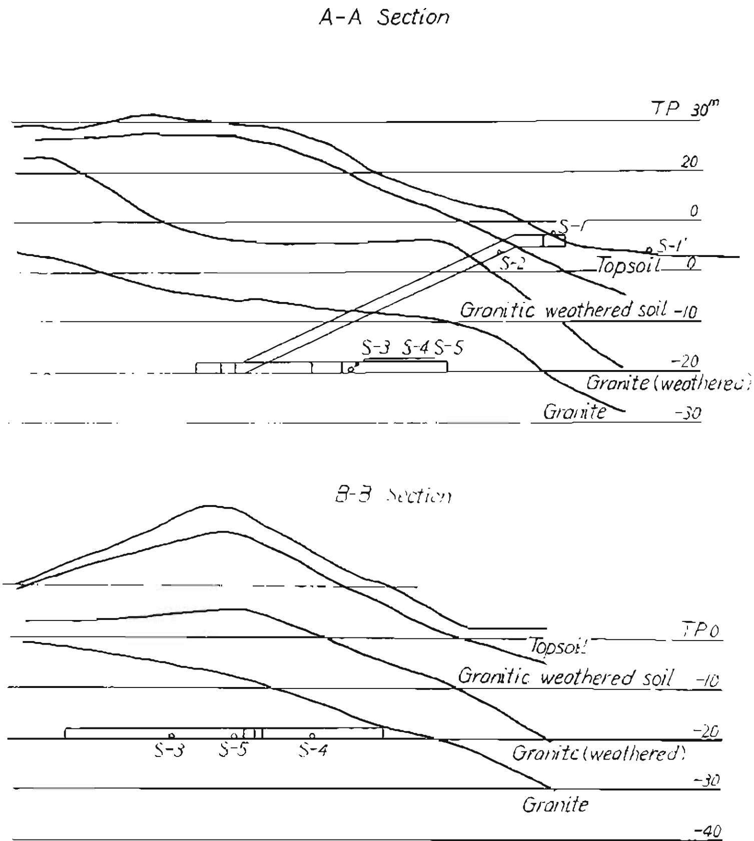



Fig. 3 Outline of geological structre of the neighbouring part of observing stations.

The rock in this site is characterized by the development of a regular joint system, its dominant direction of strike is N60°E, as shown by an arrow in Fig. 1 and dipping to 74°SE. In the case of drift, set in granite 40 m below ground, however, the degree of weathering is different in each position, S-3, S-4 and S-5. S-2 was set in the layer of granitic weathering soil.

## 4. Results of analyses from natural earthquakes

An example of a seismogram is show in Fig. 4. The greater part of the S waves were used to obtain the spectrum of the earthquake motion. Of the eight seismograms, six were of a similar district of origin and hence it can be supposed that the frequency characteristics of the earthquake motion recorded at the base will be similar. But in detail some differences will be apparent in the mechanism, path and incident circumstance with respect to the base rock and hence the statistical mean is taken to clarify the frequency characteristics of the ground. The geometrical mean of the spectrum, $A_i(\omega)\exp[i\phi_i(\omega)]$ of each earthquakes can be expressed by

$$\sqrt[n]{A_1(\omega)\cdot A_2(\omega)\cdots A_n(\omega)}\exp[i\{\phi_1(\omega)+\phi_2(\omega)+\cdots+\phi_n(\omega)\}]\cdot$$

In this case the mean of the amplitude spectrum $A_i(\omega)$ can be obtained separately from the phase spectrum.

The results are shown in Fig. 5-Fig. 6. The velocity amplitude spectra are shown along the ordinate. From the examination of the spectrum at each depth in the figure, it can be understood that the curves show a peak at 2-4 cps at S-3 and S-4, 40 m below ground, and monotonously decrease thereafter, though a slight peak can be observed at 7-8 cps. At S-1 (ground surface) and S-2 (10 m in depth) peaks are found at 2-4 cps and 7.5 cps, and a dip is found at 5-6 cps.

The ratio of the amplitude of the ground surface to that below ground is shown in Fig. 7 and it can be seen that the ratio decreases with the increase of the

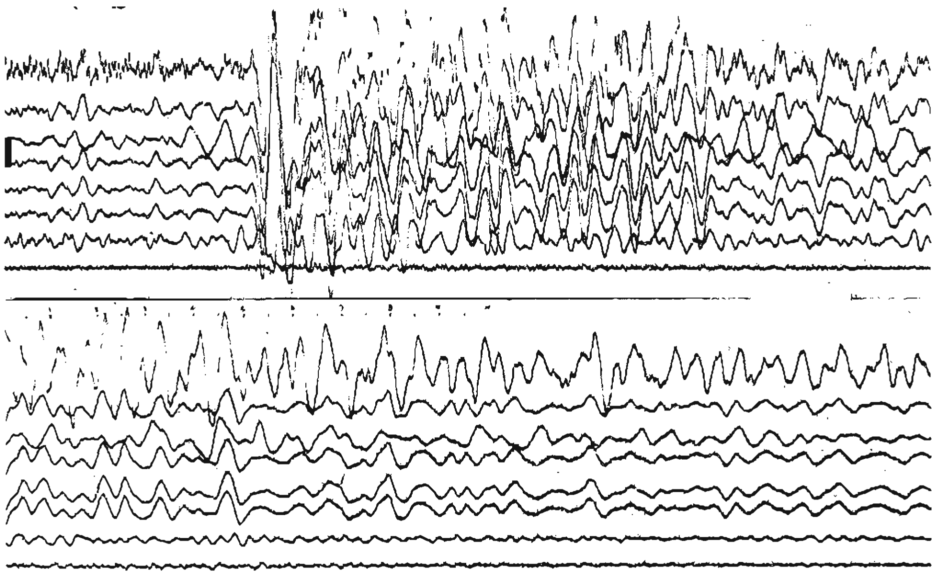


Fig. 4 Examples of the portions (S-wave part) of seismograms of natural earthquakes used in Fourier analyses.

