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The Structure of Movable Bed Configuration

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Abstract

In this paper, the authors discuss the spacial and temporal structure of the movable bed configuration in a lower flow regime from the standpoint that the structure has a close connection with the turbulence structure in open channels.

The authors measured the bed configuration along the longitudinal lines located at different distances from the side wall at various stages of growth of the bed forms. The measurments were conducted in two flumes with different widths in order to clarify the effect of the width on the bed configuration. The turbulence structures were also measured in the flumes with fixed and movable beds at the various stages of growth of the bed forms by using a small propeller-type current meter. Comparison between the spectra of bed configuration and those of turbulence reveal intereresting characteristics as follows :

(1) The structure of the bed configuration in a lower flow regime may be classified by three different origins in turbulence structure, which are horizontal turbulence characterized by a scale of width, vertical turbulence characterized by water depth, and longitudinal streaking.

(2) The range of the wave length of the alternating bar which may correspond to horizontal turbulence is about 2~10 times the width B, and the eminent wave length is $4B$.

(3) The range of the wave length of dunes which corresponds to vertical turbulence spreads to about 10 times the depth h, and its statistical structure composes some "inertia subrange" in time and space, which has a similarity independent of external conditions. The wave length of dunes accompanied with surface water wave, which occurs when F_r is nearly 1, is about 4~5 h.

(4) Another bed configuration is longitudinal streaking, the interval of which is $2h$ laterally.

(5) The above three components are able to coexist.

(6) The variation of turbulence structure has a close connection with the process of bed configuration. The energy power density of turbulence decreases relatively in the range of which the spectrum of bed congiguration shows the maximum increase, as the bed configuration comes to reach its equilibrium state.

1. Introduction

It is well known that the various regimes of bed forms take place on a movable bed according to the flow condition. This phenomenon has a very important effect on the mechanics of sand movement, channel resistance and channel stability. Therefore, extensive investigations have been conducted experimentally and theoretically to determine the geometrical and hydraulical properties of bed forms and to clarify the origin of the various types of bed forms.

Among those experiments, D. B. Simons' and E. V. Richardson's[1] are very extensive. They divided the forms of bed roughness into two regimes of flow, the lower flow regime and upper flow regime from the standpoint of their resistance to flow. They also studied the variable that determines the form of bed roughness. Concerning the theoretical study, J. F. Kennedy[2] investigated the stability of the fluid-bed interface and the characteristics of the bed features by using an analytical model that is based on the potential flow over a two-dimensional, moving wavy bed with a sinusoidal profile of varying amplitude. A similar analysis was also developed by T. Hayashi[3]. Many other analytical models[4,5] have been proposed in order to investigate the stability and the characteristics of bed forms. On the other hand, it is well known that bed forms are composed of various waves with different wave numbers and amplitudes. The statistical properties of sand waves have attracted many researcher's attention. Nordin and Algert[6], Ashida and Tanaka[7] discussed the characteristics of the spectra of sand waves. M. Hino[8] derived the "-3 power law" of sand wave spectrum based on a dimensional consideration for a large wave number equilibrium subrange where the spectral form was assumed to be governed predominantly by the angle of repose of sand particles and the wave number. The authors consider that the spectral analysis of sand waves is an effective tool for clarifying not only their characteristics but also their origin and formation. It may be a similar fact that both turbulence and sand waves are expressed by a statistical law which governs the energy equilibrium process. Therefore, the sand wave structure has a close connection with the turbulence structure in open channels. In other words, the compoents of sand waves are considered to be well divided corresponding to the turbulence structure or to the structure of hydrodynamical instability of open channel flow.

In this paper, the authors discuss the spacial and temporal variation of the structure of the movable bed configuration in a low flow regime with a knowledge of the turbulence structure in open channels.

The measurements of bed configuration are made along the longitudinal lines located at different distances from the side wall at various stages of the growth of the bed forms in two flumes with a different width. On the other hand, the measurements of trubulence are conducted in the same flumes with fixed and movable beds at the various stages of the growth of the bed forms. The mechanism of sand waves is discussed from after consideration of the spectral structure of sand waves in space and time and its relation to turbulence structure during the developement of the sand waves.

2. Experiments for Bed Configuration

2.1 Experimental procedure

Experiments were conducted in two flumes with different dimensions as shown in Table 1 in order to clarify the effect of channel width on bed configuration, in other words to show which range of the frequency component of sand waves is controlled by flume width.

