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On the Effect of Atmospheric Pressure upon Ground Tilt

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Abstract

The Fourier transforms of the tiltmetric and barometric records at the Oura and Akibasan stations in Wakayama City have been obtained and compared to investigate the effect of atmospheric pressure on the ground tilt observed by tiltmeters of the horizontal pendulum type. The result shows that the amplitude ratio of ground tilt to pressure increases as the frequency increases and that the phase difference between them also varies slightly with the frequency. This suggests that pressure influence on the tiltmetric record consists of two parts. One is a local term which is proportional to the change of pressure and possibly related to purely local influence including instrumental disturbance. The other is proportional to the time derivative which may also be either related to the purely local effect of the pressure or related to ground deformation due to the extensive loading effect of the pressure. The same treatment has been carried out as to the tiltmetric and barometric data observed at the Amagase Observatory in Kyoto, the result of which shows that there is no contradiction of the above inference for the interpretation of the atmospheric disturbance at Amagase.

1. Introduction

Ground is generally deformed by the change of atmospheric pressure, and hence tiltmetric records are more or less disturbed by it so long as the observations of the ground are carried out in a tunnel near the earth's surface. Since G. H. Darwin[1] treated ground deformation due to the loading effect of a sinusoidal atmospheric pressure theoretically, many workers[2] have made theoretical and observational investigations of pressure effects on ground deformation. It is absolutely necessary for the analysis of earth-tides and detection of substantial ground deformation due to geotectonic origins to clarify the relation between the change of atmospheric pressure and ground deformation due to it, and to elucidate the mechanism of appearance of pressure disturbance.

Atmospheric pressure is considered to disturb tiltmetric or extensometric records in the following ways, (1) tilt or bending of the earth's crust due to loading effect, (2) purely local deformation of the ground or observational site which is related to minor geological circumstances, such as fracture zones and heterogeneity of the structure, and is related also to the geographical situation of the site including the shape of the tunnel and configuration of instruments in it, (3) underground water level, and so on.

However, even if we confine ourselves to the problem of loading effect only, there must be many factors which control the phenomenon and make treatment of this problem complicated. For example, if the block movement of the earth's crust prevails as pointed out by E. Nishimura[3], then we should take the extent of the blocks and distribution of the load by the pressure on them into consideration. Furthermore, it is doubtful whether the earth's crust behaves as a perfect elastic body or not, namely whether the linearity of the relation between the loading and consequent ground deformation holds or whether some other effect enters into the deformetion of the ground. As to the local effect, instrumental disturbance and underground water level, the pressure effect is expected to show different aspects at respective stations and instruments, and individual examination for quantitative treatments of the disturbance will be required for each station and instrument.

On the tiltmetric records at Oura and Akibasan, small fluctuations which correspond fairly well to the change of atmospheric pressure are observed, and simple treatments were carried out for these fluctuations in the previous report[1]. In the present article, we have obtained the Fourier transforms of the tiltmetric and barometric data and compared them in order to find the relation between them in more detail. Since the extensometric records have also been disturbed by the pressure in the case of Oura, they have been analysed incidentally.

2. Data and Analysis

Descriptions of the instruments and sensitivities are given in Table 1. Deflections of the image points from the barograph, tiltmeters and extensometers were recorded on the same photographic paper, the recording speed was 30 mm/hour, and the lamps were switched off every ten minutes for the time mark.

Table 1. Descriptions of the instruments.

Station	Instrument	Azimuth	Sensitivity	Symbol
Oura	Tiltmeter	E-W	0.0036″/㎜	Ao
(135° 09′.5E 34° 11′.2N)		N-S	0.0040″/㎜	Bo
	Extensometer	E-W	2.8×10^{-9}/㎜	Eeo
	(L=500㎝)	N-S	4.7×10^{-9}/㎜	Eno
	Barometer		0.18 mb/㎜	Po
Akibasan	Tiltmeter	E-W	0.0032″/㎜	Aa
(135° 10′.4E 34° 11′.8N)		N-S	0.0033″/㎜	Ba
	Barometer		0.11mb/㎜	Pa

Several data intervals in which the change of atmospheric pressure was especially large have been picked out and values for every minute were read out by a comparator. The data intervals used for the analysis are listed in Table 2. Examples of the tiltmetric and extensometric records and barograms are shown in Figs. 1 and 2. After eliminating the linear drift, their Fourier transforms have been calculated by the following conventional formula,

$$F(\omega) = \frac{1}{T} \int_{-T}^{T} f(t) e^{-j\omega t} dt$$

where $f(t)$ is the tiltmetric or extensometric record or barogram, and T is half of the time interval analysed. In the present study, the periodic range has

Table 2. The time intervals analysed.

