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Mechanical Behavior of Soils under Shearing Stress

(In the Case of Cohesionless Soil)

by Sakuro MURAYAMA

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Introduction

This is a brief abstract to serve as a quick report on the theoretical approach to the mechanical behavior of soils including sands and clays. A full description of this subject will be published in another paper in the near future.

1. Stress-Strain Behavior of Sand in the Elastic State

In the case of sandy soil, the elastic state means the state where no residual strain is left after the applied stress is removed. Such an elastic state can be obtained with a sandy specimen in a triaxial compression apparatus. One of the examples is shown in Fig. 1. This figure shows a relation of the axial strain ε and the deviatoric stress $(\sigma_1-\sigma_3)$ of saturated sand obtained by repetitional loading under drainage condition whose maximum deviatoric stress was 4.5 kg/cm^2 while the mean principal stress σ_m $(\sigma_m=(\sigma_1+2\sigma_3)/3=4.0$ kg/cm$^2)$ was always kept constant. The physical properties of the sand were as follows : grain shape : angular, specific gravity of sand grain : 2.73, uniformity coefficient : 1.85, initial void ratio : 0.73. The specimen had a cylindrical form with a height of h =8.0 cm and a diameter of 3.5 cm.

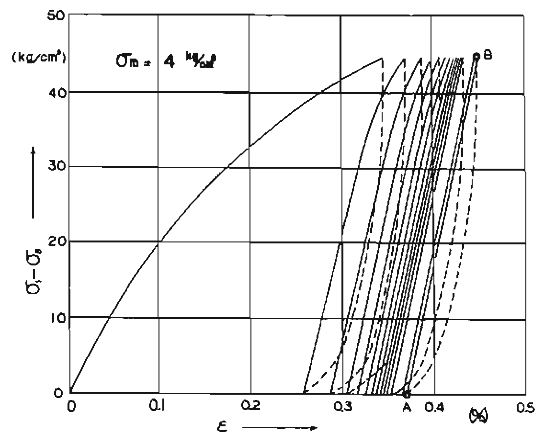

Fig. 1 Relation between axial strain and deviatoric stress under repetitional compression test with constant σ_m.

From the figure the following may be observed.

(a) Each cycle of the loading and unloading curve makes a hysteresis loop. The residual strain at the removal of the load becomes smaller as the loading repetition proceeds. The relationship between the axial deformation Δh_n and the number of repetitions n is shown in Fig. 2.

(b) As the applied deviatoric stress is repeated, the straight line part which appeares in the lower part of the loading curve becomes longer. As the upper end of the straight line indicates the elastic limit, it can be said that the elastic limit of sand increases with the number of repetitions. One of the noteworthy characteristics of the elastic part is that each straight line of the loading curve is parallel to each other.

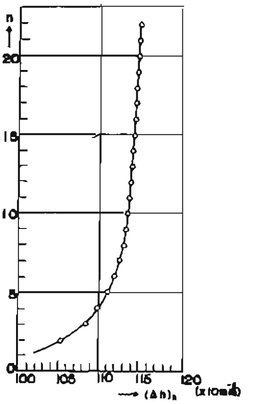

Fig. 2 The relationship between axial deformation Δh and number of repetitions n.

(c) Finally, at the 23rd repetitional loop in this test, the stress-strain hysteresis loop was fixed at the same path, and the sand specimen reached the so-called elastic state. The loading curve of the final loop is represented by the straight line AB in Fig. 1 and the elastic limit in the final loop coincides with point B. At the final elastic state, the volumetric increase due to dilatancy was only 0.1 cc at the end of loading. As the volumetric change is thus negligible, the axial strain ε can be assumed to be directly proportional to the shearing strain γ.

(d) The unloading curve does not coincide with the loading curve. The unloading curve shows a curved form, but each curve is parallel to each other within a permissible degree.

(e) The modulus of elasticity of the sand can be obtained from the inclination of the straight line AB. But this magnitude is of the order of 10^3 kg/cm^2 and is far smaller than the modulus of elasticity of sand grains whose magnitude is of the order of 10^6 kg/cm^2. Therefore, such an elasticity of sand can not be identical with the elasticity of sand grains.

2. Analytical Solution of Stress-Strain Relation of Sand in the Elastic State

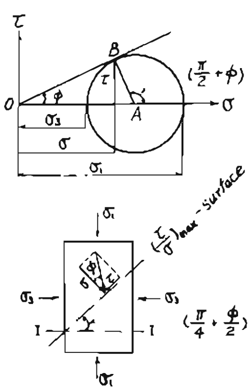

Fig. 3 Mobilized plane where (τ/σ) is maximum under triaxial compression.