The grain-size accumulation curve of sand used is as shown in Fig. 1, the mean diameter $d_m=0.71$mm, shieve coefficient $S_o=1.30$, specific weight $\sigma=2.65$ and void ratio $\lambda=45\%$.

The experiments were made giving a constant discharge on the initial smooth

Table 1 Dimension of flumes used in experiment

Flume	width B(cm)	Length L (m)	Sediment supply
A	60	150	No supply
B	20	18	Recirculating system

bed with a constant slope. When the experiments continued, bed forms came to reach the equilibrium state according to flow condition and bed material. During the above process, the variations of bed elevation were measured along the longitudinal lines located at the center and near both side walls of the flumes at three or four stages of the process.

The measurements were conducted after stopping the flowing water and after the measurement the experiment was continued with the same discharge.

The measuring interval was decided to be 10 cm for flume A and 2.5 cm for flume B, and the measuring reach was selected as 60~100 m for flume A and net 15 m for flume B to avoid the non-uniform effect of the downstream and the upstream end. The unit in measuring the elevation was 0.5 mm.

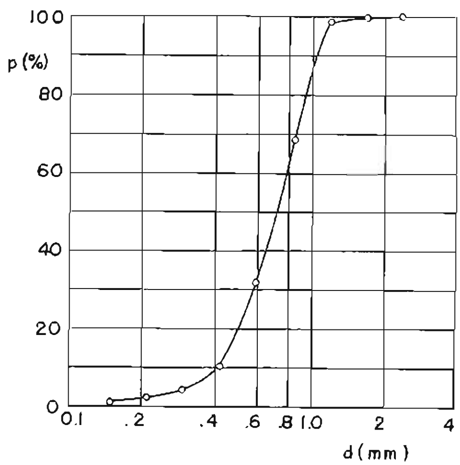


Fig. 1 Grain-Size accumulation curve of sand used.

The experimental conditions are shown in Table 2.

Table 2 Experimental conditions.

Exp.	Discharge $Q(l/s)$	Slope i	Depth h(cm)	Mean velocity U(cm/s)	Shear velocity u_*(cm/s)	Froude number F_r	Manning coeff. $n(m^{-1/3}s)$	Relative roughness k_s(cm)
A−1	20	1/500	7.1	46.6	3.34	0.56	0.0160	0.228
2	20	1/500	8.6	38.8	3.62	0.42	0.0216	1.010
3	10	1/500	4.0	41.7	2.62	0.67	0.0122	0.0658
4	10	1/500	3.8	43.9	2.58	0.72	0.0123	0.0411
5	30	1/500	15.1	33.1	4.45	0.27	0.0356	5.68
6	5	1/500	2.65	31.7	2.19	0.26	0.0118	0.0823
B−1	3.0	1/200	3.09	53.1	3.16	1.00	0.0116	0.0294
2	4.6	1/500	4.72	51.8	2.37	0.78	0.0099	0.00536
3	6.4	1/300	5.33	62.8	3.37	0.89	0.0122	0.0214
4	2.6	1/500	3.02	43.1	2.13	0.79	0.0084	0.00840

In the Table, h and u are those for the final stage of each experiment, and u_*, n and k_s are calculated from the hydraulic radius.

The experiments under the same conditions, such as A-1 and A-2 or A-3 and A-4, were conducted to discuss the reappearance of the phenomenon.

2.2 Experimental results and their analyses

The properties of bed forms obtained by the experiments are shown in Table 3 in which Λ and λ are the dominant wave length of the alternating bar and dunes respectively and v_Λ and v_λ are their travelling velocity observed in the experiments.

Table 3 Experimental results of bed form properties.

Exp.	Length of Alternating bar Λ(cm)	Length of Dune λ(cm)	Ratio of B/Λ	Ratio of h/λ	Travelling velocity v_Λ (cm/min)	Travelling velocity v_λ (cm/min)	Standard deviation σ_z(cm)	Bed configulation
A—1	260	100	0.23	0.069	2~3		0.601	Irregular→Alternating bars
2	280	120	0.21	0.071			1.144	Irregular
3	240		0.25				0.488	Alternating bars
4	350		0.172				0.443	Alternating bars
5	280	150	0.21	0.111			1.827	Irregular
6	—	—	—	—	—	—	—	Flat
B—1	80	14*)	0.25	0.22*)	10	16*)	0.269	Dunes and Alternating bars
2	—	60	—	0.079		11	0.187	Dunes and a Longitudinal Streaking
3	85	18*)	0.23	0.29*)		30*)	0.332	Irregular
4	—		—		—		—	Two Longitudinal Streakings and Dunes

*) Dunes accompanied with water surface waves.

