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Kyoto University
Study on the Relation between Local Earthquakes and Minute Ground Deformation

Part 4. On Spectral Structures of the Tiltgrams Observed at Akibasan, Wakayama City

By Torao Tanaka

(Manuscript received December 25, 1967)

Abstract

Spectral structures of the tiltgrams obtained by tiltmeters of the horizontal pendulum type at the Akibasan station in Wakayama City are discussed, especially in connection with meteorological changes. It is shown that the change of a period from 1 to 33 days in the ground tilt at the station is scarcely influenced by changes in the atmospheric temperature, pressure, its time derivative and loading of the water in the neighboring sea. It is also shown that the effect of precipitation on the ground tilt is on the contrary very remarkable and that the ground tilt caused by an impulsive precipitation is approximately represented by $at \exp(-\beta t)$ with equal values of $\beta$ for both tilting components of the E-W and N-S directions. Corrected spectra for the disturbances are given, which suggest predominance of several oscillatory tilting motions of the ground.

Introduction

It is a well known fact that crustal deformations are observed before and after large earthquakes near their epicentral regions and that continuous observations of the movement of the ground are regarded as one of the promising ways of predicting the occurrence of earthquakes. However, the ground near the surface is also deformed by other external sources such as meteorological changes, loading effects due to sea water, underground water conditions and so on. Therefore it is essential for identification of the substantial deformation caused by the internal origin of the earth relating to earthquake occurrences to investigate the above-mentioned disturbing phenomena as circumstantially as possible and to bring their generating mechanism to light.

In the previous work\(^1\), Fourier spectra of the tiltmetric results observed at the Akibasan station were shown regarding the periodic range from 11 to 33 days. It is advantageous to investigate spectral structures of records observed by tiltmeters, extensometers or gravimeters, in order to obtain knowledge about the nature of the secular movements of the ground or that of the drift curves, and to construct appropriate filters for data processing. In this paper, a rough estimate of the contamination of the tilting movements due to meteorological origins in the tiltmetric data has been made from the spectral structures of the period from 1 to 33 days under some simplified assumptions.
Fourier Spectra

In the previous paper\(^1\) we calculated Fourier spectra of the tiltgrams observed at Akibasan by the tiltmeters of the horizontal pendulum type and the results are reproduced in Figs. 1 and 2, together with those for the shorter periodic range from 1 to 11 days. The duration of the analysis is one year, from July 31, 15h00m., 1960, to July 31, 14h00m, 1961 (UT). In these figures, the angular frequency and period are given in cycle per \(2\pi\) hours and day.

These spectra are distorted by various disturbing factors, the major ones among which are thought to be the loading by sea water, atmospheric pressure, temperature and precipitation. Hence, in order to examine the loading effect of sea water on the spectrum of the tiltmetric record, we have calculated the Fourier spectrum of the tidal record observed at the Wakayama Harbour Tidal Station which is run by the Wakayama Meteorological Observatory. Similarly Fourier spectra of the temperature, atmospheric pressure, time derivative of the pressure and precipitation have also been obtained, the results of which are shown in Figs. 3 to 7. The spectrum of the time derivative of the pressure, mb/hour, is used tentatively in place of that of its spatial derivative for appraising approximately the influence due to it. All of the durations of the analysis are of course equal to that of the tiltgram, being namely from July 31, 15h00m, 1960, to July 31, 14h00m, 1961 (UT). The barogram used in this analysis is that observed at Akibasan by an aneroid type barometer and the rest of the meteorological data is that which was observed at the Wakayama Meteorological Observatory. In the present case we have provisionally adopted the intensity of precipitation, mm per one hour, to represent its pattern, which is considered as the time derivative of precipitation. Since the distance from

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Fig. 1. Spectral amplitude \(A(\omega)\) and phase \(\phi(\omega)\) of the E-W component tilt observed at Akibasan. The cross represents the corrected values for oceanic tides, temperature, atmospheric pressure and its time gradient.
Fig. 2. Spectral amplitude and phase of the N-S component tilt observed at Akibasan. The cross represents the corrected values for oceanic tides, temperature, atmospheric pressure and its time gradient.

Fig. 3. Spectral amplitude and phase of the sea level at Wakayama Harbour.
Fig. 4. Spectral amplitude and phase of the temperature at the Wakayama Meteorological Observatory.

