Bull. Disas. Prev. Res. Inst., Kyoto Univ., Vol. 18, Part 3, No. 145, February, 1969

An Experimental Study on the Inelastic Behavior of Steel Frames with a Rectangular Cross-Section Subjected to Vertical and Horizontal Loading

By Minoru WAKABAYASHI, Taijiro NONAKA and Shosuke MORINO

(Manuscript received December 3, 1968)

Abstract

An experimental study was made of the behavior of single bay three-storied frames, using 1/30-scale models with rectangular cross-sections. Frame models were cut from a sheet of mild steel. A varying horizontal force and constant vertical loads were applied at the top of columns. The horizontal force-displacement relations show the remarkable dependence of frame behavior on the existence of vertical loads; the behavior is characterized by a gradually increasing lateral displacement, first under a growing load and later under a diminishing load, the slope continuously decreasing in the horizontal force-displacement relation.

Elastic-perfectly plastic type of analysis underestimated the restoring force in the large deformation range. An approximate elastic-strain hardening analysis was attempted in order to gain access to the real behavior of frames, and a reasonable agreement is seen with the experimental results.

1. Introduction

It is often necessary or advisable to rely on sufficient ductility or the deformation capacity of a structure beyond the elastic limit without causing a catastolophic failure, when we design a tall building to withstand intense earthquake excitations. The response analysis of a structure requires the establishment of the load-deformation relation from which the restoring forces are determined at various deformation stages. The recent development of the plastic analysis of structures has clarified the ultimate state or the real strength of a structure[1], under such circumstances that the assumptions of ample ductility and perfect plasticity are pertinent and that instability or geometrical change is negligible ; it facilitates the approximate determination of the restoring-force characteristics in the elastic-plastic range under such circumstances. The change in geometry, however, may be extremely important and may have either a strengthening or a weakening effect[2]. The state of framed structures during earthquake motion is characterized by constant vertical loads and varying horizontal forces. In multi-storied frames, the accumulation of dead loads amounts to a large compressive axial force in the columns of the lower stories, and this somehow induces an unstable feature in the restoring-force characteristics. The adoption of high strength steel accelerates the importance of the destabilizing effect in moderm multi-storied frames. Not a few theoretical studies have been made on this point on the basis of perfect plasticity. Extensive reviews on these studies are found in the literature[3,4]. Onat has shown, however, theoretically that even a slight amount of strain hardening considerably increases the load necessary for quasi-static deformations, when the compressive axial forces are negligible[2]. The effects of instability and strain hardening are incorporated approximately for the behavior of pitched roof frames by assuming a rigid-plastic-rigid or rigid-plastic-strain hardening type of stress-strain relation[5,6]. Davies related the increment in moment to the rotation across a plastic hinge assuming the relation for rigid-plastic-rigid material[7]. Wakabayashi, Nonaka and Matsui took an approximate account to the strain-hardening effect, after the arising of the mechanism state, which perfect plasticity predicts, for one bay, one- and two-storied rectangular frames, to be in reasonable agreement with experimental results[8]. At present, no rigorous solution seems to be available in the elastic, plastic, strain-hardening range, except for a cruciform frame for which analysis was made on the basis of a trilinear moment-curvature relation[9]. This approach involves too much labor in computation to be applied to multi-storied frames.

More urgent is the need for experimental studies on the restoring-force characteristics of multi-storied frames under large vertical loads. Former tests with wide-flange sections involved some ambiguities in connection with beam-to-column connections and with residual stresses rocked in the frame models[8~12]. In the tests to be described in this paper, a frame model of single bay, three stories was cut out from a sheet of metal, having rectangular cross-sections, so that a nice continuity of connections, uniformity of model and absence of residual stresses could be assured. Theoretical assessment is also made of the horizontal force-displacement relation, persuing the plastic-hinge concept and relating the incremental moment to the rotation across a plastic hinge, based on an assumed distribution of plastic strains at the plastic hinge.

2. Description of the Test

2.1 Test Program

The model was a single bay, three-storied, rigid-jointed frame with a rectangular cross-section. A constant vertical load was applied symmetrically on the top of the upper columns, and a varying horizontal force was applied in a quasi-static manner at top floor level.

The experimental program consisted of twelve tests. Specimens were designed to the common dimensions as shown in Fig. 1. The beam depth and vertical load were varied so that there were six combinations as shown in Table 1. Two different materials, A and B, were used for each combination. The first two figures of the specimen number refer to the beam-to-column section modulus ratio and the last two to the vertical load-column yield load ratio. A test specimen was taken by machining from a 60 mm-thick steel sneet, which was annealed, so that the thickness constituted the width of a frame member. A typical stress-strain relation, which was observed in a tensile test, is shown in Fig. 2. The average material properties are as shown in Table 2.

2.2 Experimental Apparatus

The experimental arrangements were similar to those in ealier experi-

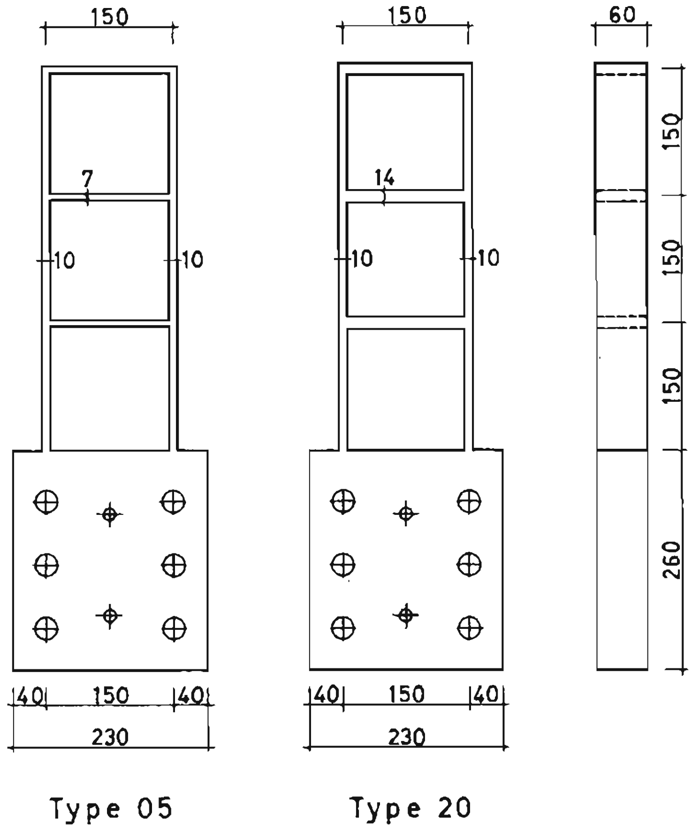


Fig. 1. Test Specimens (Unit : mm).