The writer [1] (1964) has applied a statistical method to the mobilizing mechanism in order to deduce the following stress-strain relation in the elastic state for sand subjected to deviatoric loading.

The shearing deformation of sand under triaxial compression $(\sigma_1 > \sigma_2 = \sigma_3)$ is caused along the plane where (τ/σ) is the maximum as shown in Fig. 3. Therefore,

$$\phi = \tan^{-1}\left(\frac{\tau}{\sigma}\right)_{max} = \sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right), \quad \cdots\cdots\cdots\cdots(1)$$

where τ : shearing stress along the $(\tau/\sigma)_{max}$ -plane, σ : normal stress on the same plane, ϕ : mobilized internal resisting angle of sand mass.

But this sliding plane is a common sliding plane along which numerous mobilizing sand grains assemble. While each individual grain existing along this common sliding plane actually mobilizes along each individual contacting surface of the adjacent grain in its own direction.

Fig. 4 shows a sand grain which is ready to slide along the surface of the adjacent grain touching only at point A. The slope angle of the individual sliding surface against the $(\tau/\sigma)_{max}$-plane is expressed by θ_i. Where suffix i means the value for each individual grain. The grain is acted on by external forces through other contacting points and the resultant force of those external forces is expressed by f_{ri} in the figure. The intercepting angle between f_{ri} and the normal to the common sliding plane is denoted by β_i. Then the sliding condition of the grain can be expressed by

$$\theta_i - \beta_i + \delta \equiv y < 0, \qquad \cdots\cdots(2)$$

where δ is the frictional angle between the grain surfaces and is assumed as a constant for every grain on the average.

If the number of grains per unit area of sand mass N is large enough and the orientation of the grains is quite random and the sand is packed homogeneously, β or the arithmetic mean of β_i becomes as follows:

$$\beta = \phi \quad . \qquad \cdots\cdots(3)$$

If the applied force is reduced, the grain slides back from the final position to its initial position as far as the following condition is satisfied.

$$\beta_i < \theta_i - \delta \quad . \qquad \cdots\cdots(4)$$

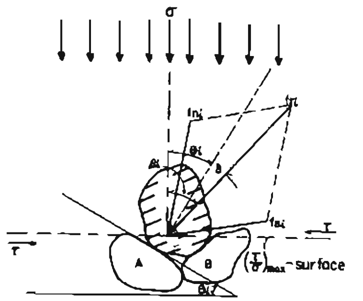

Fig. 4 Forces acting on a sand particle ready to move along the surface of the adjacent particle.

Such a reciprocating motion of grains may cause the hysteresis loop on the stress-strain diagram in the elastic state by a cycle of loading and unloading.

As the magnitudes of β_i and θ_i vary at each particle, each one may be subjected to a certain frequency distribution function. In this paper they are assumed to be represented by the following Gaussian distribution functions.

$$\left.\begin{aligned} f(\beta_i) &= \frac{1}{\sqrt{2\pi}\cdot\rho_1}\, exp\left\{-\frac{(\beta_i-\beta)^2}{2\rho_1^2}\right\}\ , \\ \varphi(\theta_i+\delta) &= \frac{1}{\sqrt{2\pi}\cdot\rho_2}\, exp\left(-\frac{\{(\theta_i+\delta)-(\theta+\delta)\}^2}{2\rho_2^2}\right)\ , \\ &\text{where} \\ \beta &= \sum^{N}\beta_i/N, \quad \theta=\sum^{N}\theta_i/N \ . \end{aligned}\right\} \qquad \cdots\cdots(5)$$

In the above equations, β and θ are mean values and ρ_1 and ρ_2 are standard deviations for the respective distribution functions.

The probability of the mobilization of particles P can be deduced as follows applying a statistical method to Eqs. (2) and (5).

$$\left.\begin{aligned} P &= \int_{-\infty}^{-(\alpha/\rho)} \frac{1}{\sqrt{2\pi}}\, exp\left(-\frac{t^2}{2}\right) dt = F\left(-\frac{\alpha}{\rho}\right)\ , \\ &\text{where} \\ \rho &= \sqrt{\rho_1^2+\rho_2^2}, \quad \alpha=\theta+\delta-\beta\ , \\ t &= (y-\alpha)/\rho \ . \end{aligned}\right\} \qquad \cdots\cdots(6)$$

Therefore P is represented by the shaded area of the Gaussian distribution curve or the ordinate at $-\alpha/\rho$ of its cumulative distribution curve as shown in Fig. 5.

If P_0 expresses the probability of the mobilization of particles under all-round uniform pressure, the increment of the probability due to the application of deviatoric stress can be expressed by $(P-P_0)$.