Fig. 5. Spectral amplitude and phase of the atmospheric pressure at Akibasan.
Fig. 6. Spectral amplitude and phase of time gradient of the pressure at Akibasan.

Fig. 7. Spectral amplitude and phase of the intensity of precipitation at the Wakayama Meteorological Observatory.
Akibasan to the Wakayama Meteorological Observatory is about 3 km, the temperature and precipitation would not be, strictly speaking, equal to those at Akibasan. However, we have not practiced meteorological observations except for pressure at Akibasan and could not help using these data, although it is desirable to use values observed at the same place where observations of the ground movement have been carried out.

As seen in the figures, each spectrum shows its own characteristic feature. The tidal record spectrum shows a gradual decrease in amplitude as the angular frequency increases and no prominent peaks except that for the period of 32.1 days. In the temperature spectrum, several peaks of amplitude are seen in this range. Remarkable peaks for periods of 26.9, 13.9 and 9.4 days are seen in the amplitude spectrum of the pressure and a similar tendency is also recognized in that of its time derivative, since the latter was obtained by multiplying \( j \omega \) by the former, as is well known. Comparing the precipitation spectrum with those of the ground tilt, it is obvious that they bear a striking resemblance to each other, not only as to the coincidence of peaks but also as to all aspects of the amplitude and phase spectra. From this, we can deduce that the deformation of the ground due to precipitation has an important influence on the tiltmetric record at Akibasan in the present frequency range.

**Reduction of the Fourier Spectra**

An attempt is made under some simplified assumptions in this section to obtain an estimate of the above-mentioned disturbances and to know the general tendency of the Fourier spectrum of the ground tilt free from them.

Let \( f(t) \) be a sequence of observed values which relate to the ground deformations, for example, an observed ground strain or tilt in an arbitrary direction obtained from an extensometer or tiltmeter. Since \( f(t) \) is usually disturbed by meteorological and oceanic effects, it is reasonable to consider \( f(t) \) as a function including these disturbances as parameters. Thus, \( f(t) \) is represented as

\[
\mathbf{A} = \mathbf{f}(t) = \{f(t, a(t), b(t), c(t), \ldots)\}
\]

where \( a(t), b(t), c(t), \ldots \) are sequences of the disturbing factors which bring about the ground deformation. It may be assumed that they affect it independently of one another, so that we can then express the deformation \( f(t) \) as the sum of the deformations caused by these factors. Accordingly, the above equation may be rewritten as follows;

\[
f(t) = g(t) + \{a(t)\} + \{b(t)\} + \{c(t)\} + \ldots
\]

where \( g(t) \) represents the sequence of the ground deformation due to internal origins within the earth only and the bracket means the sequence corresponding to the deformation caused by each disturbing factor. When the condition that the deformations \( \{a(t)\}, \{b(t)\}, \{c(t)\}, \ldots \) are proportional to the amount of changes of the disturbing factors \( a(t), b(t), c(t), \ldots \), respectively is satisfied, then we get

\[
f(t) = g(t) + \epsilon_1 \cdot a(t) + \epsilon_2 \cdot b(t) + \epsilon_3 \cdot c(t) + \ldots
\]
Strictly speaking, this is not always the case but the coefficients $c_1$, $c_2$, $c_3$, ..., are regarded as variables with time $t$ and frequency $\omega$ of the changes of the factors. Indeed, the mode of the daily tilting diagram of the ground caused by the temperature shows seasonal variations at Akibasan for example. However, no definite representation has yet been obtained about these coefficients, and we assume that equation (1) holds as an approximation.

We determined the provisional values for the relation between the ground tilt observed at Akibasan and oceanic tides, temperature, atmospheric pressure and its time derivative in the previous paper. The procedure of determination is given briefly as follows.