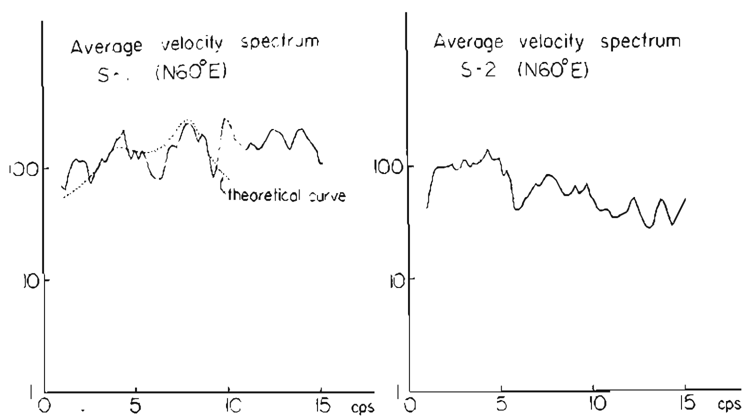



Fig. 5

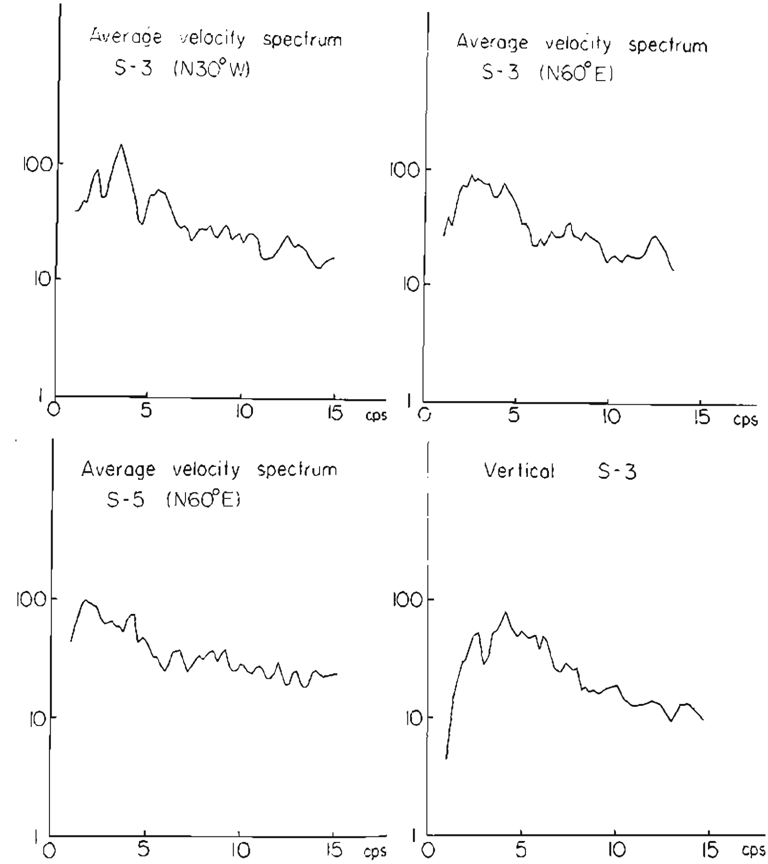



Fig. 5 Average of amplitude spectra of displacement velocity

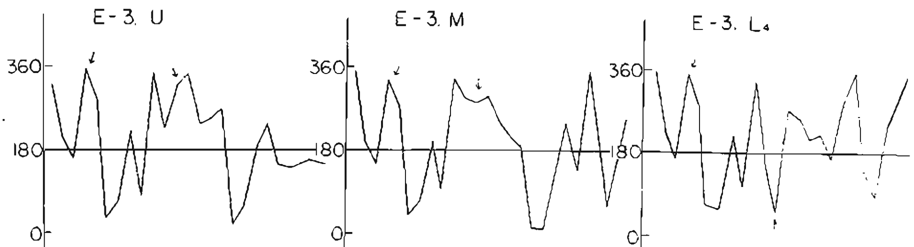



Fig. 6 Examples of phase spectra of displacement velocity
U : S-1, M : S-2, $L_4$ : S-3

frequency. The phase differences between S-1, S-2 and S-3 were calculated from the phase spectra in order to examine the relations between the vibration at the ground surface and that below ground. The relative mean difference of phase between the ground surface and the granite layer in the range 2-4 cps was 20°, in the ranges 7-8 cps was about 30° between S-1 and S-2 and on the contrary, was about 90° between S-2 and S-3 and hence the phase tends to be reversed.