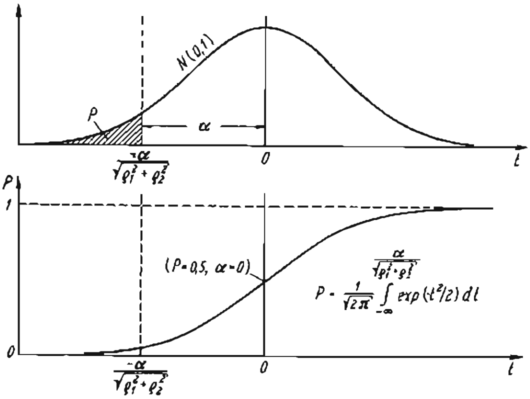

Fig. 5 Gaussian distribution curve (above) and its cumulative distribution curve(below).

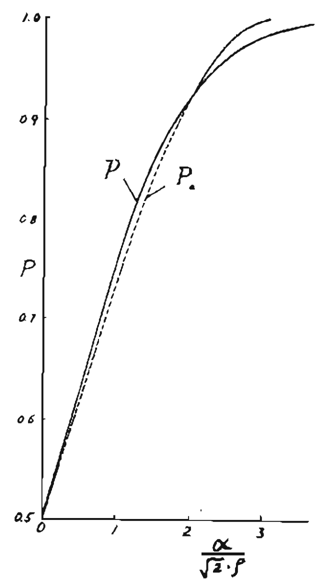

Fig. 6 Proposed sinusoidal curve to approximate the cumulative distribution curve.

$$P-P_0=F(-\alpha/\rho)-F(-\alpha_0/\rho),$$

where ....(7)

$$\alpha_0=\theta+\delta \quad .$$

As an approximate equation of Eq. (6) the following sinusoidal function is adopted.

$$P=F\left(-\frac{\alpha}{\rho}\right)\doteqdot\frac{1}{2}+\frac{1}{2}\sin\left(-\frac{\sqrt{2}}{\rho}\cdot\frac{\alpha}{2}\right)=P_a \quad . \quad \cdots\cdots\cdots(8)$$

The degree of approximation between P and P_a is shown in Fig. 6. From the preceding relations, $P-P_0$ can be approximated as

$$\frac{P-P_0}{\cos\beta}=W\cdot z\fallingdotseq P \quad ,$$

where

$$W=\frac{1}{6\rho}\cdot\cos\frac{\theta+\delta}{2} \quad , \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

$$z=\frac{\sigma_1-\sigma_3}{\sigma_m} \quad ,$$

As θ is the slope angle of the sliding surface and δ is the frictional angle between the particle surface, $(\theta+\delta)$ relates the internal resistance against shearing deformation of sand. Since the denser the sand grains interlock, the larger the angle θ becomes, the value of W decreases in proportion as the structure of the sand becomes more compact. Therefore W may be designated as the Structural Factor. In Eq. (9) z is the stress ratio of deviatoric stress and mean principal stress.

In a sand mass in the elastic state, every particle mobilized is assumed to slide a certain finite displacement λ_e on the average and then stop there half way along the slope. Moreover, every angular displacement λ/d (where d : average interparticle distance between sand particles) is assumed to contribute by $c\lambda/d$ (where c : coefficient of contribution, constant) to overall shearing strain along the common sliding plane γ_β. Therefore, γ_β is given by

$$\gamma_\beta = c(\lambda_e/d) \cdot N \cdot (P-P_0) = A_e \cdot (P-P_0) \quad . \qquad \cdots\cdots(10)$$

Using the relation of shearing strain, the maximum strain γ can be obtained as follows :

$$\gamma = \gamma_\beta/\cos\beta = A_e \cdot (P-P_0)/\cos\beta \quad ,$$

where $\qquad \cdots\cdots(11)$

$$A_e = c\lambda_e N/d \quad .$$

Substituting Eq. (9) into Eq. (11), we obtain,

$$\gamma = A_e \cdot W \cdot z, \qquad z = (\sigma_1 - \sigma_3)/\sigma_m \quad . \qquad \cdots\cdots(12)$$

The above is the relation between maximum shearing strain and deviatoric stress under the loading process. Hence the compliance is represented by AW/σ_m.

As described in the preceeding section, the loading stress-strain lines below the elastic limit are parallel to each other as far as the mean principal stress is kept constant. Therefore it can be said that the compliance within the region of the elastic state should be constant as far as σ_m is kept at the same intensity.

Hence ρ, θ and W are constants as far as σ_m remains constant.