Let $f_E(t)$ and $f_N(t)$ be the E-W and N-S components of the ground tilt at Akibasan, and similarly $w(t)$ the sea level recorded at the Wakayama Harbour Tidal Station, $y(t)$ the temperature observed at the Wakayama Meteorological Observatory, and, $p(t)$ and $p'(t)$ the atmospheric pressure and its time derivative, respectively. In the following we shall denote the E-W and N-S components of the ground tilt by the suffixes $E$ and $N$, and take the increase of $f_E(t)$ and $f_N(t)$ as the west and southward tiltings, respectively. After eliminating the drift from each record by Pertzev's method, the mean daily variations were calculated from one month's data. Then, forming $c_E$ and $c_N$, the ratios of the amplitudes of the $M_2$ constituent of $f_R(t)$ and $f_N(t)$ to that of $w(t)$, we subtracted the products $c_E \cdot w(t)_{\text{mean}}$ and $c_N \cdot w(t)_{\text{mean}}$ from $f_E(t)_{\text{mean}}$ and $f_N(t)_{\text{mean}}$, respectively, where the subscript $\text{mean}$ denotes the mean daily variation. In this case, it was assumed that the oceanic influence on the ground tilt at Akibasan was proportional only to the change of sea level at Wakayama Harbour independently of the periods of the tidal constituents and that the ratio was equal to that of $M_2$, namely $c_E$ or $c_N$. Considering that $f_E(t)_{\text{mean}} - c_E \cdot w(t)_{\text{mean}}$ and $f_N(t)_{\text{mean}} - c_N \cdot w(t)_{\text{mean}}$ consist of the mean daily variations of the ground tilts caused by those of the atmospheric pressure, its time derivative and temperature, we determined the coefficients for the three factors by the method of least squares. The mean values of these coefficients obtained from the one year's data from August, 1960 to July, 1961, are shown in Table 1, together with the ratios of the $M_2$ amplitude of the ground tilt to that of the oceanic tides.

Assuming that these values are constant to time $t$ and independent of the frequency $\omega$ of change of the disturbing factors as mentioned above, then the

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<td>E-W component</td>
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<tr>
<td>Sea level at Wakayama Harbour : $w(t)$</td>
<td>0.00035 (second/cm)</td>
</tr>
<tr>
<td>Temperature at the Wakayama Meteorological Observatory : $y(t)$</td>
<td>0.0039 (second/°C)</td>
</tr>
<tr>
<td>Atmospheric pressure at Akibasan : $p(t)$</td>
<td>-0.0014 (second/mb)</td>
</tr>
<tr>
<td>Time derivative of the pressure at Akibasan : $p'(t)$</td>
<td>0.0038 (second/mb/hour)</td>
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ground tilts to be observed at Akibasan may be written, according to (1), as

\[ f_E(t) = 0.00035w(t) + 0.0039y(t) - 0.0014p(t) + 0.0038p'(t) + [r(t)]_E + g_E(t), \]

\[ f_Y(t) = 0.00020w(t) - 0.0029y(t) + 0.000047p(t) - 0.0028p'(t) + [r(t)]_Y + g_Y(t), \]

where \([r(t)]\) is the ground tilt caused by precipitation, and \(g(t)\) is the residual term including the substantial tilting movement of the ground. The reason why we particularly discriminate only the ground tilting due to precipitation from the other disturbances is that the relation between them is not so simple as to be represented by proportional expression, which will be discussed later in some detail.

Firstly, in order to evaluate the influences due to changes in the sea level \(w(t)\), temperature \(y(t)\), atmospheric pressure \(p(t)\) and its time derivative \(p'(t)\), we have calculated the Fourier transforms

\[
F_E'(\omega) = \mathcal{F}f_E(t) + G_E(\omega) = F_E(\omega) - 0.00035W(\omega) - 0.0039Y(\omega) + 0.0014P(\omega) - 0.0038P'(\omega),
\]

\[
F_Y'(\omega) = \mathcal{F}f_Y(t) + G_Y(\omega) = F_Y(\omega) - 0.00020W(\omega) + 0.00029Y(\omega) - 0.000047P(\omega) + 0.0028P'(\omega),
\]

using the values given in the figures, where \(F(\omega)\), \(W(\omega)\), \(Y(\omega)\), \(P(\omega)\), \(P'(\omega)\), \(\mathcal{F}W(\omega)\) and \(G(\omega)\) are the Fourier transforms of \(f(t)\), \(w(t)\), \(y(t)\), \(p(t)\), \(p'(t)\), \([r(t)]_E\) and \(g(t)\), respectively. The results are shown by the crosses in Figs. 1 and 2. As seen in these figures, the corrected spectra \(F_E'(\omega)\) and \(F_Y'(\omega)\) show only a slight difference in comparison with the original ones. Even though there are errors due to the approximations in the results, it seldom seems to happen that distortions of the spectra are two or three times as large as those in the present results. From this it is concluded that the observed ground tilts in the period from 1 to 33 days are scarcely disturbed by the loading effect of the near sea water, temperature, pressure and its time derivative, if the relation between these disturbing factors and consequent ground tilt determined from the periodic range within one day is assumed to be extrapolated to the phenomena of the present range with which we are concerned.

