Mechanics of the Successive Saltation of a Sand Particle on a Granular Bed in a Turbulent Stream

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Abstract

Most of the saltating sand particles on a granular bed in a turbulent stream have successively continued the saltation motion. In this paper, such a saltation motion is defined as successive saltation. A theory of the successive saltation of a single sand particle on a granular bed is proposed on the basis of the equation of motion for the saltation and the dynamic characteristics of collision between a saltaing sand particle and bed sand particles. The theoretical relationships for the saltation height and distance and for their distribution characteristics are in good agreement with the results of the experiments.

1. Introduction

One of the crucial problems in sediment transport mechanics is to establish the mechanics of the motion of sand particles on a granular bed in a turbulent stream. Although many investigations of sediment transport mechanics have been carried out, the mechanics of the motion of sand particles have not yet been established.

In 1951, Kawamura[1] proposed a theory of sand movement by wind, based on the equation of motion of a sand particle applying the drag forces acting on the particle to the equation of motion, but neglecting the virtual mass force because the force is usually very small compared with the drag force in a wind stream. Recently Owen[2] studied the mechanism of saltation of sand particles by wind to discover the velocity profiles in a saltation layer and the rate of sediment transport.

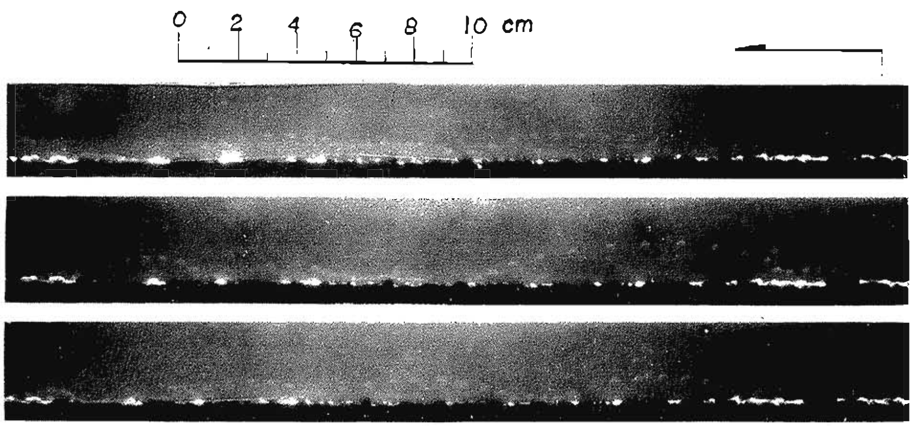
Yalin[3] recently proposed, on the other hand, a theory of saltation of sand particles in a turbulent water stream taking into consideration the uplift force on a sand particle in the vicinity of the granular bed, as measured by Einstein and Sammi[4] and by Chepil[5], based on the mechanics of sand movement by wind, and he also proposed a formula for the rate of sediment transport. In Japan, Kishi and Fukuoka[6] more recently carried out a basic experiment on the first saltation of a single spherical particle from the beginning of movement in a turbulent stream and modified Yalin's theory of saltation taking into consideration the virtual mass force. Tsuchiya and Sumino[7] also performed the same experiments as those done by Kishi and Fukuoka to discover the mechanism of successive saltation of a spherical sand particle from the beginning of movement including the first saltation.

In the author's preceding paper, an approach to establish the mechanics of saltation of a single sand particle in a turbulent stream, including the sliding and rolling motion, was presented, based on a different concept of the motion of a sand particle from the theories of Yalin and of Kishi and Fukuoka, after considering the equation of motion of an accelerating spherical particle in fluid. It was found that the theoretical relationships of the rolling distance and distribution characteristics and the velocity of a sand particle are in good agreement with the experimental results. In addition, a theory of the first saltation from the beginning of movement by collision of the rolling sand particle with a bed sand particle is established. The theoretical relationships of the saltation height of a sand particle and the distribution characteristics are in good agreement with the results of the experiments.

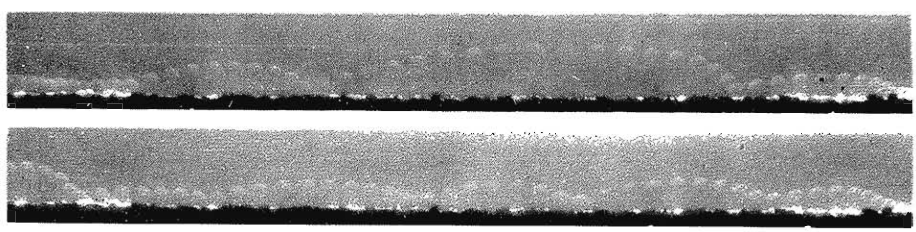
In this paper, a theory on the successive saltation of a single sand particle on a granular bed in a turbulent stream is established, based on the same concept as that of the preceding paper, taking into consideration the mechanism of collision between a saltating sand particle and bed sand particles.

2. Theory of the Saltation of a Sand Particle

Photo. 1 shows some photographs of the saltation of a single spherical particle on a granular bed made of sand grains of the same size as the particle in a turbulent stream. Since the Reynolds number becomes very high in the motion of a sand particle in a turbulent stream in general, the quadratic formula for drag



(a) $d=0.382$ cm, $\sigma/\rho=1.502$, $u_*=5.94$ cm/sec



(b) $d=0.593$ cm, $\sigma/\rho=1.343$, $u_*=6.55$ cm/sec

Photo. 1 Some photographs of the saltation of a spherical sand particle in a turbulent stream.