To express the relations in the elastic state, the preceding equations are rewritten adding the suffix e.

$$\gamma_e = A_e \cdot (P-P_0)/\cos\beta = A_e \cdot W_e \cdot z, \qquad \frac{P-P_0}{\cos\beta} = W_e \cdot z \cdot P_e \ ,$$

$$W_e = \frac{1}{6\rho_e} \cdot \cos\frac{\theta_e + \delta}{2} \qquad \text{(const)} \quad . \qquad \cdots\cdots(13)$$

γ_e is the maximum shearing strain of the sand in the state. P_e is the probability of the mobilization of particles which is directly effective to the maximum shearing strain in the elastic state.

3. Disintegration of Sand Particles due to Deviatoric Stress larger than the Elastic Limit

If sand is applied by a deviatoric stress z larger than its elastic limit z_{el} under a constant σ_m, the structure of the sand begins to "disintegrate" and some sand particles mobilize over the peaks of adjacent particles. As a result of this mobilization, the sand behaves as plastic deformation. In this case Eq. (9) is also valid. Therefore if z ($>z_{el}$) is increased, then θ decreases and consequently W increases. Since P is the product of W by z, P may increase acceleratively with the increase of z. The influence of z on P or $\bar{P}$, which means the influence of z on the distribution functions expressed by Eq. (5), may be assumed as follows.

(a) If $z \leq z_{el}$ or sand is in the elastic state, we obtain from Eq. (13).

$$\frac{d\bar{P}_e}{dz} = W_e \ , \qquad \text{(const)} \ .$$

(b) When $z \geq z_{el}$, it may be supposed that the nearer z approaches a certain large value of z_∞, the smaller the value of $(z_\infty - z)$ becomes and the larger $d\bar{P}/dz$ becomes. Therefore, it may be expressed as

$$\frac{d\bar{P}}{dz} = c_1 \cdot \frac{1}{z_\infty - z} \qquad (c_1 \text{ : coefficient})$$

(c) The resistance of sand particles against mobilization is not uniformly distributed with respect to z. As the sand particles which resist against mobilization are forced to mobilize in higher rate by the applied z of higher intensity, the above presented coefficient c_1 can not remain a constant and it may also be supposed to be inversely proportional to $(z_\infty - z)$. Therefore from the above assumptions, the following may be obtained.

$$\frac{d\bar{P}}{dz} = k \cdot \frac{1}{(z_\infty - z)^2} \ , \qquad (k \text{ : const}) \qquad \cdots\cdots(14)$$

Integrating Eq. (14) and denoting $\bar{P}$ at the elastic limit as P_{el}, P in the disintegration stage is given by,

$$P^* = \frac{k}{z^*_\infty} \cdot \frac{z^*}{z^*_\infty - z^*} \ ,$$

where $\qquad \cdots\cdots(15)$

$$P^* = \bar{P} - \bar{P}_{el}, \qquad z^*_\infty = z_\infty - z_{el} \ ,$$

$$z^* = z - z_{el}, \qquad z^*_\infty - z^*_\infty = z_\infty - z$$

When the sand enters the disintegrating stage, passing through the elastic state, sand particles slide over the peaks of the adjacent particles and settle there. Therefore the sliding distance of this stage is longer than that of the elastic state. This distance is denoted as λ_p and is supposed as the distance which depends on the initial condition of compaction as far as the applied σ_m is kept constant. As the contribution of the sliding distance to the overall shearing strain may be assumed to be the same as that in the case of the elastic state, the shearing strain of the sand at this stage can be expressed by the following relation similar to Eq. (11).

$$\left.\begin{aligned} \gamma_p &= A_p \cdot p^* \ , \\ A_p &= c \cdot \lambda_p \cdot N/d \ , \end{aligned}\right\} \qquad \cdots\cdots(16)$$

where γ_p is the maximum shearing strain measured from that of the elastic limit. Therefore the total maximum shearing strain is given by

$$\gamma = \gamma_e + \gamma_p \qquad \cdots\cdots(17)$$

From Eqs. (15) and (16), γ_p is obtained as

$$\gamma_p=\frac{A_p\cdot k}{z^*_\infty}\cdot\frac{z^*}{z^*_\infty-z^*},$$

or $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ ......(18)

$$\frac{z^*}{\gamma_p}=\frac{z^*_\infty}{A_p\cdot k}\cdot(z^*_\infty-z^*).$$

From the latter equation of Eq. (18), z^*/γ_p decreases with the increase of z.