It is well known that heavy rainfall brings about ground deformations near the surface, and this is no exception in the case of Akibasan. The disturbance remains even after rainfall has ceased, and it generally needs two or three weeks before the disturbance fades away. The shape of the changes in the ground tilt and strain appearing on the records seems to be expressed by \(at \exp(-\beta t)\).

At Ide Observatory, M. Takada observed that the effect of rainfall on the ground tilt and strain generally appeared when the amount exceeded 15 mm, reached its peak two days after rainfall commenced and lasted two or three weeks. He thereupon adopted triangles with their vertices at two days after rainfall and bases of 15 or 20 days long as an approximate expression of the effect. S. Takemoto showed that the ground tilt and strain due to rainfall were given by the sum of two exponential functions, namely \(at \exp(-\beta t) + \gamma t \exp(-\alpha t)\), at Iwakura Observatory, Kyoto. As to the extensometric record at Osakayama Observatory, I. Ozawa calculated the weighting function of pre-
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Precipitation according to the method proposed by C. Tsuboi, under the assumption that the rate of the change of the strain \( F(t) \) caused by precipitation is expressed as

\[
F(t) = \int_0^{2\pi} f(t + \alpha) \phi(\alpha) d\alpha,
\]

where \( f(t) \) and \( \phi(t) \) are the daily precipitation and weighting function, respectively, and that \( F(t) \) and \( f(t) \) are both expandible into Fourier series within the interval \( 0 \leq t \leq 2\pi \).

Taking these instances into account, it is conceivable that the phenomenon is equivalent to that of a transformation through a transducer, where its input and output are time series corresponding to precipitation and ground deformations, respectively. Therefore, we put the relation between the intensity of the precipitation \( r(t) \) and consequent ground tilt \( [r(t)] \) as

\[
[r(t)] = \int_{-\infty}^{\infty} r(t - \tau) h(\tau) d\tau,
\]

where \( h(t) \) is the impulse response of the ground tilt to the precipitation. In this case, the Fourier transform \( R(\omega) \) of \( [r(t)] \) is given by

\[
R(\omega) = \mathcal{F}(r(t)) \cdot \mathcal{F}(h(t)) = R(\omega) \cdot H(\omega),
\]

where \( R(\omega) \) and \( H(\omega) \) are the Fourier transforms of \( r(t) \) and \( h(t) \), respectively. Hence, in order to estimate the impulse response, we divide \( F_x'(\omega) \) and \( F_y'(\omega) \) by \( R(\omega) \) shown in Fig. 7, although they consist not only of the effect of the precipitation but also of the substantial crustal tilting and errors due to approximations. The results obtained are given in Fig. 8.

![Fig. 8. Ratio of the corrected spectrum of the ground tilt to that of the intensity of precipitation. ○: E-W component, ●: N-S component.](image-url)
Apparently the phase spectrum indicates a systematic aspect, from which it is deduced that the spectrum of the ground tilts is mainly distorted by the effect of precipitation. Rewriting the results on the linear coordinate of the angular frequency $\omega$ and smoothing the spectral amplitude, final results are obtained, which are shown in Fig. 9. As is seen in the figure, the phase spectra of the E-W and N-S components of the ground tilt are considered to be identical.

![Fig. 9. Ratio of the corrected spectrum of the ground tilt to that of the intensity of precipitation. Solid curves denote the amplitude and phase spectra of the impulse response $h_E(t)$ and $h_N(t)$.](image)

Now, if we assume that the response of the ground tilt to an impulsive precipitation is expressed by

$$h(t) = \begin{cases} \alpha t \exp(-\beta t), & (t \geq 0) \\ 0, & (t < 0) \end{cases},$$

then its amplitude and phase spectra, $A(\omega)$ and $\phi(\omega)$, are given as

$$A(\omega) = \frac{\alpha}{\beta^2 + \omega^2},$$

$$\phi(\omega) = \tan^{-1} \frac{2\beta \omega}{\beta^2 - \omega^2},$$

respectively. It is observed from the figure that the frequency corresponding to $\phi = \pi/2$ is 0.021, and we get $\beta = 1/48$ by (3), considering the spectral phase shift of $\pi$. Therefore, it is concluded that the ground tilt attains its maximum about 48 hours after the impulsive precipitation. With $\alpha = 0.00036$ and $\alpha = 0.00013$ for the E-W and N-S components, we obtain