Station	Time interval (JST)
Oura	Feb. 6, 08 : 00 — 11 : 00, 1961
	Mar. 2, 05 : 00 — 08 : 00, 1961
	Mar. 24, 02 : 20 — 06 : 00, 1961
	Apr. 12, 13 : 00 — 16 : 00, 1961
	(Data of the Eeo lacked)
	May 3, 18 : 30 — 21 : 50, 1961
	May 20, 14 : 50 — 18 : 00, 1961
	June 9, 04 : 00 — 08 : 00, 1961
	July 9, 19 : 00 — 23 : 00, 1961
Akibasan	Sep. 12, 11 : 00 — 13 : 10, 1960
	Sep. 14, 06 : 10 — 08 : 20, 1960
	Nov. 10, 00 : 08 — 03 : 08, 1960
	Nov. 12, 18 : 28 — 22 : 58, 1960
	Nov. 12, 22 : 58 — Nov. 13, 02 : 28, 1960
	Dec. 9, 03 : 30 — 06 : 40, 1960
	Dec. 11, 23 : 30 — Dec. 12, 01 : 30, 1960

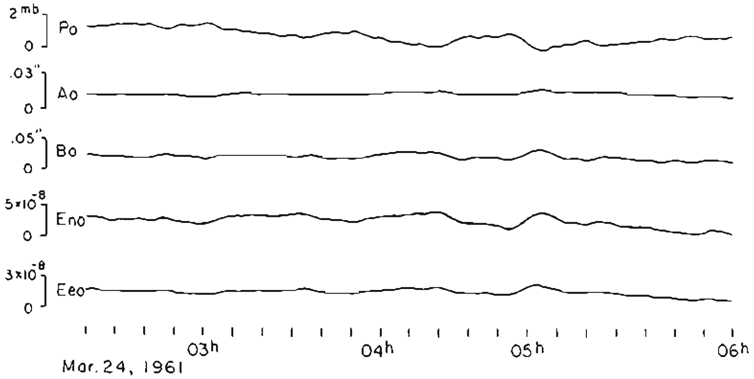

Fig. 1. Example of the record section at Oura from which the spectra were taken. Po ; atmospheric pressure, Ao ; E-W component tilt, Bo ; N-S component tilt, Eno ; N-S component strain, Eeo ; E-W component strain.

been restricted within periods of one hour and five minutes respectively according to the lengths and sampling intervals of the data used. Typical examples of the amplitude and phase spectra are shown in Figs. 3 to 10. Good correspondence between the amplitude spectra of the tiltmetric and extensometric records and those of the barograms is apparent in all cases except the N-S component tilt at Akibasan in which the correspondence is not so remarkable.

In order to reduce the effect due to truncation of the data analysed, care has been taken not to cut off the data where the pressure change was quick.

No windows have been applied before the transforms and the errors due to truncation and reading out of the records may not be disregarded when the amplitude of the deflection is very small in such a case as the N-S component tilt at Akibasan.

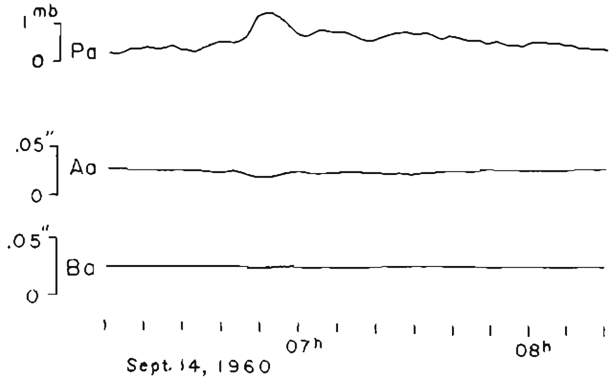

Fig. 2. Example of the record section at Akibasan from which the spectra were taken. Pa ; atmospheric pressure, Aa ; E-W component tilt, Ba ; N-S component tilt.

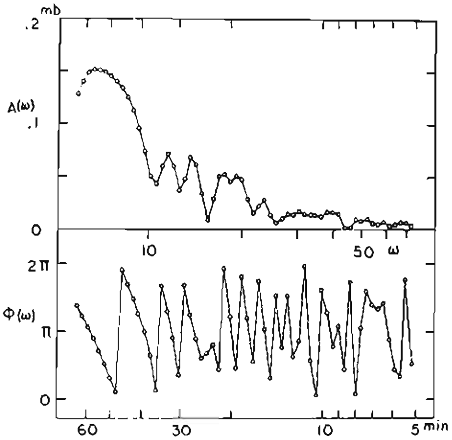

Fig. 3. Amplitude and phase spectra of the atmospheric pressure at Oura calculated from the data from Mar. 24, 02 : 20 to 06:00, 1961.