The theoretical relationship between z and γ is shown in Fig. 7, and the theoretical and experimental relationships of $z^*/\gamma_p\sim z$ are shown in Figs. 8 and 9 respectively. A similar hyperbolic representation of the stress-strain relation was presented by Kondner (1963) [2]. But there are some fundamental differences between Kondner's proposal and Eq. (18). That is : in the former proposal, the origin of the axes of the basic stress-strain relation is taken at the zero point of stress and strain instead of the elastic limit, the deviatoric stress itself is adopted as the stress unit instead of the stress ratio of z, and the ultimate strength is always predicted by the stress where the axial strain becomes infinitive. The last problem will be discussed in the next section.

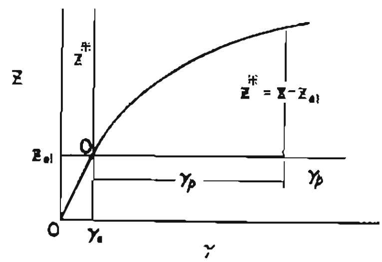

Fig. 7 Theoretical relationship between z and γ.

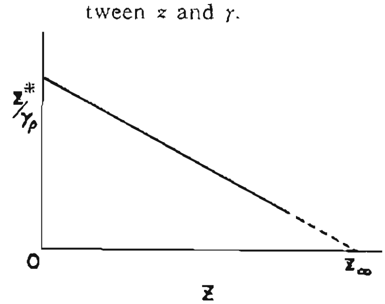

Fig. 8 Theoretical relationship of $z/\gamma-\gamma$.

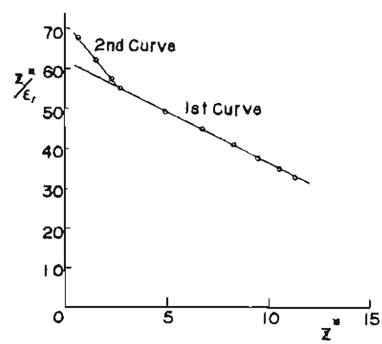

Fig. 9 Experimental linear relationship of $z/\gamma-\gamma$ constructed from the 1st and 2nd stress-strain curves at plastic state shown in Fig. 1

As for the secant modulus G_p referring to the elastic limit, the following is obtained.

$$G_p=\frac{z^*}{\gamma_p}\cdot\sigma_m=\frac{z^*_\infty\cdot\sigma_m}{A_p\cdot k}(z_\infty-z).\quad\cdots\cdots(19)$$

Therefore the real secant modulus should be composed of G_p and the elastic modulus of the elastic region.

As for the structural factor W, the following is obtained.

$$W\cdot z=\mu W_{el}\cdot z_{el}+W_0\frac{z_\infty}{z_\infty-z_{el}}(z-z_{el}),$$
$$W_0=\frac{k}{z_\infty}\cdot\frac{1}{z_\infty-z},\qquad \mu=\frac{A_e}{A_p}<1\quad\cdots\cdots(20)$$

W_0 is the structural factor of the sand whose elastic limit is zero ($z_{el}=0$). If z_{el}, is negligibly small compared to z_∞, W is given by

$$W\fallingdotseq W_0-(W_0-\mu W_e)(z_{el}/z).\quad\cdots\cdots\cdots\cdots(21)$$

4. Failure of Sands

In this report distinct terms of "disintegration" and "failure" are adopted. In the process of disintegration the elastic state of sand changes into the plastic state but the progress of mobilization of sand particles is limited to a finite degree provided the external stress ratio is kept constant. While in the case of failure, plastic mobilization of sand particles proceeds continuously even when the external stress ratio is kept constant.

The criterion for this failure phenomenon of sand can be founded on the following two conditions. The first criterion is a condition where the rate of volumetric increase due to the disintegration vanishes and the whole of the external input energy is consumed only by the continuation of internal work for the mobilization of particles. As the structural factor W increases with the progress of disintegration, it reaches its maximum value at the failure stage.

This maximum structural factor is denoted by $W_{f.1}$. The failure strength expressed by the stress ratio determined by the first criterion $z_{f.1}$ can be obtained by substituting $W_{f.1}$ into W in Eq. (20). But its actuality should be checked by comparing it with the failure strength given by the second criterion. Such failure as defined by the first criterion is often observed in tests with loose sand.

The second criterion is a condition where the probability of the mobilization of particles P reaches 0.5. This criterion may be a new proposal for the failure of soil. During the process of disintegration, sand particles mobilize alternatively and consequently their inter-particle forces are exchanged with each other. If the number of mobilizing particles is less than that of stationary ones this alternation is able to proceed smoothly, but if the case becomes the reverse proportion the alternation becomes discontinuous and vibratory irregular shearing deformation with shock becomes predominant. Due to such irregular jumping mobilization of particles, the structure of the sand disintegrates acceleratively to increase the structural factor. When the increased structural factor reaches $W_{f.1}$ then the failure defined by the first criterion immediately follows. Therefore when the probability P reaches 0.5 on the way of loading of the stress-controlled test, failure follows very quickly after the accelerative disintegration of the sand. While on the strain-controlled test, applied external stress is reduced to a value sufficient to keep the deformation at a uniform rate. Such a failure as defined by the second criterion is often observed in tests with dense sand.