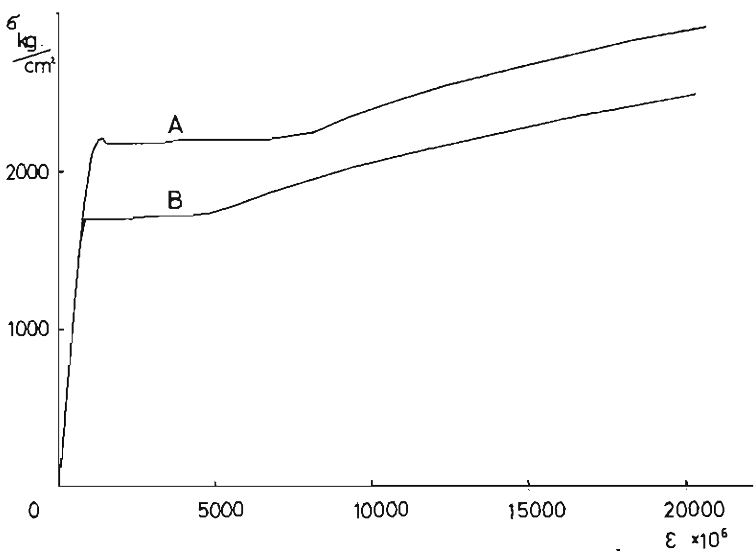


Fig. 2. Stress-Strain Relations.

TABLE 1.
Test program.

Specimen Number	z_B/z_C	P_y/P
0500 A 0500 B	0.49	0
0520 A 0520 B	0.49	0.2
0540 A 0540 B	0.49	0.4
2000 A 2000 B	1.96	0
2020 A 2020 B	1.96	0.2
2040 A 2040 B	1.96	0.4

z_B: Section modulus of beam.
z_C: Section modulus of column.
P : Vertical load.
P_y: Column yield load.

TABLE 2.
Properties of the material.

	Series A	Series B
σ_y (kg/cm²)	2174	1698
σ_B (kg/cm²)	4425	3910
ε_{sh} $\times 10^6$	5263	4977
Elongation (%)	35.3	38.3

σ_y : Yield stress.
σ_B : Ultimate stress.
ε_{sh} : Strain at which strain-hardening starts.

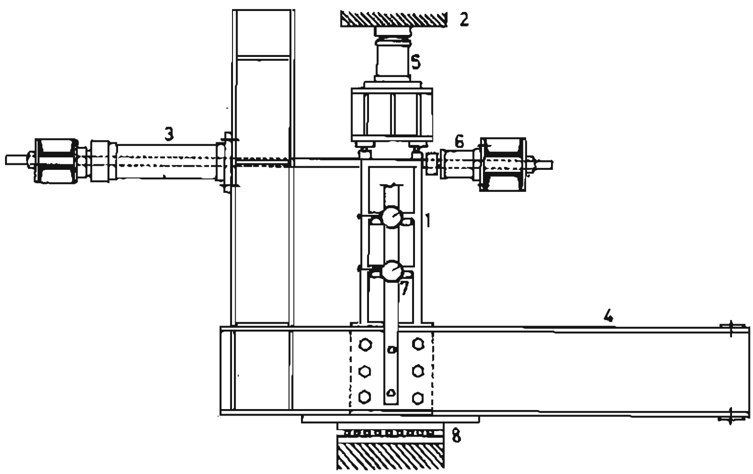


Fig. 3. Experimental Apparatus.
1. Specimen 2. Testing machine (100 ton capacity) 3. Oil jack (10 ton capacity) 4. Loading frame 5. Load cell 6. Load cell 7. Dial gage 8. Roller bearings

ments[8,13]. The general arrangement is shown in Fig. 3. The test specimen was fixed on the L-type supporting frame at the frame foot with 6 high-tension bolts. The supporting frame was set on an oil pressure testing machine. Rollers were placed between the supporting frame and the testing machine bed, in order to allow the supporting frame to move horizontally. The vertical load, measured by the testing machine dial and a load cell, was first set to and maintained at the assigned value during a test. The horizontal force was then slowly varied by adjusting the oil jack, and measured by a load cell. The load measurements were checked by means of wire strain gauges pasted near the beam-to-column connections.

3. Theoretical Analysis

We seek for the relation between the horizontal force H and the displacement δ of the frame in equilibrium under the loading condition shown in Fig. 4. The vertical load P is constant in direction and magnitude. First an elastic-perfectly plastic type of analysis is briefly introduced and then an elastic-strain hardening analysis attempted.

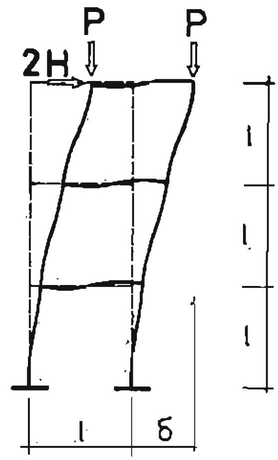


Fig. 4. Frame and Loading.

3.1 Elastic-Perfectly Plastic Analysis

The main assumptions are as follows:

1° The frame is composed of one-dimensional members with centroidal axes as member lines.

2° Although the geometrical change is taken into account, the deflections are so small that the cosine of the slope angle can be approximated as unity.

3° No out-of-plane deformation or local buckling phenomena occur.

4° Members are subjected to flexural deformation only, the axial and shearing deformation being negligible.

5° The change in axial forces is negligible. Beams have no axial force, and columns have constant axial force P.

6° The effect of shear on the fully plastic condition is negligible.

7° The moment-curvature relation is elastic-perfectly plastic, and an indefinite change of curvature can occur under the full plastic moment which, together with the axial force, satisfies a full plastic condition.

Because of the anti-symmetric property, which is based on the assumption 5°, we only have to consider the right or left half of the frame with hinges on the mid-span. When the loads are so small that the condition of linear elasticity is satisfied throughout the frame, the problem is solved by applying the slope-deflection equations with consideration of the axial force, which relate, for a member, the end-moment and shear with the end rotations and relative displacement between the ends[8]. Equilibrium conditions of moment at each joint and of horizontal force on each story, provide sufficient numbers of equations for the unknown quantities.

When the load attain such a magnitude that a full plastic moment is reached at a member end*, forming a plastic hinge there, the end moment is set, in magnitude, to the full plastic moment of that member. We proceed to determine the elastic deformation of the rest of the frame till full plastic moment

* Because of the loading condition, the bending moment takes an extreme value at the member end. An actual frame member has finite depth, and the dimensions of a joint panel are not negligibly small in comparison with the member length. In the analysis, therefore, it is assumed that a member has the fictitious full plastic moment which would occur at the beam-to-column joint of the center lines on the basis of linear distribution of moment, when the end section has the full plastic moment of that member.