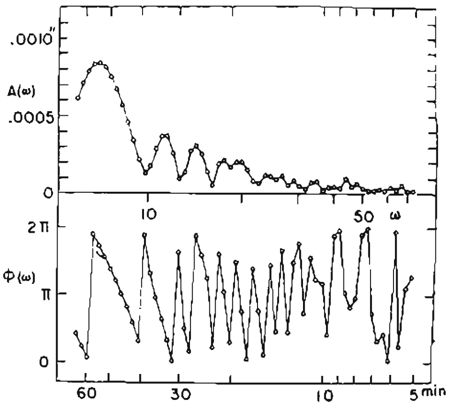

Fig. 4. Amplitude and phase spectra of the E-W component tilt at Oura calculated from the data from Mar. 24, 02:20 to 06:00, 1961.

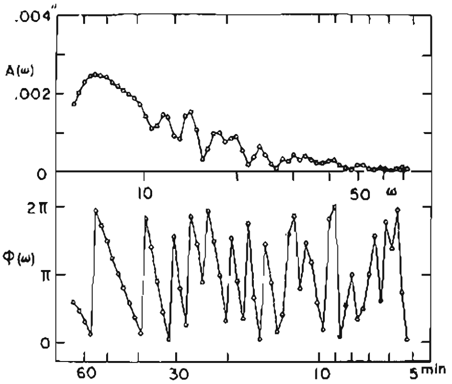

Fig. 5. Amplitude and phase spectra of the N-S component tilt at Oura calculated from the data from Mar. 24, 02 : 20 to 06:00, 1961.

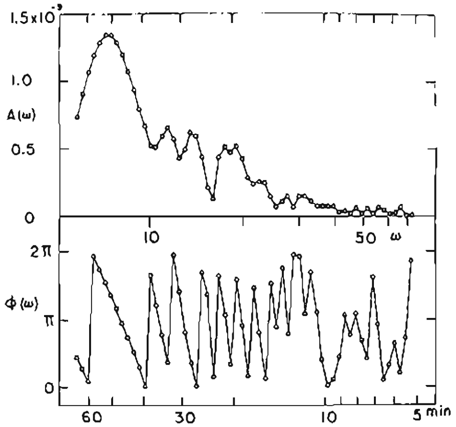

Fig. 6. Amplitude and phase spectra of the E-W component strain at Oura calculated from the data from Mar. 24, 02:20 to 06:00, 1961.

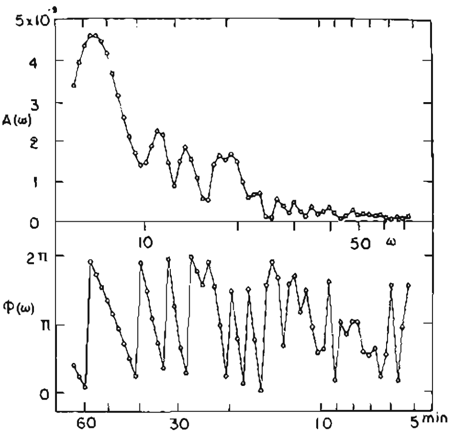

Fig. 7. Amplitude and phase spectra of the N-S component strain at Oura calculated from the data from Mar. 24, 02:20 to 06:00, 1961.

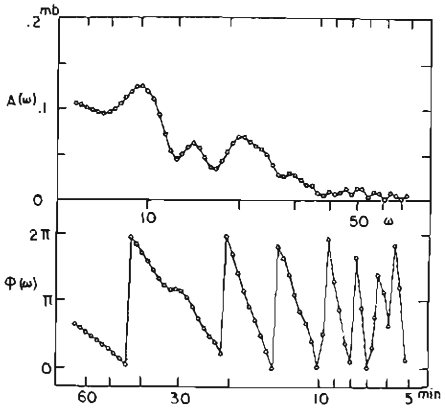

Fig. 8. Amplitude and phase spectra of the atmospheric pressure at Akibasan calculated from the data from Sep. 14, 06:10 to 08 :20, 1960.

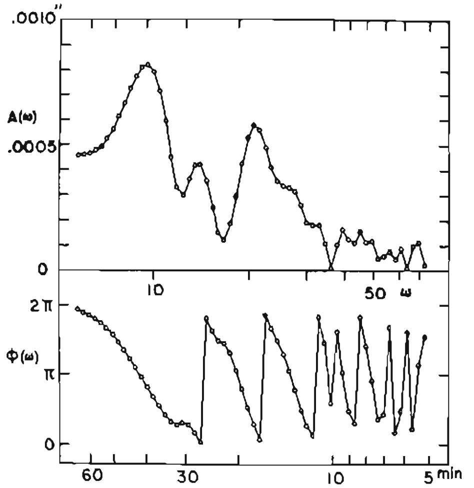

Fig. 9. Amplitude and phase spectra of the E-W component tilt at Akibasan calculated from the data from Sep. 14, 06:10 to 08:20, 1960.