Thus the failure strength defined by the second criterion $z_{f.2}$ which gives the peak strength of the sand is determined by the stress ratio where P is equal to 0.5. The structural factor at the failure denoted by $W_{f.2}$ is obtained by Eq. (9) as follows by neglecting the small value of P_0.

$$W_{f.2} = \frac{1}{2\cdot z_{f.2}\cdot\cos\beta} ,$$

or ....(22)

$$z_{f.2} = \frac{1}{2\cdot W_{f.2}\cdot\cos\beta} .$$

The maximum value of $z_{f.2}$ can be obtained by sand whose elastic limit coincides with $z_{f.2}$. Similarly from Eq. (13),

$$(z_{2\cdot f})_{max} \doteqdot (z_{el})_{max} = \frac{1}{2\cdot W_e\cdot \cos\beta_{el}} \quad . \qquad \cdots\cdots(23)$$

This is the maximum failure strength ratio which can be realized with the sand.

From Eqs. (22) and (23), we get

$$W_{f\cdot 2}\cdot z_{f\cdot 2}\cdot \cos\beta = W_e(z_{el})_{max}\cdot \cos\beta_{el} \quad . \qquad \cdots\cdots(24)$$

In the vicinity of the failure strength $\cos\beta$ may be approximately equal to $\cos\beta_{el}$. Hense,

$$W_{f\cdot 2}\cdot z_{f\cdot 2} \doteqdot W_e\cdot (z_{el})_{max} = \mathrm{const} \quad . \qquad \cdots\cdots(25)$$

This equation represents the locus where the condition of failure of the second criterion is satisfied. The relationships expressed by Eqs. (20), (21) and (25) are illustrated together in Fig. 10. Applying the above and the preceding relations and neglecting the small elastic strain compared with that at failure, the relation of stress-strain after the peak stress at failure can be represented as follows.

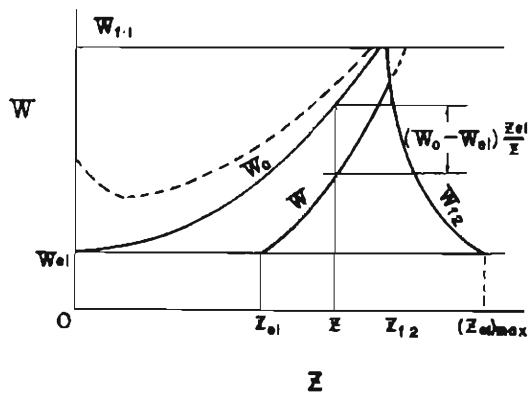

Fig. 10 Relationship between W and z, ($W_{el} \doteqdot W_e$).

$$\left.\begin{aligned} &\gamma-\gamma_{f\cdot 2} = \frac{B}{z_\infty - z_{f\cdot 2}}\cdot\frac{z_{f\cdot 2}-z}{z_\infty - z}\,, \\ &\text{or} \\ &\frac{z_{f\cdot 2}-z}{\gamma-\gamma_{f\cdot 2}} = \frac{(z_\infty - z_{f\cdot 2})}{B}\left\{(z_\infty - z_{f\cdot 2}) + (z_{f\cdot 2}-z)\right\}, \end{aligned}\right\} \qquad \cdots\cdots(26)$$

where B is constant and $\gamma_{f\cdot 2}$ is the maximum shearing strain at failure.

5. Residual Strain due to Repetitional Stress Application to Sand

On the repetitional loading, the residual strain at each end of unloading decreases with the increase of the number of repetitions and consequently the sand becomes more elastic at the end of unloading. As described in Section 1, since each stress-strain curve on the unloading process is parallel to each other even at the cycle in the elastic state within a permissible degree, the probability of the mobilization of particles under unloading can be assumed to be constant in spite of the number of repetitional loading cycles as far as σ_m is kept constant. Therefore the probability for the stabilized particles at the end of unloading which causes the residual strain is expressed by $(W\cdot z - W_e\cdot z)\cos\beta$.

Here it may be convenient to introduce a new concept of the latent plastic particle tentatively designated which has a possibility of sliding on the adjacent particle surface over its peak and then settling at a stable orientation of minimum potential under a certain stress ratio. Such latent plastic particles are distributed at random in the sand and the probability of their existence among particles in the sand as a whole is denoted by R. Then the decrement in R due to each cycle of loading and unloading whose maximum stress is z can be expressed as follows:

$$-\frac{\Delta R}{\Delta n} = R\cdot(W\cdot z - W_e\cdot z) \quad .$$

where n is the number of cycle of repetitional loading and Δn means a unit cycle of repetitional loading.