forces is applicable to the equation of motion. It is assumed that the size of the sand particle is so large that the effect of turbulence on the motion is not taken into consideration. Although the hodograph space can be used in establishing the equation of motion of a sand particle, the equations of motion are assumed to be established in the vertical and horizontal directions respectively, because the saltation height is assumed to not be so high. Neglecting the Basset term which is one of the virtual mass forces, the equations of motion of a sand particle can be written as

$$dW/dt=\mp(3/4)C_{D1}W^2/(\sigma/\rho+1/2)d-(\sigma/\rho-1)g/(\sigma/\rho+1/2) \tag{1}$$

in the vertical direction, and

$$dU/dt=(3/4)C_{D2}(u-U)^2/(\sigma/\rho+1/2)d \tag{2}$$

in the horizontal direction respectively, in which W is the velocity component of the sand particle in the vertical direction, U the component in the horizontal direction, C_{D1} the drag coefficient of the sand particle in the vertical direction, C_{D2} the drag coefficient in the horizontal direction, σ the density of the sand particle, ρ the density of fluid, d the diameter of the sand particle and g the acceleration of gravity. And u in Eq. (2) denotes the representative velocity in the saltation motion of the sand particle. Since the effect of the velocity profile can be neglected in this case where the saltation height does not become so high, the velocity is assumed to be a constant written as

$$u=A_r u_* \tag{3}$$

in which u_* is the shear velocity and A_r a constant which becomes 8.5 in the case of fully developed turbulent flow. In addition, the compound notation in Eq. (1) becomes positive in the upwards motion of the sand particle and negative in the downwards motion respectively.

Let the following dimensionless quantities be introduced into the equation of motion,

$$\left.\begin{aligned}&\bar{W}=W/u_*,\quad \bar{U}=U/u_*,\\ &K^2=(4/3)\{(\sigma/\rho-1)gd/u_*^2\}(1/C_{D1})\\ &\tau_1=(3/4)\{C_{D1}/(\sigma/\rho+1/2)\}(u_*t/d)\\ &\tau_2=(3/4)\{C_{D2}/(\sigma/\rho+1/2)\}(u_*t/d)\end{aligned}\right\} \tag{4}$$

and Eqs. (1) and (2) can be rewritten respectively as

$$d\bar{W}/d\tau_1=\mp\bar{W}^2-K^2 \tag{5}$$

$$d\bar{U}/d\tau_2=(\bar{u}-\bar{U})^2 \tag{6}$$

Introducing the dimensionless form defined by

$$\xi=x/d,\quad \eta=z/d \tag{7}$$

in which x denotes the coordinate of which the origin is at the point of the beginning of movement, and z the ordinate, the solution can be reduced as follows:-

(1) *Motion in the vertical direction*

In the upward motion of a sand particle, Eq. (1) can be written as

$$d\bar{W}/d\tau_1=-(\bar{W}^2+K^2) \tag{8}$$

Integration of Eq. (8) with respect to τ_1 with the initial condition that $\bar{W}=\bar{W}_0$ at $\tau_1=0$ yields

$$\bar{W}=K\{(\bar{W}_0/K)-\tan K\tau_1\}/\{1+(\bar{W}_0/K)\tan K\tau_1\} \tag{9}$$

Integrating once more Eq. (9) with respect to τ_1 with the initial condition that $\eta=0$ at $\tau_1=0$, the trajectory in the vertical direction can be written as

$$\eta=(4/3)\{(\sigma/\rho+1/2)/C_{D1}\}\log\{\cos K\tau_1+(\bar{W}_0/K)\sin K\tau_1\} \tag{10}$$

Since the saltation height becomes maximum when $\bar{W}=0$ in Eq. (9), the time $\tau_{1,0}$ can be expressed as

$$\tau_{1,0}=(1/K)\tan^{-1}(\bar{W}_0/K) \tag{11}$$

Therefore, the maximum saltation height H can be written by putting Eq. (11) into Eq. (10) as

$$\bar{H}=(2/3)\{(\sigma/\rho+1/2)/C_{D1}\}\log\{1+(\bar{W}_0/K)^2\} \tag{12}$$

in which $\bar{H}$ denotes H/d. When $\bar{W}_0/K\ll 1$, Eq. (12) can be approximated by

$$\bar{H}\fallingdotseq(2/3)\{(\sigma/\rho+1/2)/C_{D1}\}(\bar{W}_0/K)^2 \tag{13}$$

On the other hand, Eq. (1) can be written in the downward motion of a sand particle as

$$d\bar{W}/d\tau_1'=\bar{W}^2-K^2 \tag{14}$$

in which τ_1' denotes the dimensionless time expressed by the same form as τ_1 to be taken $\tau_1'=0$ at $\eta=\bar{H}$. Integration of Eq. (14) with respect to τ_1' with the initial condition that $\bar{W}=0$ at $\tau_1'=0$ yields

$$\bar{W}=-K\tan(K\tau_1') \tag{15}$$

Integrating once more Eq. (15) with the initial condition that $\eta=\bar{H}$ at $\tau_1'=0$, the expression for η in the downward motion can be written as

$$\eta=\bar{H}-(4/3)\{(\sigma/\rho+1/2)/C'_{D1}\}\log\{\cosh(K\tau_1')\} \tag{16}$$

in which C'_{D1} denotes the drag coefficient corresponding to C_{D1} in the downward motion. The arriving time of the sand particle at the bed $\tau_{2,0}$ can be obtained by taking $\tau_1'=\tau_{2,0}$ as

$$\tau_{2,0}=(1/K)\cosh^{-1}\{1+(\bar{W}_0/K)^2\}^{C'_{D1}/2C_{D1}} \tag{17}$$

Therefore, the vertical component of velocity $\bar{W}_1$ just before arriving at the bed can be expressed by putting Eq. (17) into Eq. (15) as

$$\bar{W}_1=-K[\{1+(\bar{W}_0/K)^2\}^{C'_{D1}/2C_{D1}}-1]^{1/2}\{1+(\bar{W}_0/K)^2\}^{C'_{D1}/2C_{D1}} \tag{18}$$

$$=-K(\bar{W}_0/K)/\{1+(\bar{W}_0/K)^2\} \text{ for } C'_{D1}=C_{D1} \tag{18'}$$

It is assumed that C_{D1} is nearly equal to C'_{D1} in the range of a high Reynolds number. Then considering the case where $\bar{W}_0/K\ll 1$, Eqs. (17) and (18) can be approximated by

$$\tau_{2,0}\fallingdotseq(1/K)(\bar{W}_0/K) \quad \text{and} \quad \bar{W}_1\fallingdotseq\bar{W}_0 \tag{19}$$

respectively.