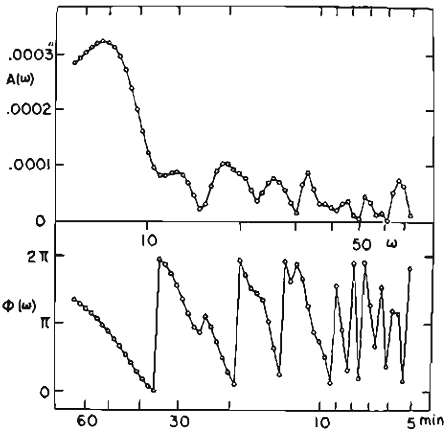

Fig. 10. Amplitude and phase spectra of the N-S component tilt at Akibasan calculated from the data from Sep. 14, 06:10 to 08:20, 1960.

3. Spectral Ratios and Discussion

Under the assumption that the pressure disturbance is represented by the transformation through a linear system and expressed as the convolution of the pressure and proper weight functions, amplitude ratios and phase differences have been calculated for each tilt and strain component. It is needless to say, however, that the disturbance caused by the pressure on the tiltmetric and extensometric data can not be fully represented by the convolution of the pressure change at one fixed point and weight function but spatial information about the pressure distribution is necessary to depict the response to the pressure change. Although we have been obliged to use only the barographic data at Oura and Akibasan in the present study, the above assumption will require some modification in the light of more extensive materials available, if possible.

Now, since meteorological effects other than those due to pressure and earth-tides also distort the spectra obtained from the tiltmetric and extensometric records, we shall deal with the amplitude ratios and phase differences with regard only to the amplitude spectral peaks for which good correspondence is seen between the spectra of the tiltmetric or extensometric records and barograms. The amplitude ratios and phase differences thus obtained are shown in Figs. 11 to 16. As seen in the figures, all components of tilts and strains at Oura and the E-W component tilt at Akibasan show fairly systematical aspects. In the cases of the E-W component tilt and N-S component strain at Oura, the amplitude ratios are almost constant and independent of the frequency in the present periodic range, and phase differences are also constant and nearly equal to π. Therefore it is concluded that the disturbance due to atmospheric pressure which appeared on the records of these components is almost proportional to the barometric change at Oura and can be expressed as the curves multiplied by

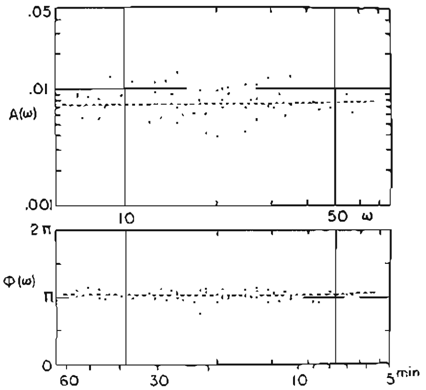

Fig. 11. Amplitude ratio and phase difference of the E-W component tilt at Oura to the atmospheric pressure. Unit of the ordinate of the ratio is second of arc/mb.

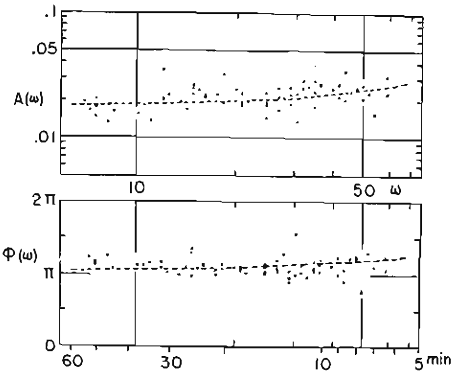

Fig. 12. Amplitude ratio and phase difference of the N-S component tilt at Oura to the atmospheric pressure. Unit of the ordinate of the ratio is second of arc/mb.

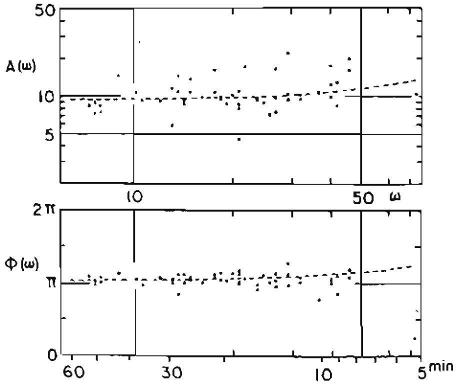

Fig. 13. Amplitude ratio and phase difference of the E-W component strain at Oura to the atmospheric pressure. Unit of the ordinate of the ratio is $\times 10^{-9}$/mb.