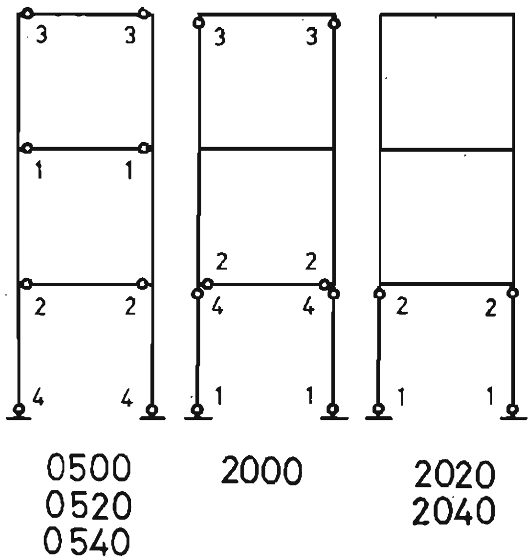


Fig. 5. Location and Order of Plastic Hinge Formation.

is reached at another point. A similar process is continued. Fig. 5 shows the order of the formation of plastic hinges in the frames tested.

It is interesting to note that in the frames in which the beams are stronger than the columns, the columns of the upper two stories decrease their inclination after the arising of the mechanism state, although the horizontal displacement of the top story keeps increasing because the lowest columns gain much inclination (Fig. 7(b)).

3.2 *Elastic-Strain Hardening Analysis*

Observing the stress-strain relation in Fig. 2, we see that the aforementioned assumption of perfectly plastic behavior will not fully describe the true behavior of a frame over a large deformation range because of the appreciable strain-hardening effect. Since a complete analysis for the frame of strain-hardening material is extremely difficult owing to computational complexities, we shall now make an attempt to include the strain-hardening effect in an approximate manner. Assumptions 1° to 6° of the elastic-perfectly plastic type analysis are retained. Instead of assumption 7°, we assume that the moment-curvature relation is elastic and linear until the full plastic moment

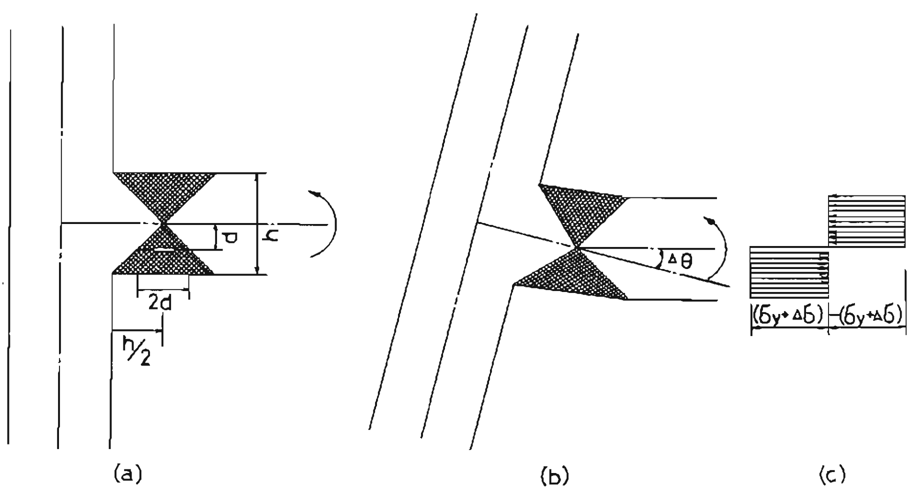


Fig. 6. Assumption of Strain Hardening Plastic Hinge.

is reached, when a plastic hinge is formed, whence the full plastic moment increases in accordance with the discontinuity in slope $\Delta\theta$ across the plastic hinge. Plastic strains are assumed to occur, when a full plastic moment is reached, only at plastic hinges.

Suppose that the beam end of Fig. 6 reaches the state of full plasticity in pure bending. We assume that a plastic hinge is formed at a point half the beam depth apart from the end-section, and that plastic strains are uniformly distributed in the isosceles-right triangular region of the tention and compression sides, Fig. 6(a). There is a jump $\Delta\theta$ in the slope angle across the plastic hinge, Fig. 6(b). Velocity fields are assumed in the triangular regions so as to match this relative rotation. The extentional plastic strain $\Delta\epsilon$ of a horizontal element on tension side at distance d from the neutral axis (Fig. 6(a)) is obtained as the ratio of the relative displacement component $d\cdot\Delta\theta$ in the horizontal direction to length $2d$ to be $\Delta\theta/2$. Similarly on the compression side $\Delta\epsilon=-\Delta\theta/2$. These uniform plastic strains provide uniform stress distribution, unless unloading takes place, as shown in Fig. 6(c). If the increment $\Delta\sigma$ in stress has a linear relation $\Delta\sigma=E_h\cdot\Delta\epsilon$ with the strain increment after the arising of the plastic hinge, then the moment increment ΔM is linearly related with $\Delta\theta$ as,

$$\Delta M=\frac{1}{4}bh^2\Delta\sigma=\frac{bh^2}{4}E_h\cdot\Delta\epsilon=\frac{E_h\cdot bh^2}{8}\cdot\Delta\theta, \tag{1}$$

where b is the width and h is the depth of the beam. We thus assume that the full plastic moment is increased near the beam end by the amount given from Eq. (1), and that the rest of the frame remains elastic. We also use Eq. (1) approximately for a column, where b and h are the linear dimensions of the column section and ΔM is the increment of moment from the full plastic moment under the existence of an axial force. This may be permissible when the axial force is sufficiently small as compared with the limit force in pure tension or compression.

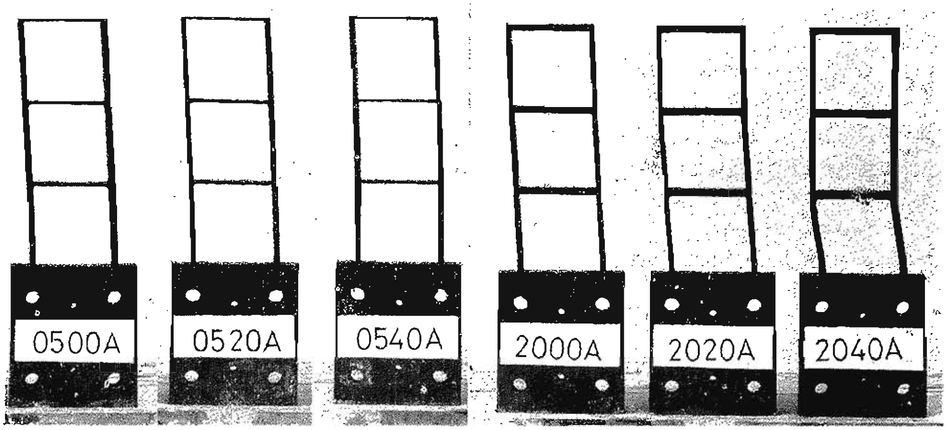
The foregoing assumption of the strain-hardening plastic hinge has several defects from the point of view of the continuum theory, but makes the strain-hardening analysis considerably simple. A similar assumption has already been made about strain-rate dependent analysis in the coexistence of bending moment and axial force[14], and reasonable agreement was seen with the experimental results[15].