The topography and the geological structure of this site are complicated and hence the stationary wave formed by the multiple reflections of SH waves were not so regular as to show a definite concidence and reversal of the phase. Accordingly, the phase coincidence (20-30°) and reversion (90°) as stated above may be regarded as the 'in-phase' and the 'out of phase' respectively and then it can be supposed that the proper vibration of 2-4 cps is the in phase from the ground surface up to the base rock and the phase of 7-8 cps is reversed at the boundary between the granitic weathering soil and the granite which is 15 m in depth.

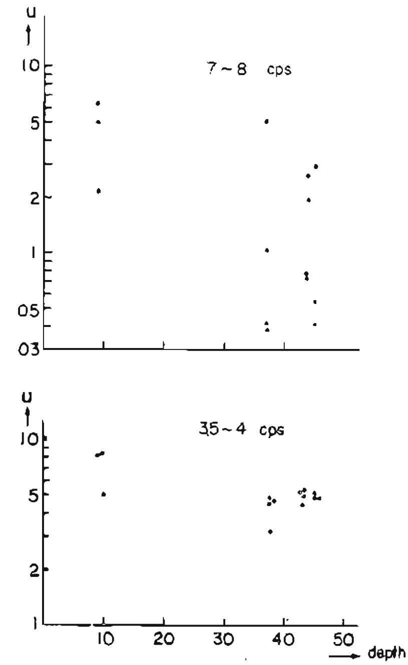



Fig. 7 Amplitude ratio in the various depth from ground surface. (The case of 3.5~4c. p. s. and 7~8c. p. s.)

## 5. Results from the analyses of vibration tests

The positions where the oscillator was set were at the entrance to the drift and in the drift, each in two directions, i. e. parallel to the joint and perpendicular to it. The record is shown in Fig. 8.

So that the wave characteristics may be checked the loci are shown in Fig. 9.

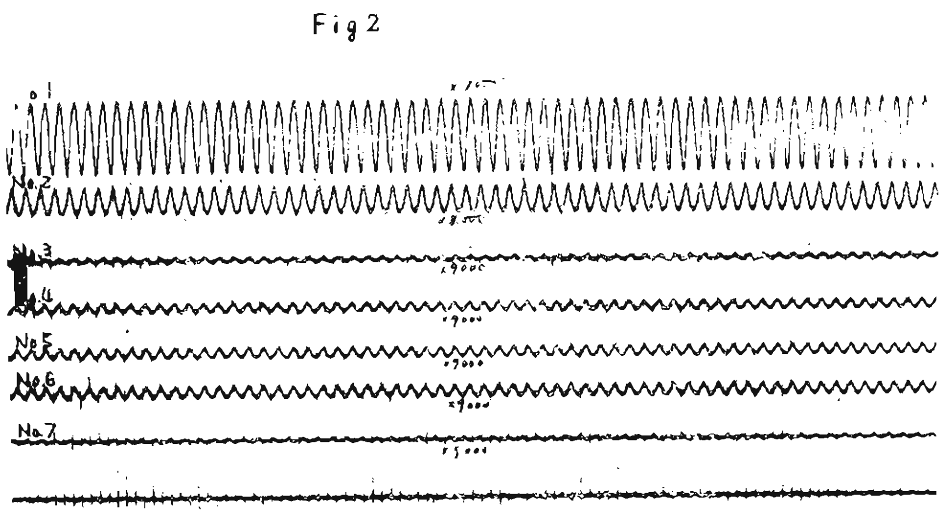



Fig. 8 Example of seismograms of vibration tests by the oscillator.

10 and 11. These were obtained from the seismograms at S-3, two in the horizontal direction and one in the vertical direction, In Fig. 11 the locus is shown at S-3 which is in the drift 40 m in depth, corresponding to the excitation of the vibration along at the same horizontal plane.

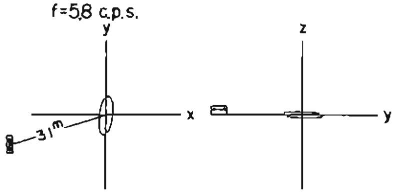



Fig. 9 Loci of oscillation at S-3 generated by the oscillator situated in drift. The oscillations were applied to normal to the direction of stratification.

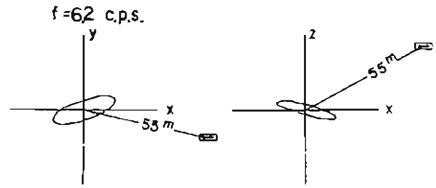



Fig. 10 Loci of oscillation at S-3 generated by the oscillator situated at the ground surface. The oscillations were applied to parallel to the direction of stratification.

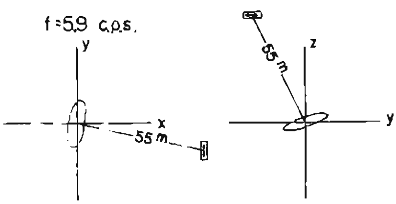



Fig. 11 Loci of oscillation at S-3 generated by the oscillator situated at the ground surface. The oscillation were applied to normal to the direction of stratification.