Besides, the more latent plastic particles there are, the more particles can be stabilized by the external stress exceeding the elastic limit. Therefore both kinds of particles may be assumed to be proportional to each other. Hence,

$$R = \frac{1}{k_r}(W - W_e)\cdot z, \quad (k_r : \text{const}) \quad . \qquad \cdots\cdots(27)$$

From the above equations we get

$$-\frac{dR}{dn} = k_r\cdot R^2 \quad . \qquad \cdots\cdots(28)$$

Integrating Eq. (28) under the condition that $R = R_0$ at $n = 0$, we get

$$R_0 - R = \frac{n}{a + b\cdot n} \quad ,$$

where $\qquad \cdots\cdots(29)$

$$a = \frac{1}{a_2\cdot R_0^2} \quad , \quad b = \frac{1}{R_0} \quad .$$

Since the contribution procedure of the decrement of the latent plastic particles to the residual shearing strain is the same as that of the increment of the mobilized particles during plastic deformation to the maximum shearing strain, the residual maximum shearing strain at the end of the nth cycle of repetitional loading γ_{rn} can be obtained as follows:

$$\gamma_{rn} = A_p\cdot(R - R_0) = A_p\cdot\frac{n}{a + bn} \quad . \qquad \cdots\cdots(30)$$

The ultimate value of the residual strain at the end of a large number of repetitions is given by

$$(\gamma_{rn})_{n\to\infty} = \frac{A_p}{b} = A_p\cdot R_0 \quad .$$

Eq. (30) has the same form as that experimentally obtained by Sasaki (1952) [8] on the settlement of soil by roller compaction. Fig. 11 is the relationship of $n/\varepsilon_n - n$ constructed from Fig. 2. From this figure the result found by experiment agrees well with the result predicted by Eq. (30).

The increase in elasticity of sand due to repetitional loading is represented as the elevation of the elastic limit z_{el}. The relation between z_{el} and n is expressed by the following equation which is obtained from Eqs. (15) and (29).

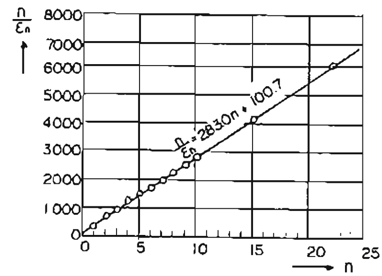

Fig. 11 Relationship of $n/\varepsilon_n - n$ constructed from Fig. 2.

$$\frac{n+1}{a+b(n+1)} - \frac{n}{a+bn} = \left\{ \frac{k}{z_\infty^*(z_\infty^* - z^*)} - We \right\} z^* \quad . \qquad \cdots\cdots(31)$$

6. Compacion of Sand due to Shearing Stress

As stated in Section 3, sand containing the stabilized particles under no shearing stress (but σ_m is definite and constant) is disintegrated by the application of the shearing stress.

However sand containing an excess amount of latent plastic particles from the beginning of loading, tends to settle in a stable orientation by the application of the shearing stress, while some of the stabilized particles in the sand are disintegrated at the same time by the shearing stress. The former behavior can be said to be the compaction process.

Therefore the existing probability of the latent plastic particle under a stress ratio z denoted by R can be classified into two kinds of particles. The one consists of the particles denoted by R_d, which is produced by the disintegration of the stabilized particles and the other consists of the particles denoted by R_c which still remains as the residual of the original compactible latent plastic particles. Hence at z

$$R = R_c + R_d \quad . \qquad \cdots\cdots(32)$$

At the beginning of the stress application where $z=0$, the relation among Rs can be represented by the suffix 0 as follows.

$$R_0 = R_{c0} \quad , \quad (\because R_{d0} = 0) \quad . \qquad \cdots\cdots(33)$$

Sand particles are mobilized by d $(R \cdot W \cdot z)$ due to the increment of dz, where W is the structural factor of the sand. As the latent plastic particles contained in the mobilized particles can be stabilized, the increment of ΔR duc to dz is expressed as

$$dR = dR_d - k_1 \cdot d(R_d \cdot W \cdot z) - k_2 \cdot d(R_c \cdot W \cdot z), \quad (k_1, k_2 \ : \text{constants}) \quad .$$