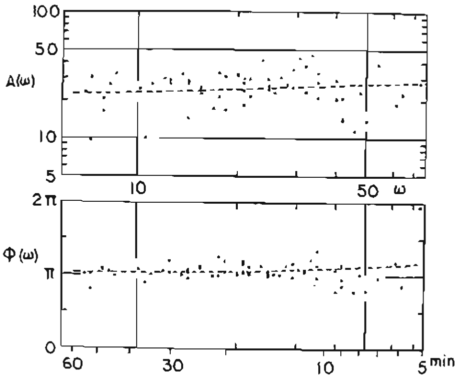

Fig. 14. Amplitude ratio and phase difference of the N-S component strain at Oura to the atmospheric pressure. Unit of the ordinate of the ratio is $\times 10^{-9}$/mb.

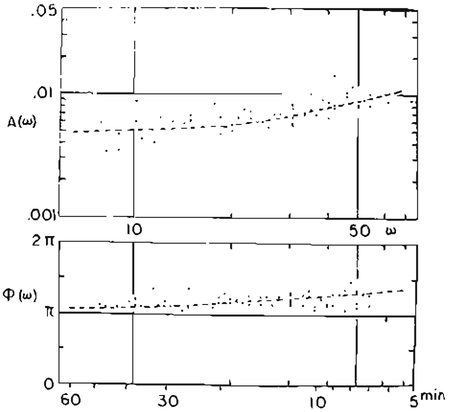

Fig. 15. Amplitude ratio and phase difference of the E-W component tilt at Akibasan to the at-mospheric pressure. Unit of the ordinate of the ratio is second of arc/mb.

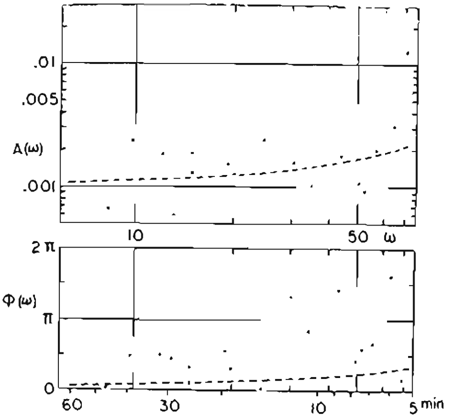

Fig. 16. Amplitude ratio and phase difference of the N-S component tilt at Akibasan to the atmospheric pressure. Unit of the ordinate of the ratio is second of arc/mb.

some constants to the barogram at Oura within the present periodic range. On the other hand, the spectrals ratios of the N-S component tilt and E-W component strain at Oura and E-W component tilt at Akibasan show some different aspects from those mentioned above, namely that in these components the amplitude ratios gradually increase as the frequency increases, and the phase differences deviate slightly from π. A similar tendency is also seen in the N-S component tilt at Akibasan, although the data points of the phase difference show considerable scattering. It is deduced from these facts that the disturbance due to atmospheric pressure on the tiltmetric and extensometric records can be expressed as the output from transformations of the pressure change through linear systems as a first approximation. However, the periodic range of the Fourier transforms calculated is insufficient to get the inverse transforms and to straightforwardly determine the forms of the weight functions in the time domain. Then, in order to interprete the frequency dependence of the spectral ratios from another viewpoint, we introduce the tiltmetric or extensometric change proportional to the time derivative of the atmospheric pressure at Oura and Akibasan. Let $p(t)$ be the sequence of the observed atmospheric pressure and $f(t)$ the disturbance due to it on the tiltmetric or extensometric record, and we simply assume that the following relation holds,

$$f(t)=\alpha\cdot p(t)+\beta\cdot dp(t)/dt, \cdots\cdots\cdots\cdots\cdots\cdots(1)$$

where α and β are constants. Then making the Fourier transform, we get

$$F(\omega)=\alpha\cdot P(\omega)+\beta\omega j\cdot P(\omega),$$

where $F(\omega)$ and $P(\omega)$ are the Fourier transforms of $f(t)$ and $P(t)$, respectively. Accordingly, the amplitude ratio $|F(\omega)/P(\omega)|$ and phase difference $\phi(\omega)$ as to $f(t)$ and $p(t)$ are given as

$$|F(\omega)P/(\omega)| = \sqrt{\alpha^2+\beta^2\omega^2} \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

$$\phi(\omega) = \tan^{-1}(-\beta\omega/\alpha) \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots(3)$$

Therefore, the frequency dependence of the spectral ratios may be explained by adding the term proportional to the time derivative of the pressure to the term proportional directly to its change. Upon this, the values of α and β for each tilt and strain component have been determined so as to satisfy the data points of the amplitude ratios by (2), and then the phase difference have been calculated by (3) using the α and β. The values of α and β thus estimated are summarized in Table 3, and the corresponding amplitude ratios and phase differences calculated from (2) and (3) are also shown in Figs. 11 to 16 by dashed lines. As seen in the figures the curves favorably satisfy the data points, and from this it is concluded that the disturbance due to atmospheric pressure is able to be approximately expressed by the sum of the direct effect which is proportional to the pressure change and second effect proportional to the time derivative of the pressure change.