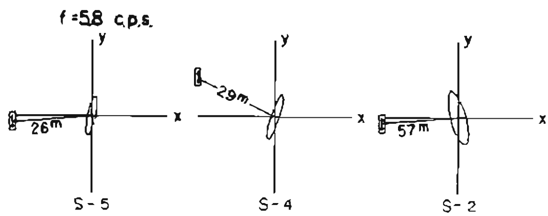


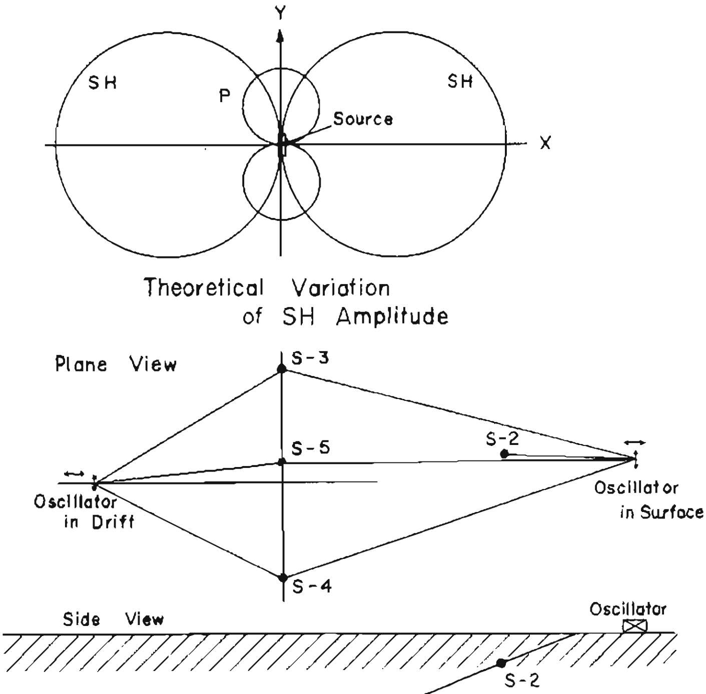

Fig. 12 Loci of oscillation at S-2, S-4, and S-5 generated by the oscillator situated in dirft.

As can be seen in the figure, the amplitude is larger in the direction of the excitation and very small in the vertical direction.

The same tendency may be observed with the surface excitation as with the underground observation, as shown in Figs. 10 and 11. In Fig. 12 the loci at S-2, S-4 and S-5 are shown. In this case the seismographs are set only in one horizontal direction. However, their loci can be drawn from two horizontal components, which correspond to the excitation of the same direction and were recorded independently, with reference to the fixed observation at S-3. The general tendencies are the same as those at S-3.

These results can be regarded as coinciding approximately with the theoretical patterns of SH propagation described by Heelen, Aidan and Cherry which were obtained under conditions where the wave was emitted from the point source or in the infinite elastic media[8)9)]. In Fig. 13 the amplitude variation of the SH wave in each direction of the propagation described by Heelen and Aidan is shown and a schematic diagram of the situation of the oscillator and seismographs is shown simultaneously.

In Figs. 14-16 the relation between the displacement and the frequency is shown. In the figures the full line shows the surface excitation, the dotted line



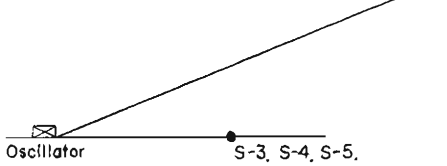




Fig. 13 Upper ; Theoretical Variation of SH-Amplitude radiated by point source in an unbound solid.
Lower ; The schematic diagram between the situation of oscillator and that of seismographs.