(2) *Motion in the horizontal direction*

Integration of Eq. (6) with the initial condition that $\bar{U}=\bar{U}_0$ at $\tau_2=0$ yields

$$\bar{U}=\bar{u}-(\bar{u}-\bar{U}_0)/\{1+(\bar{u}-\bar{U}_0)\tau_2\} \tag{20}$$

Integrating once more with respect to τ_2 with the initial condition that $\xi=0$ at $\tau_2=0$ the trajectory in the horizontal direction can be expressed as

$$\xi=(4/3)\{(\sigma/\rho+1/2)/C_{D2}\}\{\bar{u}\tau_2-\log|(\bar{u}-\bar{U}_0)\tau_2+1|\} \tag{21}$$

Putting the time $\tau_2=\tau_{1,0}+\tau_{2,0}$ into Eqs. (20) and (21), the horizontal component of velocity of the particle just before arriving at the bed $\bar{U}_1$ and the saltation distance L can be written respectively as

$$\bar{U}_1=\bar{u}-(\bar{u}-\bar{U}_0)/[1+(\bar{u}-\bar{U}_0)(\tan^{-1}(\bar{W}_0/K)+\cosh^{-1}\{1+(\bar{W}_0/K)^2\}^{1/2})/K] \tag{22}$$

$$\bar{L}=(4/3)\{(\sigma/\rho+1/2)/C_{D2}\}[\bar{u}(\tan^{-1}(\bar{W}_0/K)+\cosh^{-1}\{1+(\bar{W}_0/K)^2\}^{1/2})/K-\log|(\bar{u}-\bar{U}_0)(\tan^{-1}(\bar{W}_0/K)+\cosh^{-1}\{1+(\bar{W}_0/K)^2\}^{1/2})/K+1|] \tag{23}$$

in which $\bar{L}$ denotes L/d. When $\bar{W}_0/K\ll 1$ Eqs. (22) and (23) can be written respectively as

$$\bar{U}_1 \fallingdotseq \bar{u} - (\bar{u} - \bar{U}_0)/\{1 + 2(\bar{u} - U_0)\bar{W}_0/K^2\} \tag{24}$$

$$\bar{L} \fallingdotseq (4/3)\{(\sigma/\rho + 1/2)/C_{D2}\}\{2\bar{u}\bar{W}_0/K^2 - \log|2(\bar{u} - \bar{U}_0)\bar{W}_0/K^2 + 1|\} \tag{25}$$

3. Theory of the Collision and Rebound of a Sand Particle on Bed Particles

Fig. 1 shows a schematic diagram for the collision and rebound of a sand particle on bed particles, in which V_1 and V_2 denote the velocity vectors of a saltating sand particle before and after the collision with a bed sand particle respectively, α and β the angles between the directions of the velocity vectors and the horizontal respectively, and γ is the angle between the line connecting the center of the particle with the center of a stationary sand particle and the horizontal. Making the assumption that the effect of rotation of a saltating sand particle on the motion, the friction force in the tangential direction at the collision and the drag force acting on the particle at the rebound are small compared with the effect of only the collision, the conservation law of momemtum in the normal and tangential directions yields respectively

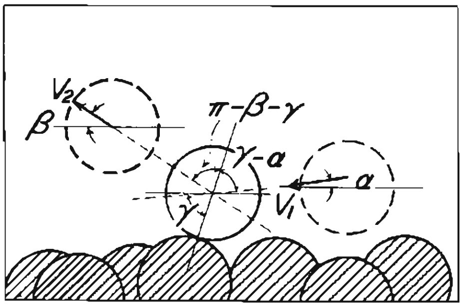


Fig. 1 Schematic diagram for collision of a sand particle with bed particles.

$$\left.\begin{aligned} -eV_1\cos(\gamma-\alpha) &= V_2\cos(\pi-\beta-\gamma) \\ V_1\sin(\gamma-\alpha) &= V_2\sin(\pi-\beta-\gamma) \end{aligned}\right\} \tag{26}$$

in which e is the coefficient of rebound of a saltating sand particle of which value is assumed to be approximately 0.8 in the case of the saltation of a sand particle on a fixed granular bed and 0.5 in the case of the saltation on a movable bed.

The value of e can be obtained by Eq. (26) as

$$e = \{(V_1\cos\alpha - V_2\cos\beta)V_2\cos\beta - (V_1\sin\alpha + V_2\sin\beta)V_2\sin\beta\}$$
$$\{(V_1\cos\alpha - V_2\cos\beta)V_1\cos\alpha + (V_1\sin\alpha + V_2\sin\beta)V_1\sin\alpha\}^{-1} \tag{27}$$

Furthermore, defining the following quantities

$$\left.\begin{aligned} &V_1\cos\alpha = U_1,\quad V_1\sin\alpha = W_1,\quad V_2\cos\beta = U_0,\quad V_2\sin\beta = W_0, \\ &U_1 = -W_1\cot\alpha = \delta_1 W_1 (\delta_1 < 0) \\ &U_0 = W_0\cot\beta = \delta_2 W_0 (\delta_2 > 0) \end{aligned}\right\} \tag{28}$$

and making the dimensionless forms for Eq. (28)

$$\left.\begin{aligned} &\bar{W}_1 = W/u_*,\quad \bar{U}_1 = U_1/u_*,\quad \bar{W}_0 = W_0/u_*, \\ &\bar{U}_0 = U_0/u_*,\quad \bar{U}_1 = \delta_1\bar{W}_1,\quad \bar{U}_0 = \delta_2\bar{W}_0 \end{aligned}\right\} \tag{29}$$

the relationship between $\bar{W}_1$ and $\bar{W}_0$ can be written as

$$\bar{W}_0 = [\{(1+e)(\delta_1\delta_2 - 1) + \sqrt{(1+e)^2(1-\delta_1\delta_2)^2 - 4e(1+\delta_1^2)(1+\delta_2^2)}\}/2$$
$$(1+\delta_2^2)]\bar{W}_1 = P_0\bar{W}_1 \quad (P_0 < 0) \tag{30}$$