In the actual analysis, it is assumed that the initial full plastic moment of a member is the one which would occur at the beam-to-column joint of the center lines on the basis of the linear distribution of moment, when the initial full plastic moment of the member is reached at the plastic hinge. Henceforth, the distance between the joint and the plastic hinge is neglected in comparison with the member length. The analysis after the formation of a plastic hinge in the frame is the same as that in the elastic-perfectly plastic type of analysis except that there are additive unknowns, $\Delta\theta$'s, and equations of type (1) which are equal in number.

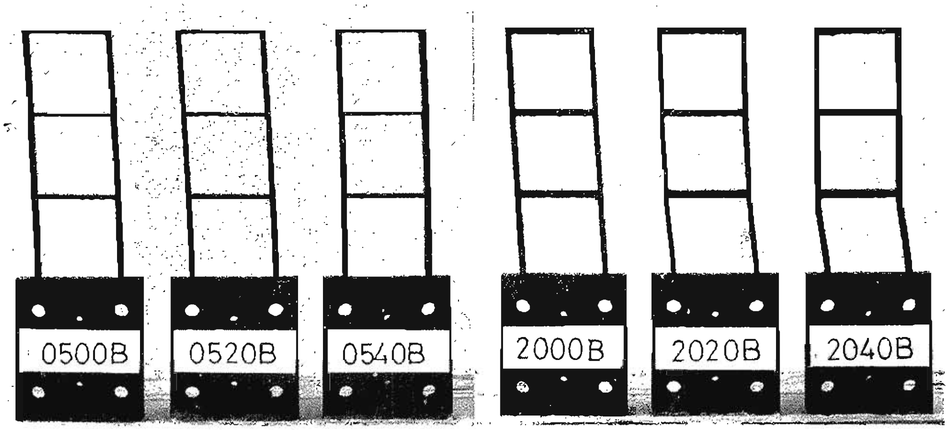
4. Results and Discussion

4.1 Deformed Shape of the Specimens

Photos. (a) to (d) show the deformed shapes of the specimens after testing. Three specimens in each photograph have the same dimensions, and each of the columns was subjected to vertical loads equal to 0, 20 and 40 percent of the column yield load, respectively, from the left.



(a) 05··A (b) 20··A



(c) 05··B (d) 20··B

Photos : Specimens after Test.

Photos. (a) and (c) show specimens of type 05 after testing. Since this type has slender beams and the restraining effect of the beams on the deformation of the columns is small, the columns tend to curve in a single curvature like a single member of full height. Large deformations are observed at both ends of each beam. In the case of specimen 0540, which was subjected to the largest value of vertical load, the horizontal displacement at the bottom beam level was reversed after a certain displacement was attained. These characteristics were found in both A and B series. Photos. (b) and (d) show specimens of type 20. Large deformations are seen at both ends of the bottom column in each specimen. This is particularly noticeable in frames which were subjected to the largest vertical load.

These characteristics of the mode of the deformation appeared most notably in the specimens which were subjected to the largest value of vertical loads, regardless of types and series. To emphasize this point, Figs. 7(a) to (d) are

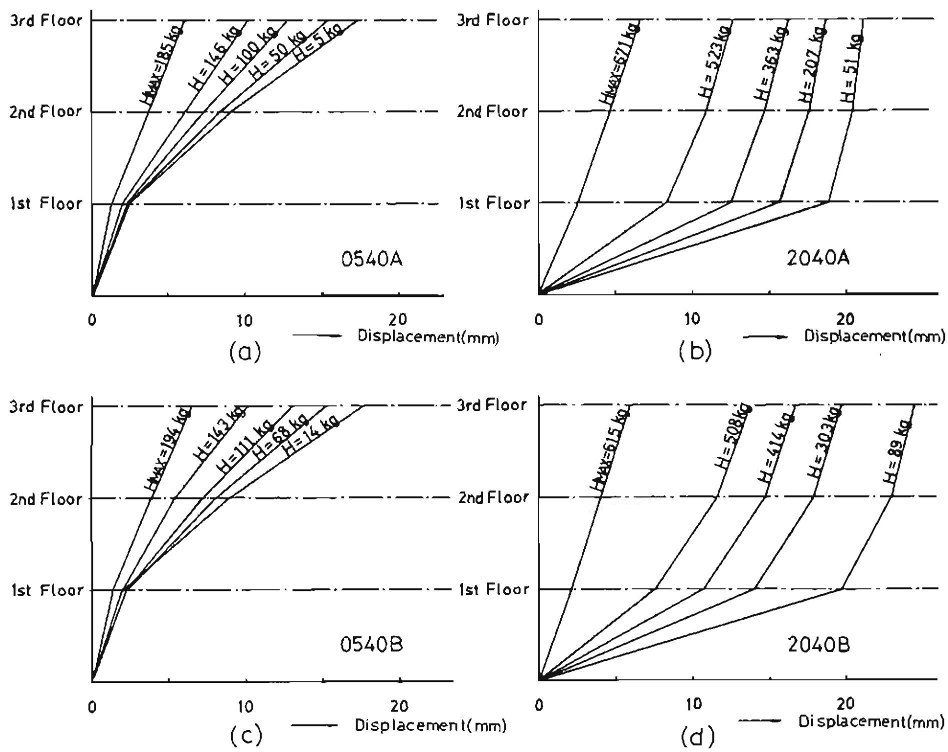
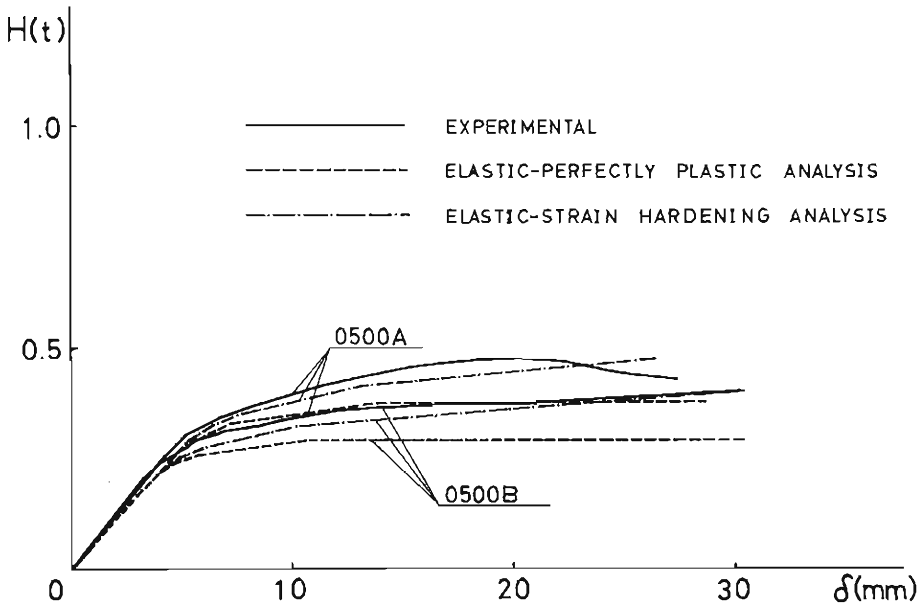