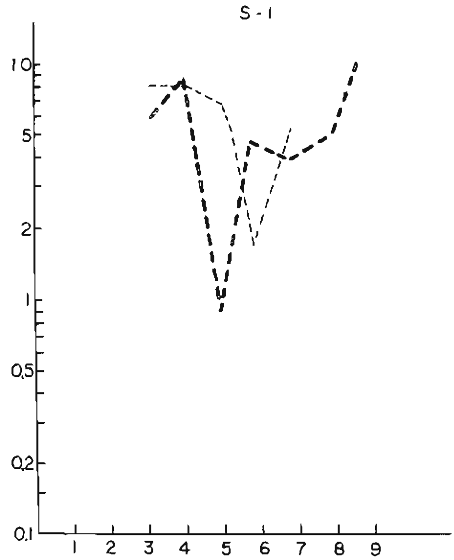



Fig. 14 Amplitude-frequency relation of vibration tests by oscillator at S-1′

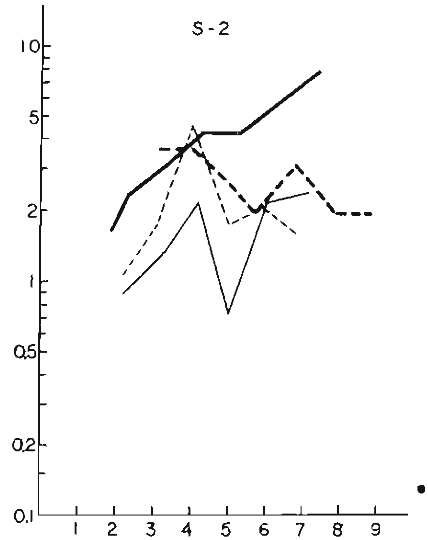



Fig. 15 Amplitude-frequency relation of vibration tests by oscillator at S-2

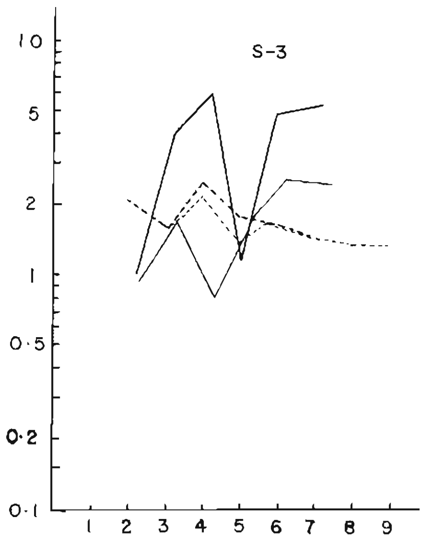


Fig. 16 Amplitude-frequency relation of vibration tests by oscillator at S-3

the excitation in the drift, the thick line excitation in the direction of the joint, the thin line the excitation perpendicular to it.

In the case of surface excitation S-2 (in the granitic weathering soil) and S-3 (in the granite layer) show two peaks at 4 cps and about 7 cps (though it is not so clear because the upper limit of frequency of the experiment is 7 cps) and the dip at 5 cps. In the case of underground excitation the peak is observed at 4 cps and the dip at 5 cps for S-1 and S-2, and there is only one peak at 4 cps for S-3 and S-5. The above facts may be explained from the considera-tion that the energy is conserved in the surface layer when the excitation is on the ground surface, but is diffused to the lower layer in the case of undergound excitation and affects the spectrum at S-3.

## 6. Vibration test by blasts

An attempt has been made to check the vibration charactesritics of the ground that are of explosive origin. For this purpose, the spectrum at the explosive