Since the values of $|\delta_1|$ and δ_2 take nearly from 2 to 10, the values of δ_1^2, δ_2^2 and $|\delta_1\delta_2|$ are assumed to be sufficiently larger than unity. Therefore Eq. (30) can be approximated by

$$\bar{W}_0 \fallingdotseq e(\delta_1/\delta_2)\bar{W}_1 \tag{31}$$

Fig. 2 shows a comparison between Eqs. (30) and (31), in which the solid

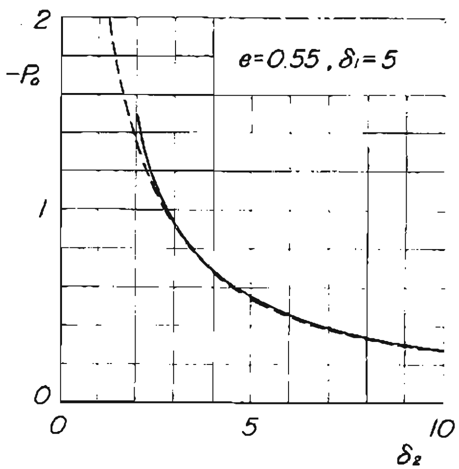


Fig. 2 Comparison between Eqs. (30) and (31).

and chain curves denote Eqs. (30) and (31) respectively. It is seen from Fig. 2 that Fq. (31) agrees well with Eq. (30) in the range where the value of δ_2 is larger than 2.5.

In addition, the expression of δ_2 in relation to γ and α can be written as

$$\begin{aligned}1/\delta_2 &= \{(1+e)\tan\gamma-(1-e\tan^2\gamma)\tan\alpha\}/\{(\tan^2\gamma-e)-(1+e)\tan\gamma\tan\alpha\}\\ &=\{(1+e)\tan\gamma+(1-e\tan^2\gamma)(1/\delta_1)\}/\{(\tan^2\gamma-e)+(1+e)(1/\delta_1)\tan\gamma\}\end{aligned} \tag{32}$$

Putting Eqs. (30) and (31) into Eq. (32), the relationships between the value of $\bar{W}_0$ or $\bar{U}$ and the values of $\tan\gamma$, $\bar{W}_1$ and $\bar{U}_1$ can finally be expressed as

$$\bar{U}_0=e\bar{U}_1 \tag{33}$$

$$\bar{W}_0=e\{(1+e)\bar{U}_1\tan\gamma+(1-e\tan^2\gamma)\bar{W}_1\}/\{(\tan^2\gamma-e)+(1+e)(\bar{W}_0/\bar{U}_1)\tan\gamma\} \tag{34}$$

Making the following abbreviations

$$b_1=(1+e)\tan\gamma,\quad b_2=(1-e\tan^2\gamma),\quad b^3=(\tan^2\gamma-e) \tag{35}$$

Eq. (34) can be rewritten as

$$\bar{W}_0=e\{b_1\bar{U}_1+b_2\bar{W}_1\}/\{b_3+b_1(\bar{W}_1/\bar{U}_1)\} \tag{36}$$

4. Theory of the Successive Saltation of a Sand Particle by Rebound

Now consider the phenomena in which a sand particle is moving downstream alternately repeating the saltation motion and rebounding motion on a granular bed both by the drag forces acting on the particle and the effect of collision of the

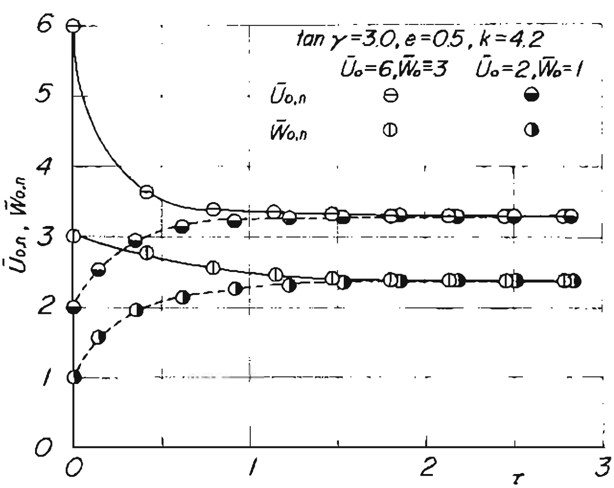


Fig. 3 Changes of velocity of a sand particle in successive saltation.

sand particle with bed sand particles. Fig. 3 shows an example of the changes of velocity of a sand particle in successive saltation, which is calculated by using Eqs. (18) and (22) for the saltation motion and Eqs. (33) and (35) for the rebounding motion and by applying the specific initial values that $\bar{U}_0=6$ and $\bar{W}_0=3$ for one of the cases and $\bar{U}_0=2$ and $\bar{W}_0=1$ for another one. In this figure, the coordinate τ is the dimensionless time which is the same expression as Eq. (4), and the circles on the curves show the initial velocities in the vertical and horizontal directions at each step of successive saltation. It is seen from the figure that the initial velocities at each step of the saltation approach a constant value at the fourth or fifth step from the beginning of movement independently of the initial velocity at the first saltation. The constant value is independent of the values of e, $\tan\gamma$ and K. Since the values of e and $\tan\gamma$ are generally characterized by the condition of granular bed and the inverse of K corresponds to the dimensionless tractive force, the constant value for the initial velocity in the successive saltation is generally characterized by only the dimensionless tractive force. In this paper, such a constant velocity of a sand particle in successive saltation is called the stationary velocity of saltation, of which the components in the vertical and horizontal directions are expressed by $\bar{W}_s$ and $\bar{U}_s$ respectively in the dimensionless form. Such a successive saltation having a stationary velocity is defined as stationary saltation.