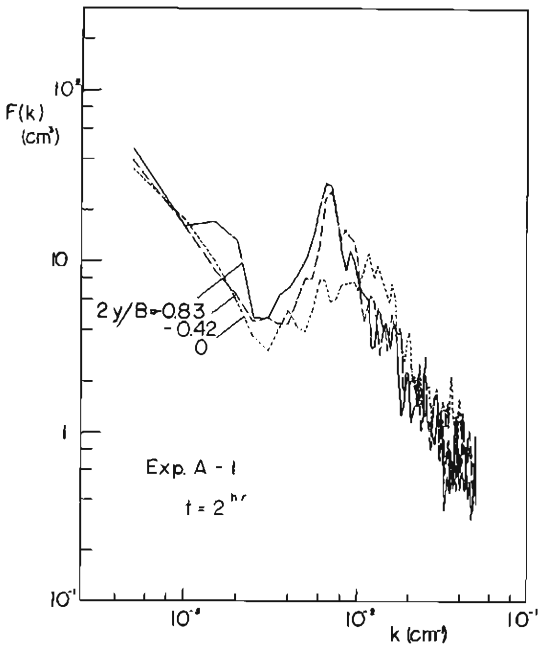


Fig 2. Lateral variation of power spectral density of bed configuration along the longitudinal line, after 2 hr. in Exp. A-1.

Photo. 1 Bed configulation to down stream, after 2 hr. in Exp. A-1.