Table 3. Values of α and β at Oura and Akibasan.

Station	α	β
Oura		
E-W Tilt	-0. 0074″/mb	+0. 000040″/mb/hour
N-S Tilt	-0, 019	+0. 00033
E-W Strain	-9.4×10^{-3}/mb	$+0.12\times10^{-9}$/mb/hour
N-S Strain	-24×10^{-9}	$+0.23\times10^{-9}$
Akibasan		
E-W Tilt	-0. 0050″/mb	+0. 00015″/mb/hour
N-S Tilt	0. 0011	-0. 000027

As to the results of ground tilt, the former is considered to be due to instrumental or purely local disturbance caused by the pressure and the latter is considered to be due to the tilt caused by the loading effect of the pressure, if the time derivative can be approximately substituted for the spatial derivative of the pressure. In this case, as is pointed out by D. Simon et al.[5], the direct effect $\alpha\cdot p(t)$ is considered to take different values at respective stations and instruments, while the second term $\beta\cdot dp(t)/dt$ may be related to the deformation extending over a fairly large area and they are supposed to be nearly equal to each other when the distance between observation stations is not so large. As shown in Table 3, however, the values of β in the same direction at the two stations are not in agreement with each other, namely the ground tends to incline nearly southwards or eastwards at Oura or Akibasan respectively, when atmospheric pressure increases. One reason for the discrepancy may be due to the substitution of the time derivative of the pressure change for the spatial derivative and leaving the azimuthal effect out of consideration. Besides, if there exists a boundary of crustal blocks between the two stations, then each block on which Oura or Akibasan is located may respond in a different manner to the loading effect of atmospheric pressure. Indeed, Nishimura[6] reported two examples of different tilting motions of the ground caused by quick pressure changes at four stations spaced out 500 - 1000 m in Beppu and suggested block movements of the ground due to its mosaic structure.

Meanwhile, it is observed on the vectorial representation of α and β shown in Fig. 17 that the tilting vector of β almost points in the opposite direction to that of α for each station. From this, another interpretation for the pressure disturbance may be drawn, namely that the ground tends to incline prevailingly in an NNW-SSE direction at Oura and WSW-ENE at Akibasan according to the pressure change, and that the tilting motion can be divided into two terms, the one proportional to the pressure change and the other proportional to its time derivative. In this case the second term is considered to relate also to the purely local disturbance of the pressure as well as the first term, and consequently the large scale crustal tilting is less important and negligible within the present periodic range from one hour to several minutes compared with the purely local effect of the pressure. When we stand on this viewpoint, the directions of the tilting vectors seem to be related to the geographical situation of the stations.

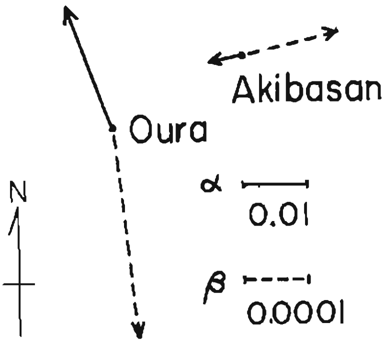

Fig. 17. Vectorial representation of α and β at Oura and Akibasan.

When the atmospheric pressure is distributed over the semi-infinite elastic body along a certain direction in the shape of a sinusoidal wave, $p = -g\rho h \cos 2\pi(x/\lambda - t/T)$, with parallel undulation, the horizontal (u) and vertical (w) displacements at the point x, z are given, following the result obtained by Darwin 7), as

$$u = \frac{g\rho h}{2\mu} z e^{-2\pi z/\lambda} \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \qquad \cdots\cdots\cdots\cdots(4)$$

$$w = \frac{g\rho h}{4\pi\mu} \lambda \left(1 + 2\pi\frac{z}{\lambda}\right) e^{-2\pi z/\lambda} \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right), \qquad \cdots\cdots\cdots\cdots(5)$$

where μ is the modulus of rigidity, g the acceleration of gravity, h the barometric height represented by the mercury column, ρ the mercury density, x the distance along the horizontal, z the depth below the surface, t the time, and λ and T the wave length and period of the sinusoidal wave. Then the tilt and horizontal strain due to the sinusoidal pressure wave are expressed as

$$\frac{dw}{dx} = -\frac{g\rho h}{2\mu}\left(1 + 2\pi\frac{z}{\lambda}\right) e^{-2\pi z/\lambda} \sin 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right) \cdots\cdots\cdots\cdots (6)$$

$$\frac{du}{dx} = \frac{\pi z g\rho h}{\lambda\mu} e^{-2\pi z/\lambda} \cos 2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right). \qquad \cdots\cdots\cdots\cdots(7)$$