The relationship of the stationary velocity with the values of e, $\tan\gamma$ and K can be reduced in an approximate form as follows:- Making the assumption that the sand particle which has alternately taken saltation and rebound keeps the stationary velocity at the k-th step in saltation, the following relationships can be obtained as

$$\bar{W}_{0,k}=Q'\bar{u},\quad \bar{U}_{0,k}=P'\bar{u} \tag{37}$$

$$\bar{W}_{0,k+1}=\bar{W}_{0,k}=Q'\bar{u},\quad \bar{U}_{0,k+1}=\bar{U}_{0,k}=P'\bar{u} \tag{38}$$

in which $Q'\bar{u}$ and $P'\bar{u}$ are the components of the stationary velocity in the vertical and horizontal directions respectively, and $\bar{W}_{0,k+1}$ and $\bar{U}_{0,k+1}$ the components in the vertical and horizontal directions at the $(k+1)$-th step in saltation.

Putting Eq. (37) into Eqs. (18′) and (22), the relationships can be reduced to

$$\bar{W}_{1,k}=-(Q'\bar{u}/K)^2/\{1+(Q'\bar{u}/K)^2\} \tag{39}$$

$$\bar{U}_{1,k}=\bar{u}-(1-P')\bar{u}/[1+\{(1-P')\bar{u}/K\}(\tan^{-1}(Q'\bar{u}/K) +\cosh^{-1}\{1+(Q'\bar{u}/K)^2\}^{1/2})] \tag{40}$$

in which $\bar{W}_{1,k}$ and $\bar{U}_{1,k}$ are the velocity components of a sand particle just before arriving at the granular bed in the vertical and horizontal directions respectively. Putting both Eqs. (39) and (40) into Eqs. (33) and (36), the relationships of $\bar{W}_{0,k+1}$ and $\bar{U}_{0,k+1}$ with P' and Q' can be reduced to

$$\bar{W}_{0,k+1}=e\bar{u}(b_1[1-(1-P')/[1+\{(1-P')(\bar{u}/K)\}\{\tan^{-1}(Q'\bar{u}/K) +\cosh^{-1}\{1+(Q'\bar{u}/K)^2\}^{1/2}]]-b_2K(Q'\bar{u}/K)/\{1+(Q'\bar{u}/K)^2\}])/(b_3-[b_1K(Q'\bar{u}/K)/\{1+(Q'\bar{u}/K)^2\}]/(1-(1-P')/[1+\{(1-P')(\bar{u}/K)\{\tan^{-1}(Q'\bar{u}/K)+\cosh^{-1}\{1+(Q'\bar{u}/K)^2\}^{1/2}]))) \tag{41}$$

$$\bar{U}_{0,k+1}=e\bar{u}(1-(1-P')/[1+\{(1-P')(\bar{u}/K)\}\{\tan^{-1}(Q'\bar{u}/K) +\cosh^{-1}\{1+(Q'\bar{u}/K)^2\}]^{1/2}) \tag{42}$$

Elimination of $\bar{W}_{0,k+1}$ and $\bar{U}_{0,k+1}$ in Eqs. (38), (41) and (42) yields

$$P'=e[1-(1-P')/[1+(1-P')(\bar{u}/K)\{\tan^{-1}(Q'\bar{u}/K) +\cosh^{-1}\{1+(Q'\bar{u}/K)^2\}^{1/2}]] \tag{43}$$

$$Q'=[b_1P'\{1+Q'^2(\bar{u}/K)^2\}^{1/2}-b_2eQ']/[b_3\{1+Q'^2(\bar{u}/K)^2\}^{1/2} -eb_1(Q'/P')] \tag{44}$$

which are dependent on the values of e, $\tan\gamma$ and K. Since the expressions of Eqs. (43) and (44) are very complicated, an approximation should be considered to formulate the saltation mathematically. Applying Eqs. (19) and (22) instead of Eqs. (18′) and (26) into the transformation, the approximate relationships corresponding to Eqs. (43) and (44) can be written as

$$P'=e[1-(1-P')/\{1+2(1-P')Q'(\bar{u}/K)^2\}] \tag{45}$$

$$Q'=e\{(b_1/e)P'-b_2Q'\}/\{b_3-b_1e(Q'/P')\} \tag{46}$$

Making the solution of Eq. (46) with respect to Q', the relation can be reduced to

$$Q'=[\{(b_2e+b_3)\pm\sqrt{(b_2e+b_3)^2-4b_1^2e}\}/2b_1e]P' \tag{47}$$

In the above equation, only the following expression is considered to be applicable under the physical condition that $Q'\leq P'$ in general.

$$Q'=[\{(b_2e+b_3)-\sqrt{(b_2e+b_3)^2-4b_1^2e}\}/2b_1e]P'=\lambda P' \tag{48}$$

in which λ is a constant which is independent of the tractive force but dependent on the condition of the granular bed, and is equal to $(1/\delta_2)$.