$$A_E(\omega) = \frac{0.00036}{(0.021)^2 + \omega^2}, \quad A_N(\omega) = \frac{0.00013}{(0.021)^2 + \omega^2},$$

$$\phi(\omega) = \tan^{-1} \frac{0.042\omega}{(0.021)^2 - \omega^2} - \pi.$$
These are shown by solid curves in Fig. 9, from which it is deduced that the impulse responses of the ground tilt to precipitation, \( h_E(t) \) and \( h_N(t) \), are given as

\[
\begin{align*}
    h_E(t) &= -0.00036 t \exp(-0.021 t), \\
    h_N(t) &= -0.00013 t \exp(-0.021 t),
\end{align*}
\]

respectively. In Fig. 10 the curves of \( h_E(t) \) and \( h_N(t) \) are shown schematically. From this we can see that the ground inclines north-eastwards after precipitation. Since the observation site is in the tunnel at the foot of the west side of the hill called Akibasan, the tilting motion contradicts apparently the tendency pointed out by K. Hosoyama that mountains and hills bulge due to precipitation. However, it must be noticed that \( h_E(t) \) and \( h_N(t) \) obtained above may be put as averages throughout the year and vary to some degree on each occasion, according to such conditions as the temperature of the rain and surface of the ground, humidity and so on.

Finally, using these impulse responses, namely by subtracting the products \( H_E(\omega) \cdot R(\omega) \) and \( H_N(\omega) \cdot R(\omega) \) from \( F_E'(\omega) \) and \( F_N'(\omega) \) respectively, we eliminate the effects of the precipitation. The results are shown in Figs. 11 and 12. We can see several peaks in these amplitude spectra; they are for the periods of 29.8, 25.2, 20.3, 17.7, 15.2 and 9.4 days in the E-W component, and 32.1, 29.8, 25.2, 21.4, 19.8, 15.2, 11.1, 9.4 and 7.5 days in the N-S component. Those for the periods of 15.2, 11.1 and 9.4 days in the latter component are especially conspicuous. However, some ambiguity remains due to the simplified assumptions in the results, and not only should the method used be further investigated but more observational data must also be employed to clarify general aspects of the long period movements of the ground and bring their origins to light.

It is to be noted that the disturbing ground tilt due to precipitation can be estimated by convolving the impulse response and intensity of the precipitation. As an example, the calculated results and observed ground tilt are compared in Fig. 13, where the linear deflection of the observed ground tilt in the E-W component has been removed for convenience of comparison. Fair coincidence is seen between the two for both components, and from this it is concluded that we can approximately eliminate the disturbance caused by precipitation.
Fig. 11. Corrected amplitude and phase spectra of the E-W component of the ground tilt at Akibasan.

Fig. 12. Corrected amplitude and phase spectra of the N-S component of the ground tilt at Akibasan.

by this procedure, which is thought to be of good use for identifying small anomalous ground deformations.

Summary

To summarize, we have made an estimate of the disturbances due to sea water and meteorological origins on the spectrum of the ground tilt observed at Akibasan, under some simplified assumptions. The spectral structures of the
ground tilt in a periodic range from 1 to 33 days are mainly distorted by the precipitation near the observation site, and the changes in meteorological factors, other than precipitation, and in the level of the nearby sea do not have so much influence on the ground tilt. The disturbance by precipitation has been determined under the assumption that it is expressed by the intensity of precipitation. The obtained responses of the ground tilt to an impulsive precipitation are given as $-0.00036 t \exp(-0.021 t)$ and $-0.00013 t \exp(-0.021 t)$ for the E-W and N-S components of the ground tilt, respectively, where the units of time and tilt are hour and second in angle, and positive values mean the westward and southward inclinations, respectively. Accordingly, the ground tilts towards the north-east by precipitation and reaches the maximum two days afterwards. These expressions can also be used to eliminate the ground tilt caused by precipitation from observed results. The corrected spectra of the ground tilt for the disturbing factors above-mentioned are also given, in which some prominent oscillating motions of the ground are observed.

Acknowledgement

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Center, University of Tokyo.

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4) loc. cit., 3).


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