Putting $\partial P/\partial t = (-2\pi g\rho h/T)\sin 2\pi(x/\lambda - t/T)$ into (6), we get

$$\frac{dw}{dx} = \frac{T}{4\pi\mu}\left(1 + 2\pi\cdot\frac{z}{\lambda}\right) e^{-2\pi z/\lambda} \cdot \frac{\partial P}{\partial t}.$$

Therefore it is to be noted that, although we have implicitly assumed small scale block movements of the ground compared with the wave length of the pressure and coefficients α and β to be independent of ω, in the case of the tilt the coefficient β is not a constant as assumed in (1) but a function of $\omega=2\pi/T$, and in the case of the horizontal strain the coefficient α becomes a function of λ, in other words, a function of the velocity of the wave v and ω, and putting α to be a function of ω the pressure disturbance might be explained by $(1/2\pi)\int_{-\infty}^{\infty}\alpha(\omega)P(\omega)e^{i\omega t}d\omega$ in place of (1).

In order to compare the pressure influence on the ground tilt at Oura and Akibasan to that of another station, the same analysis has been carried out as to the tiltmetric data observed at the Amagase Observatory in Uji City, Kyoto which is located at 34°53' N latitude and 135°50'E longitude, approximately 100 km NE of Wakayama City. As seen in the example of the barographic and tiltmetric records shown in Fig. 18, the disturbance of the pressure upon the tiltmetric record is rather small compared with those of Oura and Akibasan because of the depth of the observational site underneath the earth's surface. The sensitivities of the tiltmeters are 0.0042 and 0.0043"/mm in the E-W and N-S components respectively, and the recording speed is also 30 mm/hour. The analysed data intervals are given in Table 4. Typical examples of the Fourier spectra are shown in Figs. 19 to 21. Figs. 22 and 23 show the amplitude ratios and phase differences regarding the prominent peaks for which good correspondence is seen

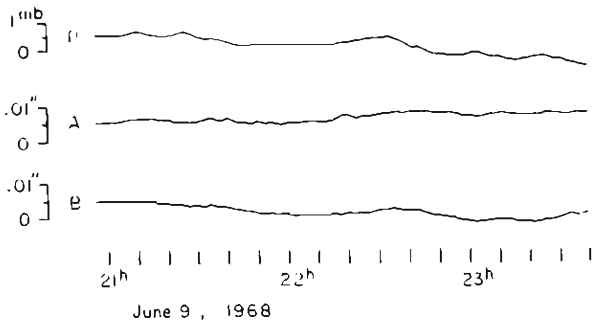

Fig. 18. Example of the record section at Amagase from which the spectra were taken. P ; atmospheric pressure. A ; E-W component tilt, B; N-S component tilt.

Table 4. The time intervals analysed.

Station	Time interval (JST)
Amagase	May 19. 06 : 00 — 08 : 51. 1968
	May 19, 20 : 59 — 23 : 43, 1968
	May 23. 00 : 00 — 02 : 52. 1968
	June 3, 20 : 57 — 23 : 40, 1968
	June 9. 20 : 56 — 23 : 40. 1968
	June 10, 05 : 57 — 08 : 49, 1968
	July 2, 05 : 58 — 08 : 50, 1968
	July 5, 23 : 58 — July 6, 02:50, 1968

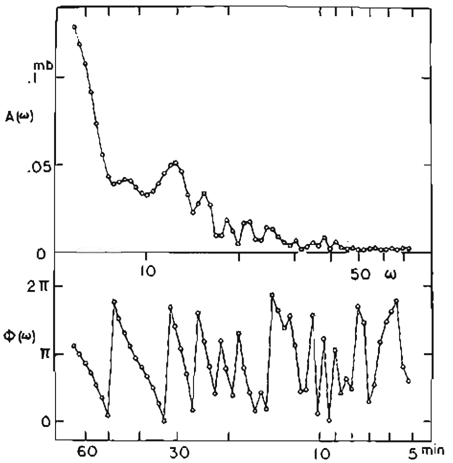

Fig. 19. Amplitude and phase spectra of the atmosheric pressure at Amagase calculated from the data from June 9, 20 : 56, to 23:40, 1968,