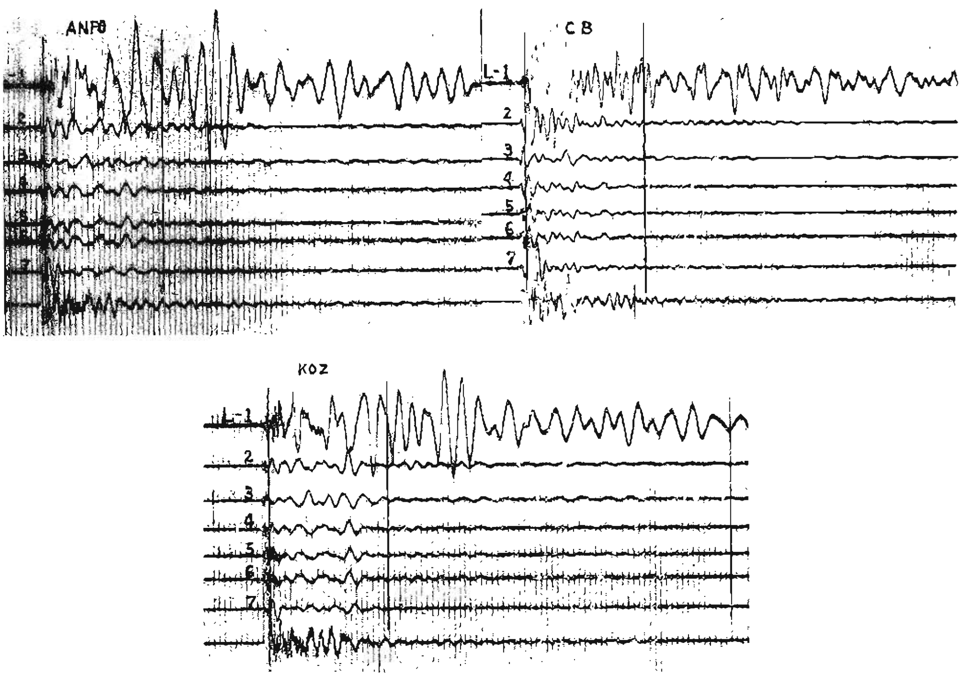


Fig. 17 Examples of the portions of seismograms of blasts

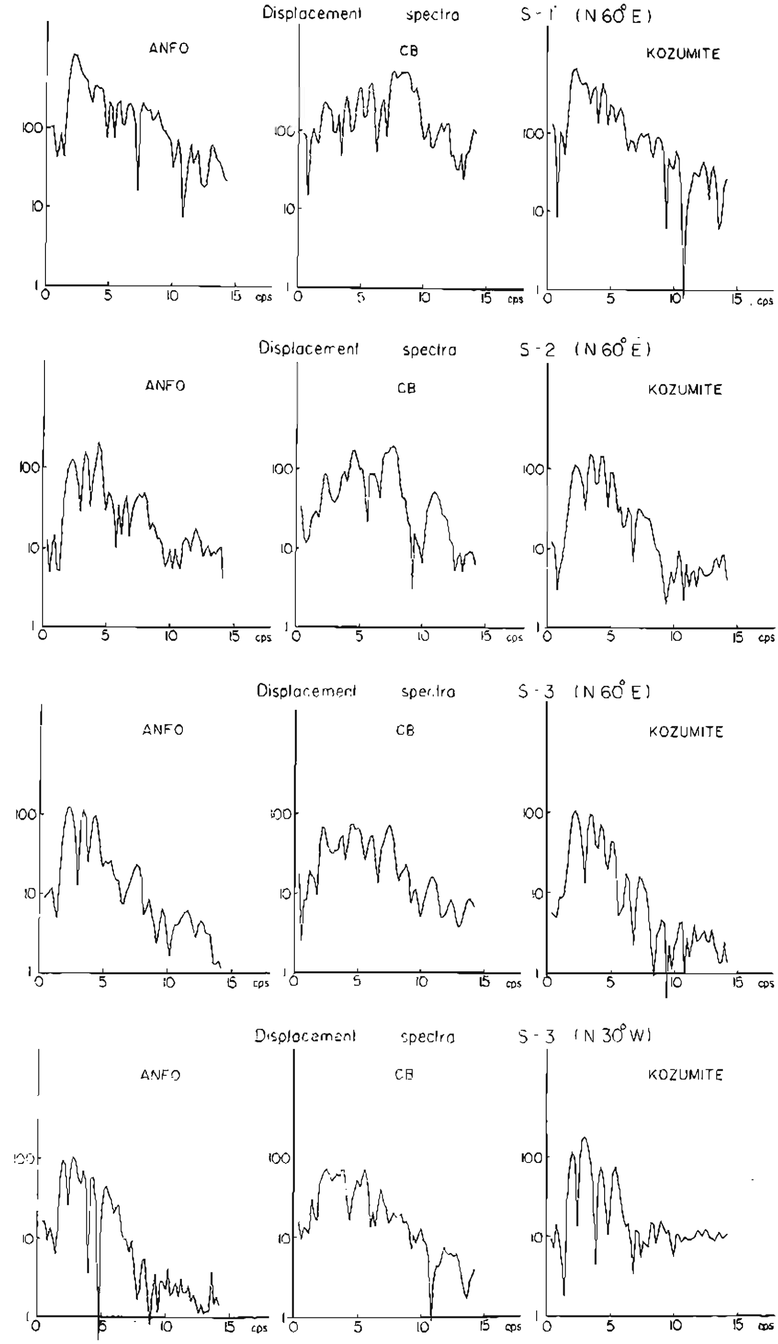



Fig. 18 Amplitude spectra of displacement by blasts

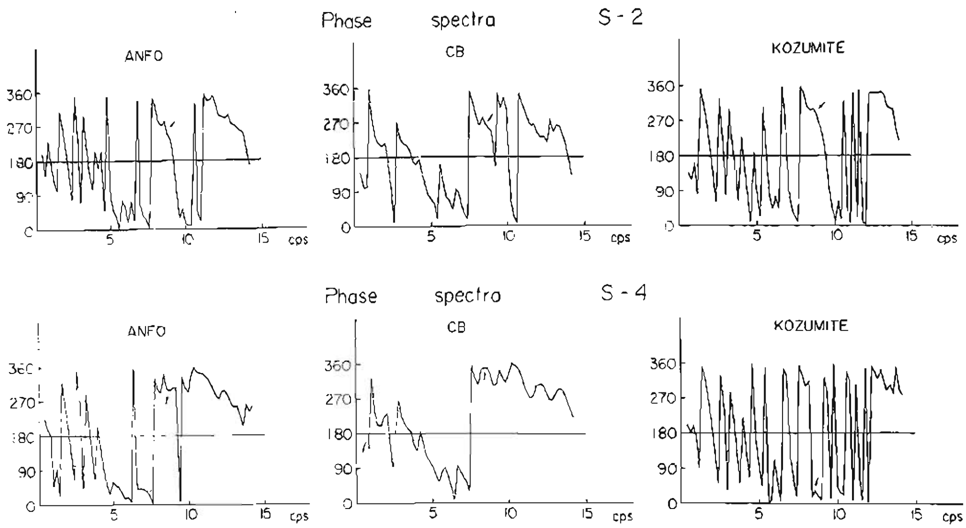



Fig. 19 Phase spectra of displacement by blasts

origin should be given and it has in fact been given theoretically as a model of spherical origin radiating P waves, It is shown that the spectrum has a gentle slope of one peak related to the amount of explosive, the depth and the geological structure of the origin.

In this case the origin was about 300 m distant from the observation point and the depth of the explosive was about 10-15 m. From the boring data at the place of blast, the sandy layer was observed up to the depth of 30 m below the ground surface. It can be assumed that the head wave reaching the ground surface was almost vertical since the propagating velocity of S waves at the granite level of the base rock was more than 2 km/sec and that in the surface layer was 0.2-1.0 km/sec.

The explosive used were ANFO explosive, C. B. explosive and Kozumite, regard being paid to the different detonation velocities of the spectrum which are 2600, 5000, and 7100 m/sec, respectively. The amount of explosive was 20 kg in each case and the records are shown in Fig. 17. The interval of the analysis was 2 sec. from the beginning of the S wave to the end of its main motion.

Some results of the analysis are shown in Fig. 18 and 19. In the figures the amplitude spectra at S-1, S-2 and S-3 in the case of ANFO, C. B. and Koz. are shown in the direction N60°E. The differences of the spectra due to the detonation velocities can be observed, since the other conditions are the same.