Putting the value of λ into Eq. (45), the equation can be rewritten as

$$P'=\{(1+e)\pm\sqrt{(1-e)^2+2(1-e)/\{\lambda(\bar{u}/K)^2\}}\}/2 \tag{49}$$

in which the compound notation should be taken as negative under the physical condition that $P'\leq 1$ describing the fact that the velocity component of a sand particle in the horizontal direction never becomes larger than that of fluid in general. Therefore, the expression for Q' can be written by applying Eqs. (48) and (49) as

$$Q'=\lambda P'=\lambda\{(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(\bar{u}/K)^2\}}\}/2 \tag{50}$$

Putting these relationships into Eq. (37), the stationary velocity components in the vertical and horizontal directions $\bar{W}_s$ and $\bar{U}_s$ can be expressed respectively as

$$\bar{W}_s=\lambda\{(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(\bar{u}/K)^2\}}\}(\bar{u}/2) \tag{51}$$

$$\bar{U}_s=\{(1+e)-\sqrt{(1-e)^2+2(1-e)/\{\lambda(\bar{u}/K)^2\}}\}(\bar{u}/2) \tag{52}$$

According to Eqs. (51) and (52), it is seen that the values of $\bar{W}_s$ and $\bar{U}_s$ are real, since $e\leq 1$ in general, and then the roots in these equations also are real. Next consider the value of K which takes the condition that $\bar{W}_s=\bar{U}_s=0$. As seen from Eqs. (51) and (52), the condition that $\bar{W}_s=\bar{U}_s=0$ can be written as

$$(1+e)-\sqrt{(1-e)^2+2(1-e)/\lambda(\bar{u}/K)^2}=0 \tag{53}$$

Therefore, the dimensionless tractive force satisfying the above relationship can be expressed as

$$u_*^2/(\sigma/\rho-1)gd=(2/3)(1/C_{D1})\{(1-e)/e\lambda\bar{u}^2\} \tag{54}$$

which is generally different from the so-called dimensionless critical tractive force.

From the above description, it is concluded that the sand particle moving downstream, repeating saltation and rebound alternately, reaches stationary velocity during the several steps of successive saltation in the case where the dimensionless tractive force is larger than the value obtained by Eq. (54), and that the components of the initial velocities in the vertical and horizontal directions in stationary saltation are expressed by Eqs. (51) and (52). And it is also found that the value of $\bar{U}_s$ approaches $e\bar{u}$ when $K\to 0$, which is equivalent to the infinite tractive force, and that the value of $\bar{W}_s$ bocomes $\lambda e\bar{u}$ in this case, the fact of which states that the components of particle velocity in the vertical and horizontal directions in the stationary saltation never exceed the values of $\lambda e\bar{u}$ and $e\bar{u}$ respectively.

5. Height and Distance of Saltation of a Sand Particle and Their Distribution Characteristics

(1) *Height and distatance of saltation of a sand particle in stationary saltation*

Although the height and distance of saltation of a sand particle are formulated by Eqs. (12) and (23) respectively in connetion with the initial velocities, without loss of generality for simplicity Eq. (13) can be used for the saltation height and the following equation which is reduced from Eq. (25) under the further assumption that $2(\bar{u}-\bar{U}_0)\bar{W}_0/K^2 \ll 1$ can also be applied to calculate the relationships of the saltation characteristics with the tractive force.

$$\bar{L} \fallingdotseq (8/3)\{(\sigma/\rho+1/2)/C_{D2}\}(\bar{U}_0\bar{W}_0/K^2) \tag{55}$$

Putting Eqs. (51) and (52) into Eqs. (13) and (55), the relationships of the height and distance of saltation with the dimensionless tractive force and the condition of the granular bed can approximately be written respectively as

$$\bar{H}_m \fallingdotseq (1/6)\lambda^2\{(\sigma/\rho+1/2)/C_{D1}\}[(1+e) - \sqrt{(1-e)^2+2(1-e)/\{\lambda(\bar{u}/K)^2\}}]^2(\bar{u}/K)^2 \tag{56}$$

$$\bar{L}_m \fallingdotseq (2/3)\lambda\{(\sigma/\rho+1/2)/C_{D1}\}[(1+e) - \sqrt{(1-e)^2+2(1-e)/\{\lambda(\bar{u}/K)^2\}}]^2(\bar{u}/K)^2 \tag{57}$$

in which $\bar{H}_m$ denotes the mean value of saltation height and $\bar{L}_m$ the mean value of saltation distnce. From Eqs. (56) and (57), the value of λ can be expressed as

$$\lambda = 4(C_{D1}/C_{D2})(\bar{H}_m/\bar{L}_m) \tag{58}$$

As described already, the drag coefficients in the vertical and horizontal directions keep nearly constant in the case where the Reynolds number of a sand particle becomes sufficiently high. Fig. 4 shows a relation between the dimensionless tractive force and the value of λ which is decided by the experimental results for the mean values of height and distance of saltation obtained by the authors[7]. Therefore, it may be considered that the results shown in Fig. 4 slightly differ from the value calculated by taking the mean value of H/L in every saltation of a sand particle. It is found from the results, however, that the value of λ keeps nearly constant, and that its value is equal to 0.4 independently of the dimensionless tractive force, within the range of the experiments. The angle of saltation at the beginning of movement with the horizontal corresponding to the value $\lambda=0.4$ is nearly equal to 22 degrees.

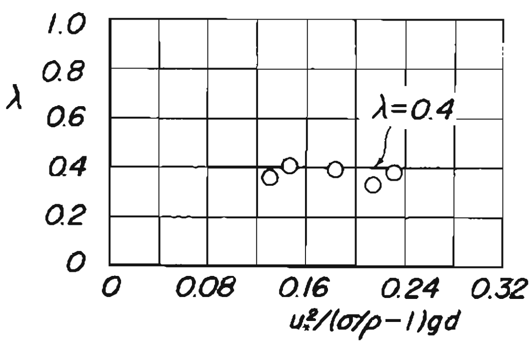


Fig. 4 Relation between the value of λ and dimensionless tractive force.