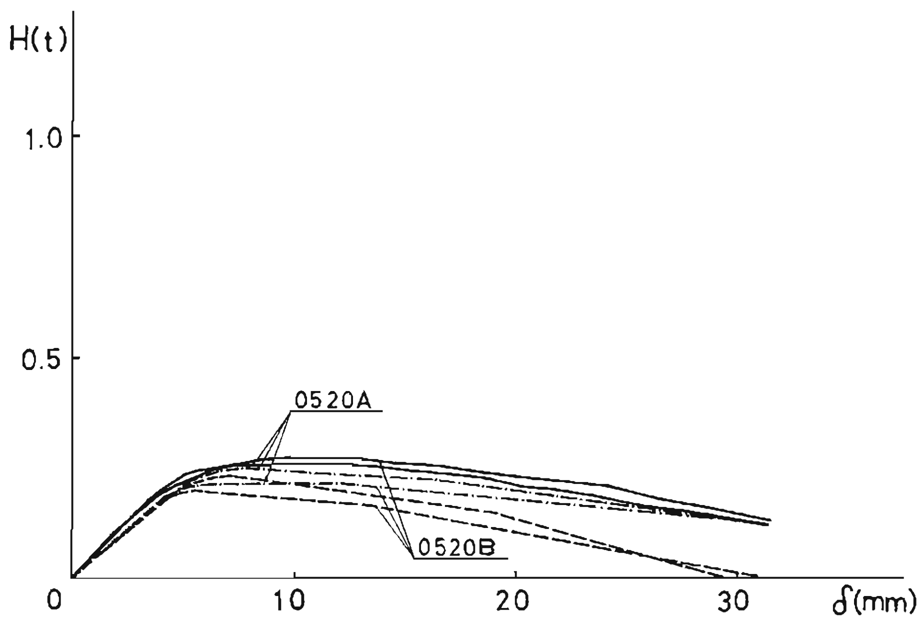
Fig. 7. Displacement Distribution.

constructed. In these figures, the displacement distributions of 4 specimens with the largest vertical load are shown with the parameter of magnitude of the horizontal force. The ordinates of these figures show the level of beams, and the abscissas show the displacements. It can be observed from these figures how the displacements at the beam levels increase with the varying horizontal load after it has reached the maximum. The characteristics of deformation described above can be seen in these figures. In addition, the locations at which large deformations were found by inspection after testing, show reasonably good agreement with the locations of plastic hinges as shown in Fig. 5, which is drawn from the results of the analysis.

4.2 Horizontal Force-Displacement Relation

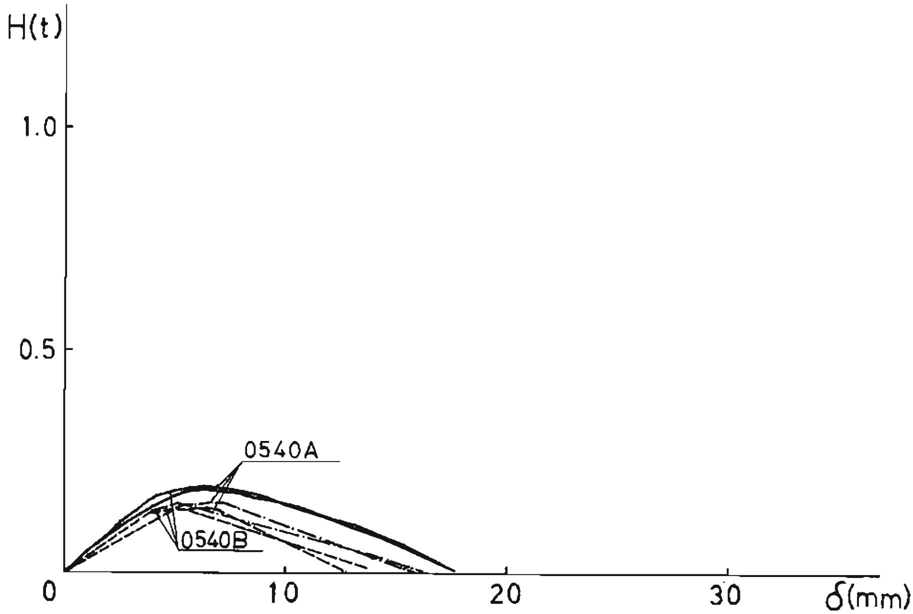


(a) 0500

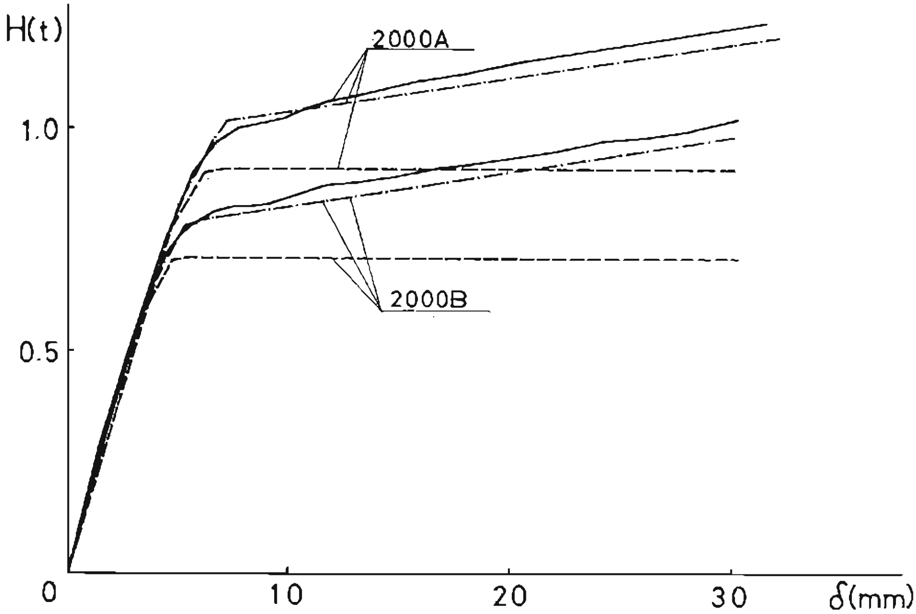


(b) 0520

Fig. 8. Horizontal Force-Displacement Relations.