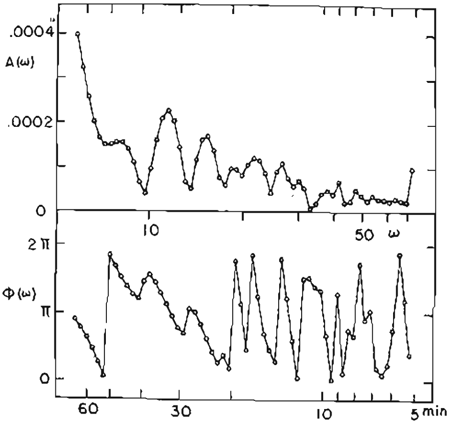

Fig. 20. Amplitude and phase spectra of the E-W component tilt at Amagase calculated from the data from June 9, 20:56 to 23:40, 1968.

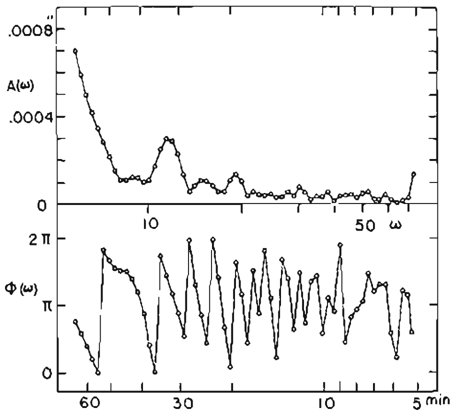

Fig. 21. Amplitude and phase spectra of the N-S component tilt at Amagase calculated from the data from June 9. 20:56 to 23:40, 1968.

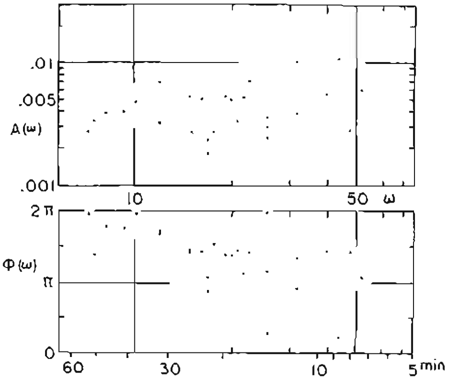

Fig. 22. Amplitude ratio and phase difference of the E-W component tilt at Amagase to the atmospheric pressure. Unit of the ordinate of the ratio is second of arc/mb.

between the pressure and ground tilt.

Although the data points scatter considerably, especially in the phase difference which is thought to be due to the smallness of the amplitude of fluctuations caused by the pressure change, a similar tendency to the case of Wakayama can be seen, at least as to the amplitude ratios.

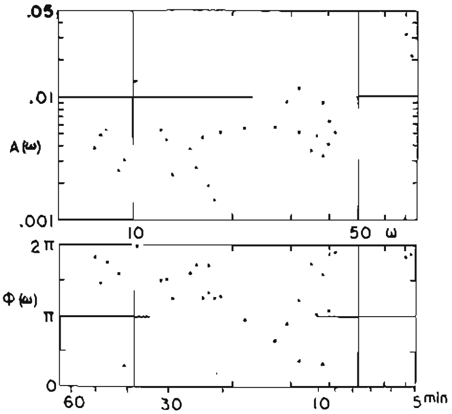

Fig. 23. Amplitude ratio and phase difference of the N-S component tilt at Amagase to the atmospheric pressure. Unit of the ordinate of the ratio is second of arc/mb.

Thus, it is quite probable that the atmospheric disturbance that appeared on the records of tiltmeters or extensometers has as a common property of the frequency dependence the fact that the amplitude ratios of the disturbance on the tiltmetric or extensometric records to the atmospheric pressure change increase as the frequency increases within the periodic range from about one hour to several minutes, and that this can be explained by considering the sum of the direct effect of the pressure proportional to the pressure change and the indirect effect proportional to the time derivative of it as the first approximation. As to the tilt, the former may be interpreted as being due to the purely local disturbance of the pressure including instrumental, and the latter due to either the crustal tilt caused by the loading effect of the pressure or purely local disturbance similar to the former. Although we have assumed the coefficients α and β in (1) to be constant and independent of ω for interpretation of the spectral ratios of the ground tilt and strain to the atmospheric pressure in the present report, β is expected to be a function of ω in the case of the ground tilt when the ground responds elastically to the loading due to atmospheric pressure. Similarly the coefficient α is expected to depend on ω in the case of the horizontal strain and the spectral ratios are possibly explained by putting the disturbance due to the pressure to be the form of $(1/2\pi)\int_{-\infty}^{\infty}\alpha(\omega)P(\omega)e^{j\omega t}d\omega$. However, it seems premature at the present stage to draw any definite conclusion about the interpretation of the spectral ratios obtained, and more extensive investigations of this problem are required.

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