As described previously, the saltating sand particle which has saltated under an arbitrary initial condition approaches stationary saltation at the fourth or fifth saltation from the beginning of movement, regardless of the initial velocity. From this point of view, the experimental results in four more saltation which were carried out by the authors[7] are compared with the theoretical relationships for the saltation height and distance in Fig. 5, in which the value of λ is assumed to be 0.4 and the value of e is decided as 0.89 in order to fit it the experimental

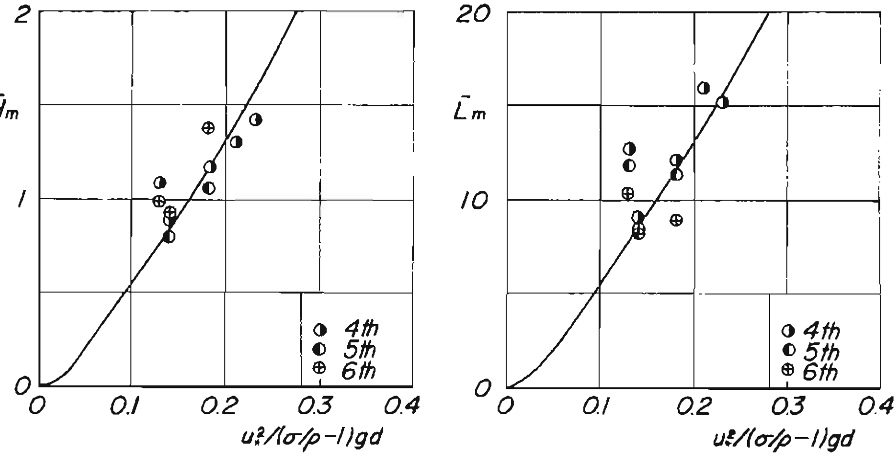


Fig. 5 Comparison between the theoretical curves of height and distance of saltation of a sand particle in stationary saltation and the experimental values.

values. It is seen from this figure that the experimental values are in good agreement with the theoretical curves within the range of the experiment and that both the saltation height and distance in stationary saltation increase rapidly with the dimensionless tractive force.

(2) *Distributions of the height and distance of saltation of a sand particle in stationary saltation*

Although the distribution characteristics of the saltation height and distance of a sand particle in stationary saltation generally depend upon the variation characteristics of the velocity of a sand particle just before and after the collision with bed particles, the dispersion characteristics of the angle of collision and the effect of turbulence on the motion of the particle, it is assumed that the distribution of height and distance of saltation is affected by the variation characteristics of the velocity. From this point of view, making the assumption that the density function of the horizontal velocity of a sand particle in stationary saltation $f_1(\bar{U})$ can be expressed by the Gaussian distribution in the form

$$f_1(\bar{U}) = (1/2\pi)(1/\sigma_{\bar{U}})\exp\{-(\bar{U}-\bar{U}_s)^2/2\sigma_{\bar{U}}^2\} \tag{59}$$

in which $\sigma_{\bar{U}}$ is the standard deviation of the dimensionless horizontal velocity $\bar{U}$, which is assumed to be

$$\sigma_{\bar{U}} = \varepsilon\bar{U}_s \tag{60}$$

in which ε is a constant, and making the assumption that the relation between the horizontal and vertical velocities can be generally expressed by

$$\bar{W} = \lambda\bar{U} \tag{61}$$

the density function of the dimensionless vertical velocity can be written as

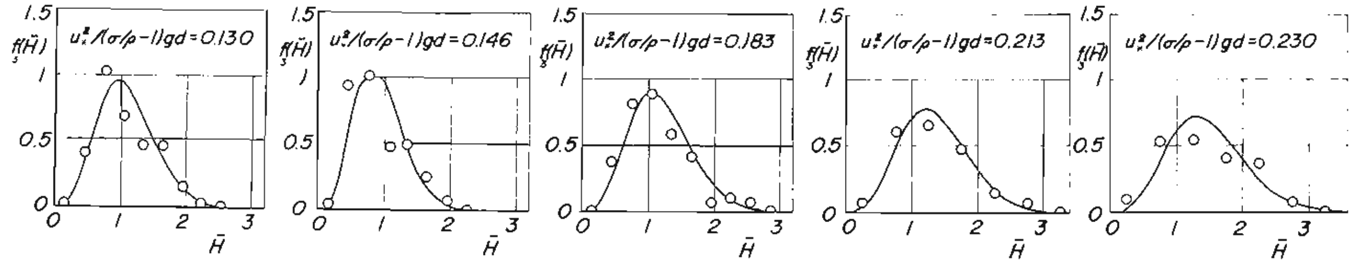


Fig. 6 Comparison between the theoretical curves of distribution of saltation height of a sand particle in stationary saltation and the experimetal values.

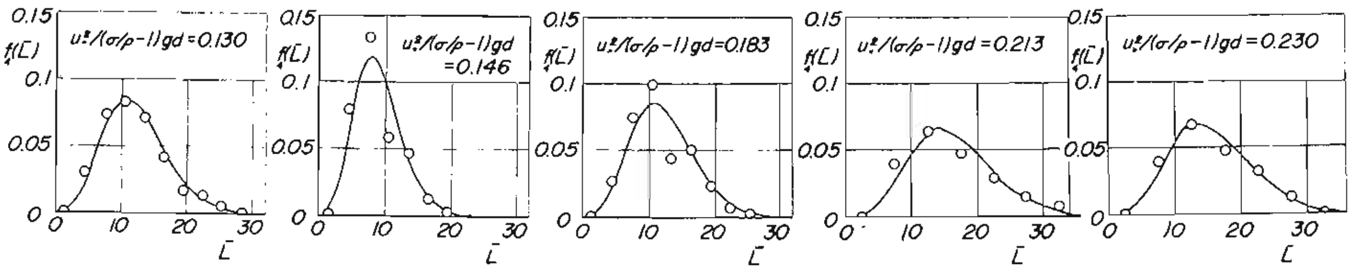


Fig. 7 Comparison between theoretical curves of distribution of saltation distance of a sand particle in stationary saltation and the experimental values.