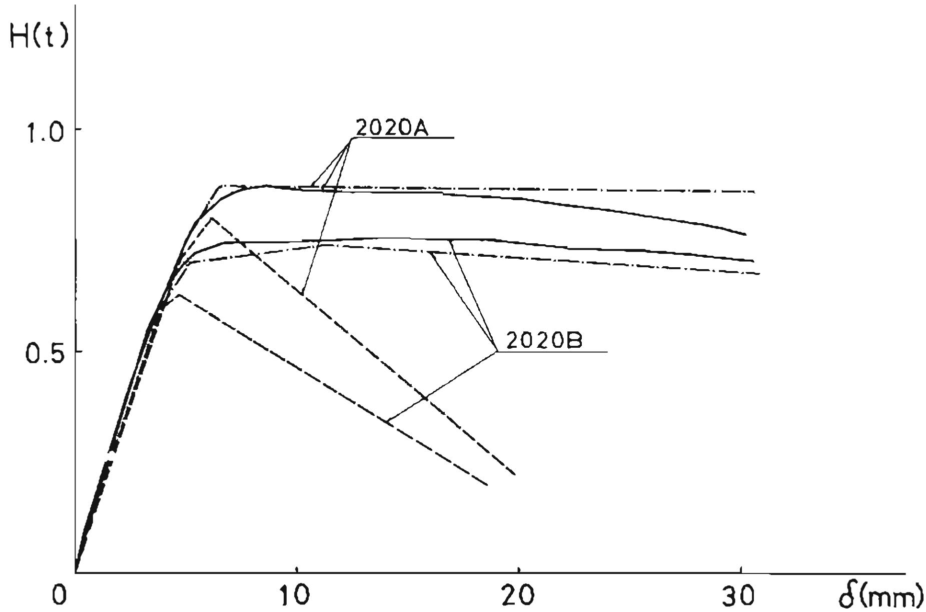


(c) 0540

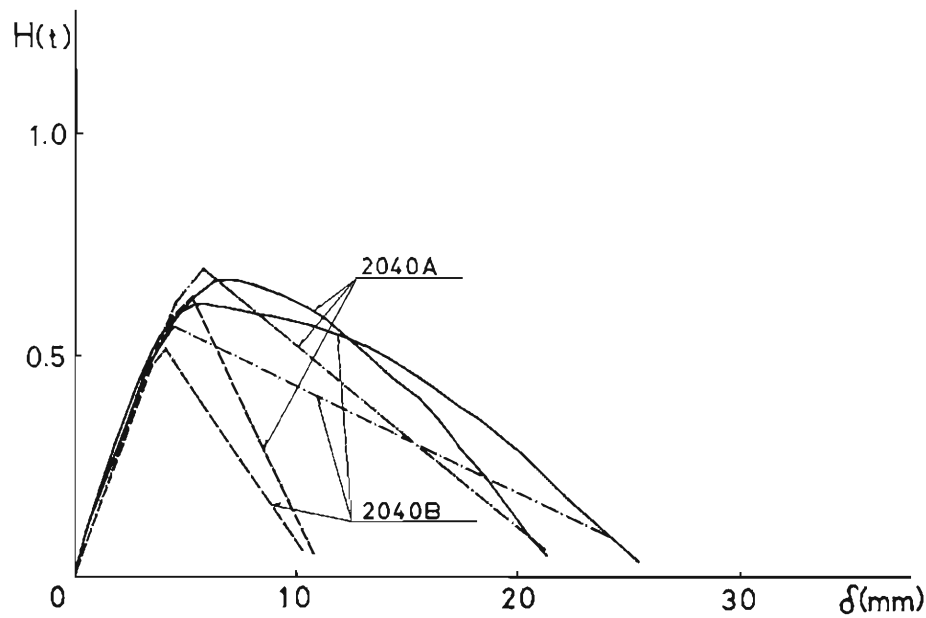


(d) 2000

Fig. 8 Horizontal Force-Displacement Relations.



(e) 2020



(f) 2040

Fig. 8 Horizontal Force-Displacement Relations.

Figs. 8(a) to (f) show the relationships between horizontal force H and displacement δ at top floor level. Each figure contains the results of two specimens which have the same vertical load-column yield load ratio. Only the properties of the material differ from each other; the yield stress of series A is higher than that of series B as shown in Fig. 2. Solid lines, dashed lines and dash-and-dot lines refer to the experiment, elastic-perfectly plastic analysis and elastic-strain hardening analysis, respectively. The two theoretical curves coincide in the elastic range. Elastic-perfectly plastic analysis under-estimates the restoring force in the large deformation range; elastic-strain hardening analysis has a reasonable agreement with experiment.

It is observed from these figures that in the absence of vertical loads the restoring force of frames continues to increase as the displacement does. In the case of the frames with a vertical load of 20% of the column yield load, the restoring force decreases slightly in type 05, while it maintains a certain value in type 20, under the deformation range give here. These observations are true for both series A and series B. This difference between type 05 and 20 is, of course, caused by the contribution of the beams to the restoring force; the slenderness ratio for a frame of type 05 is almost twice as much as that of type 20 as shown in Table 3.

4.3 The Effect of the Vertical Load

Figs. 9(a) to (d) are drawn in order to show the effect of vertical loads. In

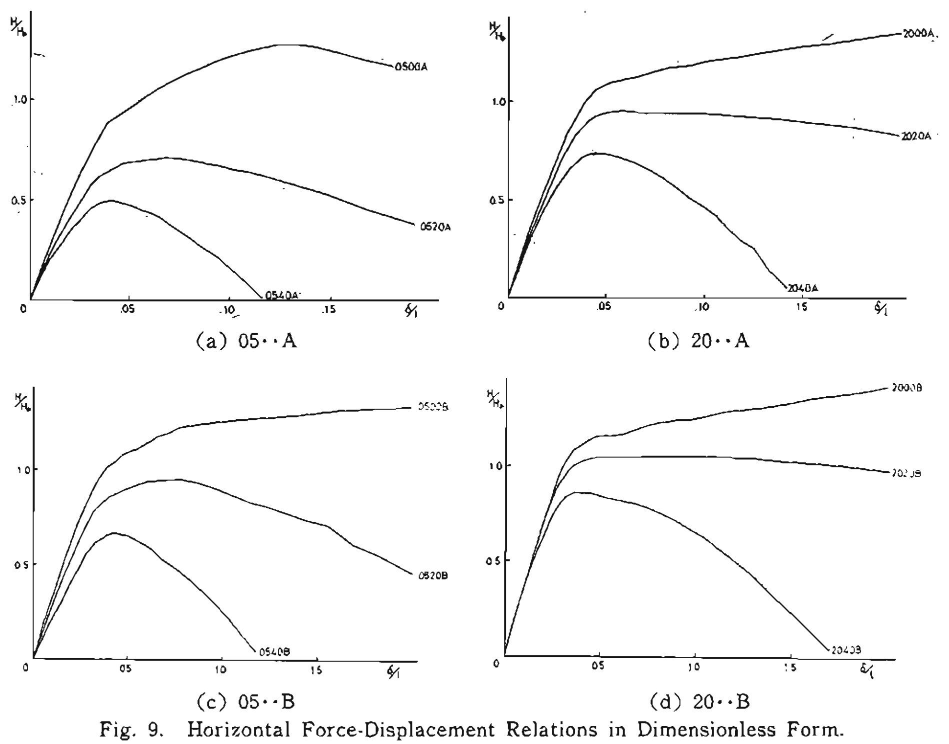


Fig. 9. Horizontal Force-Displacement Relations in Dimensionless Form.