When the relations between W and z are determined, the above eauation can be integrated. If it is allowed to show the integration only in a formal expression, it becomes

$$R = R_d - k_1 \int_0^z \frac{d(R_d \cdot Wz)}{dz} dz - k_2 \int_0^z \frac{d(R_c \cdot W \cdot z)}{dz} \cdot dz \quad ,$$

$$=R_d(z)+\{R_{c0}-R_c(z)\} \quad . \qquad \cdots\cdots(34)$$

In the above expression it can be supposed that $R_d(z)$ is the increasing function with the increase of z and $R_{c0}-R_c(z)$ the decreasing one. Therefore the relation between R and z may be represented by a curve with a minimum point at a certain value of z. Besides, as the maximum shearing strain γ can be represented by the following relation :

$$\gamma = A_p \cdot W \cdot z = A_p (R_{c0}-R) \quad .$$

W can be obtained as

$$W=\frac{R_{c0}-R}{z} \quad . \qquad \cdots\cdots(35)$$

Therefore the curve of W also may be a curve with a minimum point as shown by a dotted line in Fig 10.

7. Failure Strength of Loose Sand

It can be said that loose sand is the sand which has more compactible latent plastic particles and has no elastic limit. Since the structural factor W of such loose sand is always larger than that of dense sand, W can reach $W_{f.1}$ by less stress than that of the elastic dense sand.

Moreover as the value of W is large, the value of P at the stress where W equals $W_{f.1}$ may possibly be less than 0.5. If the above condition is satisfied the shearing strength of loose sand is determined by the first criterion of failure described in Section 3. Therefore in the case of loose sand the stress-strain curve usually has no peak point and the shearing strength is less than that of dense sand.

8. Effect of Shearing Strain on Dilatancy

When a sand particle moves along the contacting surface of the adjacent particle, it deviates from the $(\tau/\sigma)_{max}$ -plane by angle θ_i. Therefore, the interparticle distance is expanded at the rate of $\lambda \cdot \tan\theta_i/d$ (λ : distance moved along the $(\tau/\sigma)_{max}$ -plane during the expanding process) for one sliding particle in the direction perpendicular to the $(\tau/\sigma)_{max}$ -plane. If this elementary expansion is assumed to contribute to the overall expansive strain of the sand ε_n perpendicular to the $(\tau/\sigma)_{max}$ -plane with the contribution factor of c, ε_n can be written by

$$\left.\begin{aligned} &\varepsilon_n = \{c\lambda N(P-P_0)/d\}\cdot\tan\theta_e \;, \\ &\tan\theta_e=(P\cdot\tan\theta_m-P_0\cdot\tan\theta_{m0})/(P-P_0)\;. \end{aligned}\right\} \qquad \cdots\cdots(36)$$

Where $\tan\theta_m$ and $\tan\theta_{m0}$ are the mean value of $\tan\theta_i$ of the particles mobilized at P and P_0 respcetively.

If the value of θ_i is small, $\tan\theta_m$ may be obtained approximately from Eqs. (2) and (5) as the tangent of the mean value of θ_i as follows.

$$\left.\begin{aligned}
&\theta_m = \frac{1}{P}\int_{-\infty}^{0} \theta_i \cdot p(y) \cdot dy \quad , \\
&\text{where} \\
&p(y) = \frac{1}{\sqrt{2\pi}\,\rho} \cdot exp\left\{-\frac{(y-\alpha)^2}{2\rho^2}\right\} \quad , \\
&y = \theta_i + \delta - \beta_i \cdot \qquad \alpha = \theta + \delta - \beta \quad , \\
&\rho = \sqrt{\rho_1{}^2 + \rho_2{}^2} \quad .
\end{aligned}\right\} \cdots\cdots\cdots\cdots (37)$$

Similarly at the beginning of loading θ_{m0} is given by

$$\left.\begin{aligned}
&\theta_{m0} = \frac{1}{P_0}\int_{-\infty}^{0} \theta_i \cdot p(y) dy \quad , \\
&\text{where} \\
&p(y) = \frac{1}{\sqrt{2\pi}\,\rho_0} \cdot exp\left\{-\frac{(y-\alpha_0)^2}{2\rho_0{}^2}\right\} \quad , \\
&\alpha = \theta + \delta \quad , \quad \rho_0 = \sqrt{\rho_{10}{}^2 + \rho_{20}{}^2} \quad .
\end{aligned}\right\} \cdots\cdots\cdots\cdots (38)$$

Therefore, the volumetric change due to dilatancy can be obtained by solving the above equations.

Conclusion

This is a brief abstract to serve as a quick report on the theoretical study of the stress-strain behavior of cohesionless soil under a constant mean principal stress. A further study on cohesionless soil and the extension of this consideration to other kinds of soils will be reported on another opportunity.

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