$$f_2(W)=(1/2\pi)(1/\varepsilon\bar{W}_s)\exp\{-(\bar{W}-\bar{W}_s)^2/2(\varepsilon\bar{W}_s)^2\} \tag{62}$$

The relationship between the density function of saltation height $f_3(\bar{H})$ and the function of the vertical velocity $f_2(\bar{W})$ can generally be expressed as

$$f_3(\bar{H})d\bar{H}=f_2(\bar{W})d\bar{W} \tag{63}$$

Therefore, putting Eqs. (13) and (62) into Eq. (63), the density function of saltation height can be written as

$$f_3(\bar{H})=(1/2\sqrt{2\pi})(1/\varepsilon\sqrt{\bar{H}\bar{H}_m})\exp\{-(\sqrt{\bar{H}}-\sqrt{\bar{H}_m})^2/2\varepsilon^2\bar{H}_m\} \tag{64}$$

in which $\bar{H}_m$ is the mean value of saltation height which is assumed to be equivalent to the value expressed by Eq. (56).

Since the saltation distance expressed by Eq. (55) can be rewritten using Eq. (61) in order to formulate the density function of saltation distance as

$$\bar{L}\fallingdotseq(8/3)\{(\sigma/\rho+1/2)/C_{D2}\}\lambda(\bar{U}/K)^2 \tag{65}$$

applying the relation between the density function of saltation distance $f_4(\bar{L})$ and the function of horizontal velocity $f_1(\bar{U})$ expressed by

$$f_4(\bar{L})d\bar{L}=f_1(\bar{U})d\bar{U}_1 \tag{66}$$

the density function can be reduced to

$$f_4(\bar{L})=(1/2\sqrt{2\pi})(1/\varepsilon/\bar{L}\bar{L}_m)\exp\{-(\sqrt{\bar{L}}-\sqrt{\bar{L}_m})^2/2\varepsilon^2\bar{L}_m\} \tag{67}$$

in which $\bar{L}_m$ is the mean value of saltation distance which is assumed to be equal to the value expressed by Eq. (57).

Fig. 6 describes a comparison between the theoretical curves of the distribution of saltation height of a sand particle in stationary saltation and the experimental values obtained by the authors[7] and Fig. 7 also shows a comparison between the theoretical curves of the distribution of saltation distance and the experimental values. In these figures, the value of ε is assumed to be equal to 0.2 in the computation of Eqs. (64) and (67). It is seen from the comparison that both the distributions of the saltation height and distance of a sand particle in stationary saltation generally skew on the small side of the height and distance, and that the theoretical curves for the distributions are in good agreement with the experimental results although for their mean values the experimental values in the comparison are used for convenience. That is to say, the assumption for the distribution of the horizontal velocity of a sand particle in stationary saltation seems to be right for formulating the distribution characteristics of the saltation height and distance. From the above consideration it is confirmed that stationary saltation of a sand particle generally exists in a turbulent stream.

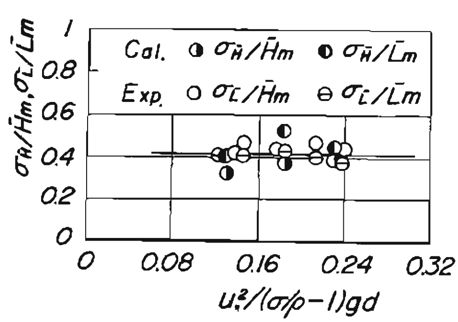


Fig. 8 Changes of values of $\sigma_{\bar{H}}/H_m$ and $\sigma_{\bar{L}}/\bar{L}_m$ with increase of dimensionless tractive force for validity of value of ε used.

In addition, although the value of ε, $\varepsilon=0.2$ was decided to fit the theoretical curve to the results of experiment, the validity of the value can be reconfirmed using the experimental results. Since the standard deviations of $f_3(\bar{H})$ and $f_4(\bar{H})$ are functions of the values of ε, $\bar{H}_m$ and $\bar{L}_m$, the values of $\sigma_{\bar{H}}$ and $\sigma_{\bar{L}}$ which are obtained from Eqs. (64) and (67), divided by the mean values are characterized by the value of ε only. Fig. 8 shows a comparison between the values of $\sigma_{\bar{H}}/\bar{H}_m$ and $\sigma_{L}/\bar{L}_m$ obtained graphically by Eqs. (64) and (67) shown in Figs. 6 and 7 and the experimental values corresponding to them. It is seen from the comparison that both values are nearly the same, but that the values are scattered. It is concluded therefore from the consideration that the distribution of the initial velocity of a sand particle in stationary saltation can be expressed by the Gaussian distribution of which the standard deviation keeps 0.2 times the mean value independently of the dimensionless tractive force.

6. Conclusion

Although the phenomena of saltation of a sand particle on a granular bed in a turbulent stream are very complicated, there generally exists stationary saltation as specially defined in this paper, which is different from the first saltation from the beginning of movement. A theory on stationary saltation, which is a type of successive saltation, is established, based on the equations of motion for the saltation of a sand particle and the dynamic relationship of the collision of a saltating sand particle with bed particles. It was concluded from the comparison between the theory and the results of the experiment that the theoretical relationships for stationary saltation are in good agreement with the results of the experiment.

Since further calculations for the successive saltation of a sand particle including any step of saltation can be carried out using an electronic computer, a theory both on the velocity profile in a saltation layer and the rate of sediment transport due to the saltation motion of sand particles will be established with the aid of some basic results of the experimental investigations on the saltation of a cloud of sand particles in a turbulent stream[10].

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Appendix

In Fig. 5 there is a mistake in the vicinity of the origin of coordinate, because the theoretical values of the saltation height and distance should vanish at the value of dimensionless tractive force expressed by Eq. (54).