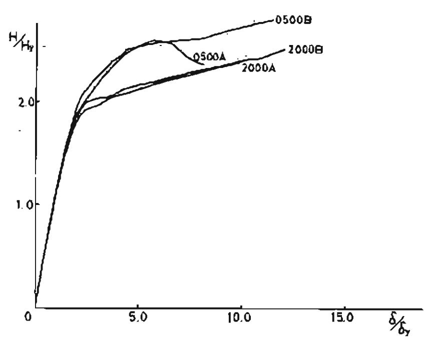
each figure, the test results are compared for specimens of the same material and the same dimensions under different values of vertical loads. The ordinate represents the ratio of horizontal restoring force H to plastic collapse load H_p which is predicted by the theory of perfect plasticity in the absence of vertical loads; the abscissa represents the ratio of the horizontal displacement δ at the top to the height $l(=15\,\mathrm{cm})$ between the beam levels.

It is clearly seen that vertical loads reduce the restoring force, and induce instability of the frames; the larger the vertical load, the smaller the slope. For all specimens without vertical load the ratio H/H_p exceeds unity, regardless of types and series, due to the effect of strain-hardening.

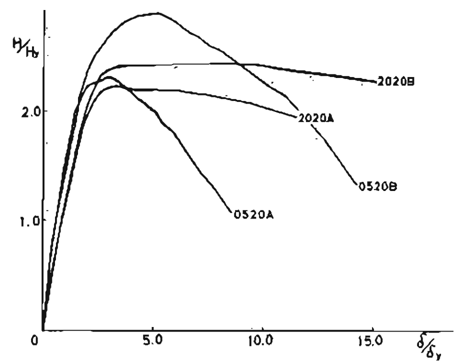
4.4 *The Effect of Dimensions and Material*

Each of the figures (a) to (c) in Fig. 10 contains the test results for the same value of vertical loads; the effects of the differences in the material and in the beam dimensions are to be observed. The ratio H/H_y is taken for the ordinate, and δ/δ_y for the abscissa. H_y and δ_y are the horizontal force and displacement at which yield point stress is reached somewhere in a specimen frame.

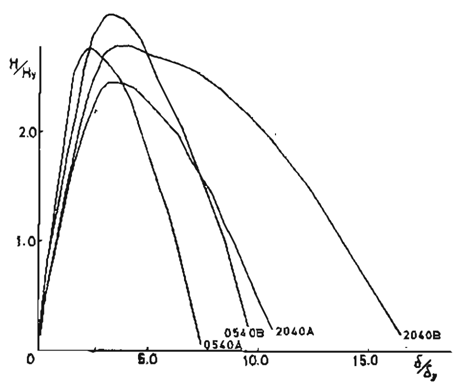
In Fig. 10(a), the results of the specimens on which no vertical force was applied are shown. It is worth noting that the difference in the curves of materials A and B, which was quite appreciable in Fig. 8, is small in this plot.



(a) $P/P_y=0$



(b) $P/P_y=0.2$



(c) $P/P_y=0.4$

Fig. 10. Horizontal Force-Displacement Relations in Dimensionless Form.

The difference between type 05 and 20 is concerned with plastic hinge formation or the collapse mechanism (Fig. 5) of perfect plasticity. Figs. 10(b) and (c) show that the instability effect of vertical loads is much more pronounced in the frame models with thinner beams than those with thicker beams. Mention is also made of the point that in the case of the former frame models under the largest vertical loads, the maximum horizontal force was reached before the frames were reduced into the collapse mechanism. This was observed not only in the elastic-perfectly plastic analysis, but also from the strain data recorded during the tests.

4.5 The Maximum Horizontal Force

In Fig. 11 is plotted the ratio of maximum horizontal force H_{max} to plastic collapse load H_p against the ratio of column load P to elastic buckling load P_{cr}. The straight line in the figure is a generalized Rankine formula,

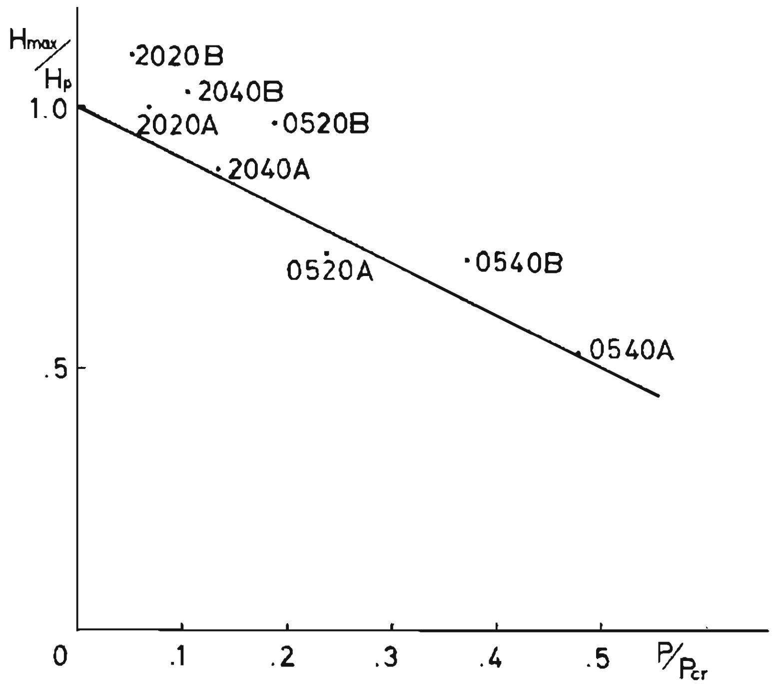


Fig. 11. Interaction Between Horizontal and Vertical Loads.

$$H_{max}/H_p+P/P_{cr}=1.0.$$

This formula seems to be a reasonably good lower bound for the maximum horizontal restoring force of frames under a constant vertical load. The actual values of H_{max}, P, P_{cr}, etc. for each frame are shown in Table 3.

4.6 Ductility Factor

Table 3 also includes the experimental ductility factors at the top beam level. This is here defined as the ratio of the top horizontal displacement at which

TABLE 3.
Experimental Results.