In Fig. 20 the ratios of the amplitude spectra, S-1′/S-3 and S-2/S-3 are shown to allow an examination of the vibration of the spectrum caused by the surface layer. The characteristics of the spectra at each point (due to the differences of detonation velocity, that is, the difference of the spectrum of the incident wave) do not appear in this figure which is expressed by the ratio of the component amplitudes. These ratios may be regarded as related to the vibrational characteristics of the ground only, and conceived as follows. When the direction of the ray path of the S wave in the upper layer is almost in the vertical direction, the direction of the particle motion will be in the horizontal plane and hence the theory of the multiple reflections of the SH wave is applied approximately in this case. By use of the geological structure presumed from the boring data, the theoretical curve of the amplitude spectrum on the surface is shown in Fig. 20.

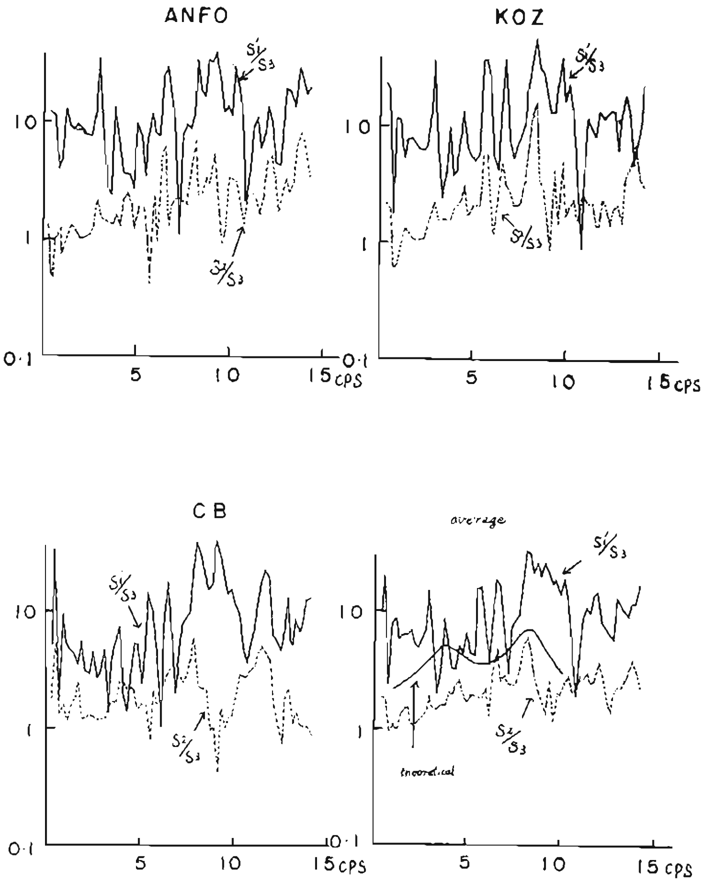



Fig. 20 Ratios of amplitude spectra: $S_1/S_3$, $S_2/S_3$

A similar tendency may be seen here, though it does not coincide perfectly because the incident condition is rather different and the surface wave is included more or less in the interval of the analysis. Comparing the results with that of the natural earthquake (shown in Fig. 5), a peak appears at 8 cps which corresponds to the vibrational characteristics of the surface layer in both cases and hence it may be possible to check the vibrational characteristics of the ground by the blast test.

## 7. Discussion and conclusion

The variations in the vertical direction of the amplitude ratios at peaks of 4 cps and 8 cps which appeared in the amplitude spectra of the natural earthquakes are shown in Fig. 7. At 4 cps the amplitude is 6–8 at a depth of 10 m and 5 at 40 m, when the amplitude at the surface is taken as 10 and these are decreasing monotonously along the vertical line and at 8 cps, the amplitude is 2–6 at 10 m and less than 1 at 40 m, though the values are rather scattered. The phase at 4 cps on the ground surface and below ground coincide, but, is reversed at 8 cps between 10 m and 40 m. From the above results it is presumed that the peak at 8 cps is affected by the boundary between 10 m and 40 m and at 4 cps by a boundary deeper than 40 m. Let us assume three parallel layers, whose thicknesses, densties, rigidities, and propagating velocities of S waves are $H_1$, $\rho_1$, $\eta_1$, $V_1$ ; $H_2$, $\rho_2$, $\eta_2$, $V_2$, respectively. An infinite harmonic wave is assumed to be incident below and to travel upwards. Taking the origin of the axis at the lowest boundary and the x-axis in the vertical direction upwards, and letting $U_0$ be the displacement of the incident wave, $U_1$ in the first layer, $U_2$ in the second layer, and $U_3$ in the third layer, we get the following equations, taking the real parts[10]:

$$U_0=\cos(pt-f_3x) \tag{1}$$

$$U_1=\frac{2R_1}{\sqrt{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}\right) \tag{2}$$

$$U_2=\frac{2R_2}{\sqrt{P^2+Q^2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}\right) \tag{3}$$

$$U_3=\frac{2P}{\sqrt{P_2+Q_2}}\cos\left(pt-\tan^{-1}\frac{Q}{P}\right) \tag{4}$$