Specimen		0500A	0520A	0540A	2000A	2020A	2040A	0500B	0520B	0540B	0500B	2020B	2040B
Specimen	Series	A						B					
	Type	05			20			05			20		
	Number	0500A	0520A	0540A	2000A	2020A	2040A	0500B	0520B	0540B	0500B	2020B	2040B
Beam Depth h (mm)		7			14			7			14		
Section Modulus Ratio z_B/z_C		0.49			1.96			0.49			1.96		
Slenderness Ratio		106.7			56.7			106.7			56.7		
Vertical Load $P(t)$		0	2.61	5.22	0	2.61	5.22	0	2.04	4.08	0	2.04	4.08
Elastic Buckling Load $P_{cr}(t)$		10.93			38.63			10.93			38.63		
P/P_y		0	0.200	0.400	0	0.200	0.400	0	0.200	0.400	0	0.200	0.400
P/P_{cr}		0	0.239	0.479	0	0.068	0.135	0	0.187	0.373	0	0.053	0.106
Ultimate Horizontal Force (t)	Experimental H_{max}	—	0.265	0.185	—	0.874	0.671	—	0.277	0.194	—	0.755	0.615
	E. P. P. Analysis H_1	0.373	0.235	0.146	0.912	0.798	0.630	0.292	0.199	0.143	0.712	0.628	0.513
	E. S. H. Analysis H_2	—	0.252	0.157	—	0.878	0.695	—	0.216	0.154	—	0.742	0.568
H_{max}/H_1		—	1.13	1.27	—	1.10	1.07	—	1.39	1.36	—	1.20	1.20
H_{max}/H_2		—	1.05	1.18	—	1.00	0.97	—	1.28	1.26	—	1.02	1.08
Ductility Factor		—	2.9	2.3	—	3.3	3.3	—	5.2	3.1	—	7.5	3.7

the horizontal force reaches its maximum to the displacement at which the first yielding occurs somewhere in the frame. The ductility decreases with the increase of the vertical loads. It is clearly shown that frames with thicker beams have more ductility. A rather peculiar thing is that the ductility factor thus defined is larger in specimens of materials B than material A which has larger yield point stress, as seen in Table 2.

5. Summary and Conclusions

In this experimental study, the horizontal force-displacement relations were observed for steel single-bay three-storey frames with rectangular sections subjected to constant vertical loads.

The horizontal force-displacement curves show an initial linear relation followed by a gradual decrease in slope, and the slope becomes negative beyond a certain amount of displacement, indicating the importance of dead loads for the horizontal restoring forces in tall buildings. The instability phenomena become significant as the vertical loads get larger and the horizontal displacements increase; even when the vertical loads are small, the instability is of considerable importance for a large deformation. It was confirmed that the instability effect is more pronounced in frames with weaker beams. The ductility was smaller in frames of material with larger yield point stress.

Theoretical assessment was also made of the elastic-perfectly plastic analysis and an approximate elastic-strain hardening analysis. The former assumed the moment-curvature relation of linear elastic type followed by the perfectly plastic type, and hence adopted the plastic-hinge concept; the latter linearly related the increment in moment after the formation of a plastic-hinge to the discontinuity in slope angle across a plastic hinge, based on an assumed distribution of plastic strains. The elastic-perfectly plastic analysis underestimated the restoring force by a considerable amount for a large deformation; the elastic-strain hardening analysis had a reasonable agreement with the experimental results. However, further studies remain to be carried out on the validity of the latter approach, before it can be applied to other frames with general dimensions under general loading condition.

Acknowledgements

This experimental study was sponsored by Takenaka Komuten Co., Ltd. The authors wish to thank Mr. Tatsuo Murota and other students of the Department of Architecture, Kyoto University, in grateful recognition of their assistance with both the testing and the laborious analytical computations.

Bibliography

1) See, e.g., Baker, J. F., Horne, M. R. and Heyman, J., "The Steel Skeleton", Vol 2, Cambridge University Press, Cambridge, 1956.
Hodge, P. G., Jr., "Plastic Analysis of Structures", McGraw-Hill, New York, 1959.
Neal, B. G., "The plastic Methods of Structural Analysis", Second Eddition, Chapman & Hall, London, 1965.
2) Onat, E. T., "On Certain Second-Order Effects in the Limit Design of Frame", Journal of the Aeronautical Sciences, Vol. 22, No. 10, Oct., 1955, pp. 681-684.
3) Ostapenko, A., "Behavior of Unbraced Frames", Lecture Note on Plastic Design of

Multi-Story Frames, Chapter 13, Fritz Enineering Laboratory Report No. 237.20, Lehigh University, 1965, pp. 13.1–13.23.

4) Sakamoto, J., "An Outline of the Theories on the Elastic-Plastic Stability of Rigid-Frames", JSSC, Vol. 3, No. 17, May, 1967, pp. 4–13 (In Japanese).

5) Horne, M. R. and Medland, I. C., "Collapse Loads of Steel Frameworks Allowing for the Effect of Strain-Hardening", Proceedings of the Institution of Civil Engineers, Vol. 33, March, 1966, pp. 381–402.

6) Horne, M. R. and Chin, M. W., "Plastic Design of Portal Frames Allowing for Frame Instability and Strain-Hardening Effects", Proceedings of the Institution of Civil Engineers, Vol. 37, May, 1967, pp. 163–182.

7) Davies, J. M., "Frame Instability and Strain Hardening in Plastic Theory", Proceedings of American Society of Civil Engineers, Vol. 92, No. ST 3, June, 1966, pp. 1–15.

8) Wakabayashi, M., Nonaka, T. and Matsui, C., "An Experimental Study on the Inelastic Behavior of Steel Frames Subjected to Vertical and Horizontal Loading", Bulletin of the Disaster Prevention Research Institute, Kyoto University, Vol. 17, Part 1, No. 119, July, 1967, pp. 27–48.

9) Wakabayashi, M., Nonaka, T. and Matsui, C., "An Experimental Study on the Horizontal Restoring Forces in Steel Frames under Large Vertical Loads", Submitted to to the Fourth World Conference on Earthquake Engineering, Chile, Jan., 1969.

10) Igarashi, S. and Taga, N., "Hysteretic Characteristics and Structural Damping of Steel Structures under Alternate Lateral Loading", Transactions of the Architectural Institute of Japan, No. 120, Feb., 1966, pp. 15–25.

11) Igarashi, S., Taga, N., Takada, S. and Koyanagi, Y., "Plastic Behavior of Steel Frames under Cyclic Loadings", Transactions of the Architectural Institute of Japan, No. 130, Dec., 1966, pp. 8–15.

12) Arnold, P., Adams, P. F. and Lu, L. W., "Experimental and Analytical Behavior of a Hybrid Frame", Fritz Engineering Laboratory Report No. 297.18, Lehigh University, May, 1966.

13) Wakabayashi, M., "The Restoring Force Characteristics of Multi-Story Frames", Bulletin of the Disaster Prevention Research Institute, Kyoto University, Vol. 14, Part 2, No. 78, Feb., 1965, pp. 29–47.

14) Nonaka, T., "Some Interaction Effects in a Problem of Plastic Beam Dynamics, Part 2: Analysis of a Structure as a System of One Degree of Freedom", Journal of Applied Mechanics, Transactions of the American Society of Mechanical Engineers, Vol. 34, Series E, No. 3, Sep., 1967, pp. 631–637.

15) Nonaka, T., "Some Interaction Effects in a Problem of Plastic Beam Dynamics, Part 3: Experimental Study" Journal of Applied Mechanics, Transactions of the American Society of Mechanical Engineers, Vol. 34, Series E, No. 3, Sept., 1967, pp. 638–643.