and

$$\left.\begin{aligned}
P&=2\cos\frac{pH_1}{V_1}\cdot\cos\frac{pH_2}{V_2}-2\alpha\sin\frac{pH_1}{V_1}\cdot\sin\frac{pH_2}{V_2}\\
Q&=2\gamma\cos\frac{pH_1}{V_1}\cdot\cos\frac{pH_2}{V_2}+2\beta\sin\frac{pH_1}{V_1}\cdot\sin\frac{pH_2}{V_2}\\
R_1&=\cos\left(1+\frac{H_2}{H_1}-\frac{x}{H_1}\right)\frac{pH_1}{V_1}\\
R_2&=\cos\frac{pH_1}{V_1}\cos\left(1-\frac{x}{H_2}\right)\frac{pH_2}{V_2}-\alpha\sin\frac{pH_1}{V_1}\cdot\sin\left(1-\frac{x}{H_2}\right)\frac{pH_2}{V_2}
\end{aligned}\right\} \tag{5}$$

$$\alpha=\frac{\rho_1V_1}{\rho_2V_2},\ \beta=\frac{\rho_1V_1}{\rho_3V_3},\ \gamma=\frac{\rho_2V_2}{\rho_3V_3},\ \frac{2\pi}{p}=T,\ \frac{2\pi}{f_3}=V_3T \tag{6}$$

As can be seen in Fig. 3, two main boundaries are observed in the present earthground, and another boundary might exist. However, two apparent predominant periods measured on the ground surface will correspond to these two boundaries, and the geological structure shown in Fig. 21 might reasonably be assumed in this case. From the fact that the wave which has a peak value at 8 cps is reversed in phase between M (S-2) and L (S-3) and its amplitude is not montonously decreasing with the depth and scattering, the boundary between the lst layer and the 2 nd layer is supposed to be inbetween the granitic weathering soil and the weathered granite. The mean velocity of the S wave is 0.4 km/sec in the first layer, 1.2 km/sec in the second layer and 2.8 km/sec in the third layer and its boundary is assumed to be at the depth of 70–80 m ; then substituting the value of $\rho_1=\rho_2=\rho_3$, $V_1 : V_2 : V_3=1 : 3 : 6$, $H_1 : H_2=1 : 4$ in eqs. (1)–(6), we obtain the vibrational mode of the earthground as shown in Fig. 21 in relation to the frequencies 3.5–4 cps and 7–8 cps and the spectrum on the ground surface when the particle velocity of the incident wave is constant. The abscissa is $\omega H_1/V_1$ and the ordinate is the displacement. This theoretically obtained spectrum almost coincides with the experimental results from the natural earthquakes, the blasts and the oscillator.

As stated above, the vibrational characteristics are obtained by the statistical method for the natural earthquake and vibration tests on the earthground, where the geological structure is relatively complicated. These three methods are different from each other, however, fairly good agreement is found as to the characteristics spectrum of the ground.

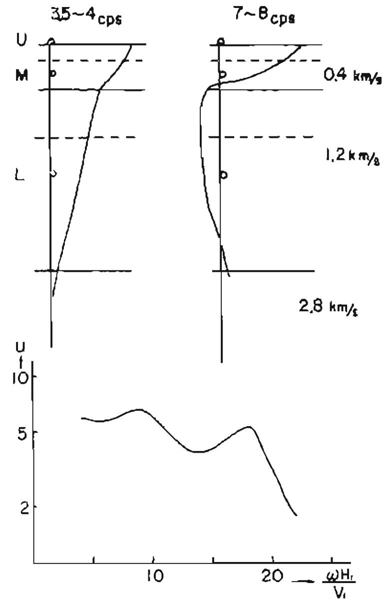



Fig. 21 Upper; Theoretical vibration mode dependent on depth in vicinity of 3.5~4 c. p. s. and of 7~8 c. p. s.
Lower; Theoretical amplitude spectrum of the ground surface.

In order to investigate vibrational characteristics, a natural earthquake has been used, but the incident earthquake wave will be different in each case and hence many seismograms have to be obtained and statistical data have to be gathered over a long period which is rather troublesome in practice. It has been made clear that the vibrational characteristics of the surface layer for the S wave can be obtained by a blast analysis in which the amplitude ratio between the ground surface and below ground is taken the observation in the boring hole is available. In this case, a seismogram near the explosive origin can not be obtained and hence only the vibrational characteristics above the point S-3 (deepest of all) can be defined. However, the spectrum of the ground motion due to earthquakes may be obtained regarding the deeper layer when the blast origin is deeper and the spectrum at the origin is given.

Vibration tests have been considered as very difficult due to the distortion of the wave form, the vibration of a higher mode caused by the rocking motion of the oscillator and the inefficiency of instruments. In this experiment the instrument has been set firmly to the base and with the improvement in efficiency a very good sine wave has been obtained. When the wave radiated from the point source such as an oscillator is calculated theoretically the relations between the vibrational characteristics and the geological structure may be explained and the earthquake motion can be calculated experimentally even at the place where the geology is complicated so long as an undistorted sine wave is available as it was in this case. However, at present the only way is to excite the ground so as to emit an SH wave and the method of observation layout is to measure its multiple reflection in the vertical direction.

In the present measuremets, the observations in the direction of the joints when excited in the same direction agree with condition and the measured wave is also of the SH type. The agreement between the spectrum of the observed wave and that of the natural earthquake shown the validity of the above consideration.

These two artificial methods for determining the vibrational characteristics of the ground may be performed over a relatively short period and with the development of experimental techniques and theories will be a useful clue for the investigation of the aseismic characteristics of the ground.

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