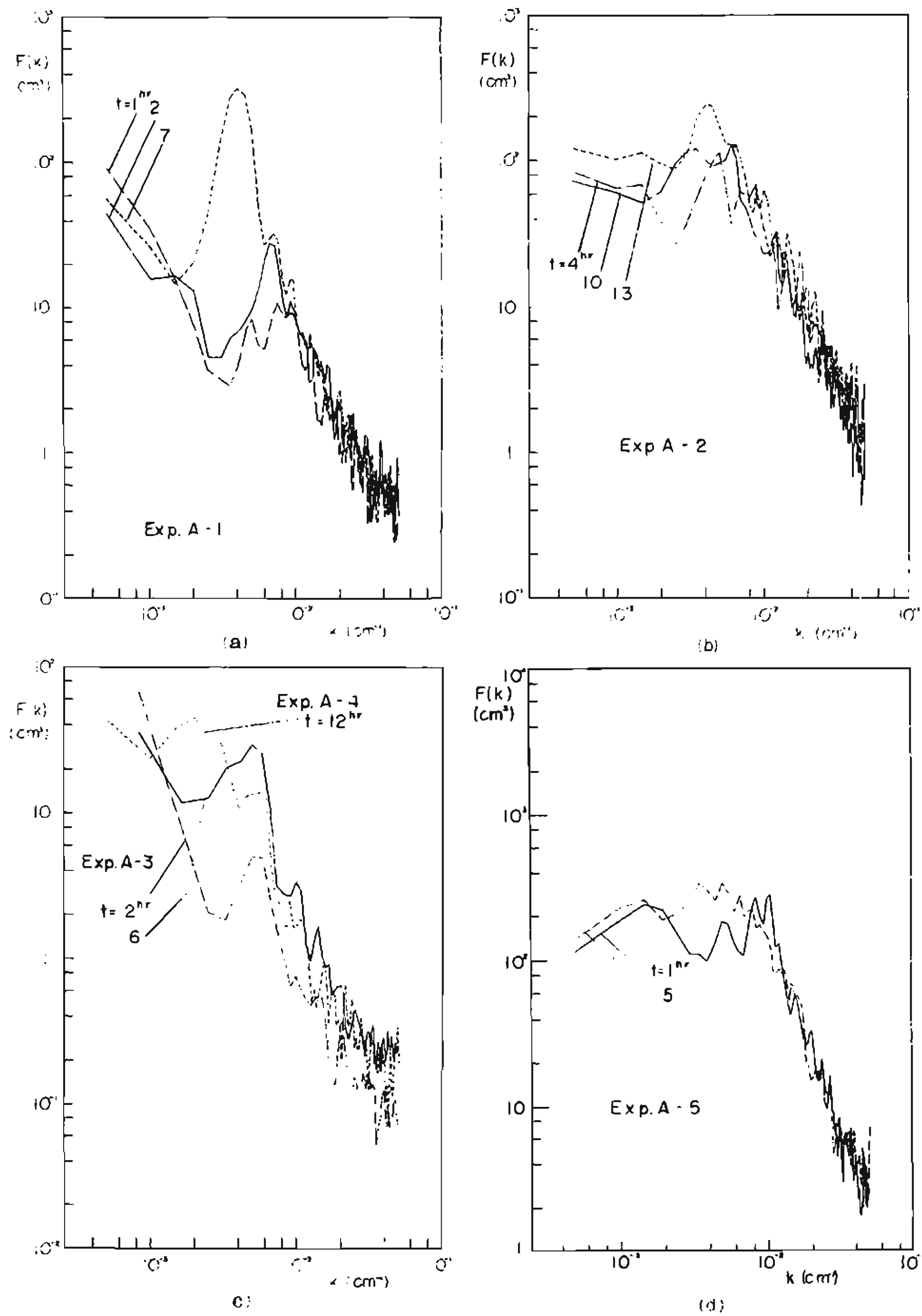


Fig. 3 Variations of power spectral density of bed configuration along the longitudinal line with time in flume A. $2y/B=-0.83$.

The spectra of the bed configuration were calculated from the data. From the variation of the spectral structure with relation to time, the process of the bed form development can be considered. And, the comparison of the spectral structure along the various longitudinal lines will show the three-dimensional characteristics of the sand waves.

(1) *Flume*−*A*

Fig. 2 shows a lateral variation of power spectral density of bed configuration along a logitudinal line after 2 hr running in Exp. A-1. The bed configuration is shown as in Photo. 1. The spectral structure for the range of high wave number of $k>10^{-2}$ cm^{-1} in this flume does not vary laterally or temporally. This means that the bed configuration in this region has two dimensional characteristics and rapidly develops with time. On the other hand, the spectral structure for the range of the non-dimensional wave number $kB=0.1\sim0.5$ varies with the position of the longitudinal line. Especially the spectrum near side walls predominantly develops with time. This shows that the bed configuration in this range has three-dimensional characteristics.

The fact that the power spectral density is high in the range of $kB<0.1$ may be due to the lower wave number component involved in the initial bed. Fig. 3 (a) $\sim$(d) show the temporal variation of the power spectral density along the longitudinal lines at $2y/B=\pm0.83$, for the experiments in flume A.

From these figures, the following facts are noticed. An equilibrium subrange exists in the high wave number region of the spectrum, in which the structure of the spectrum does not vary with time and space.

The slope of the spectrum in the above region is almost constant, but the ordinate varies with the flow conditions. From discussing the results, the above equilibrium subrange is known to correspond to the non-dimensional wave number $kh>0.1$. From the observation it may easily be noticed that the bed configuration in the above region correponds to dunes and ripples by the traditional classification.

It can be considered that the rather regular pattern of bed configuration develops with the increase of time, from the temporal increasing of power spectral density near the side wall as shown in Fig. 3 and also from the coherence as shown in Fig. 4.

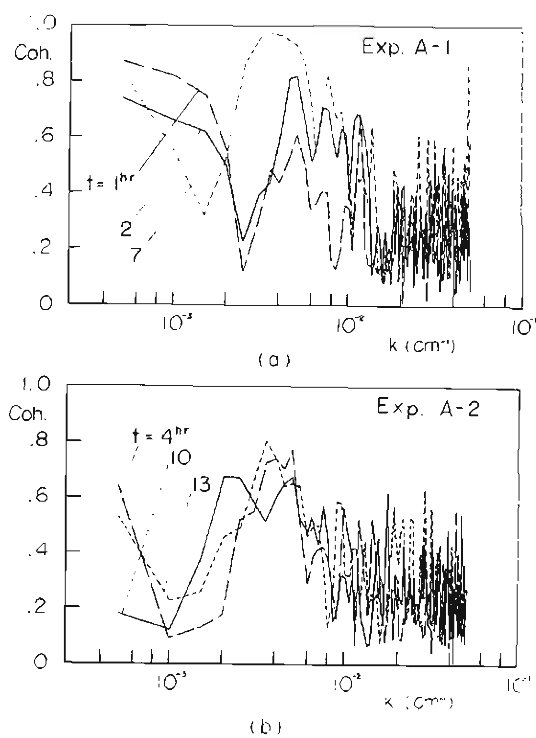


Fig. 4 Variations of coherence between $2y/B=\pm0.83$ with time in flume A.

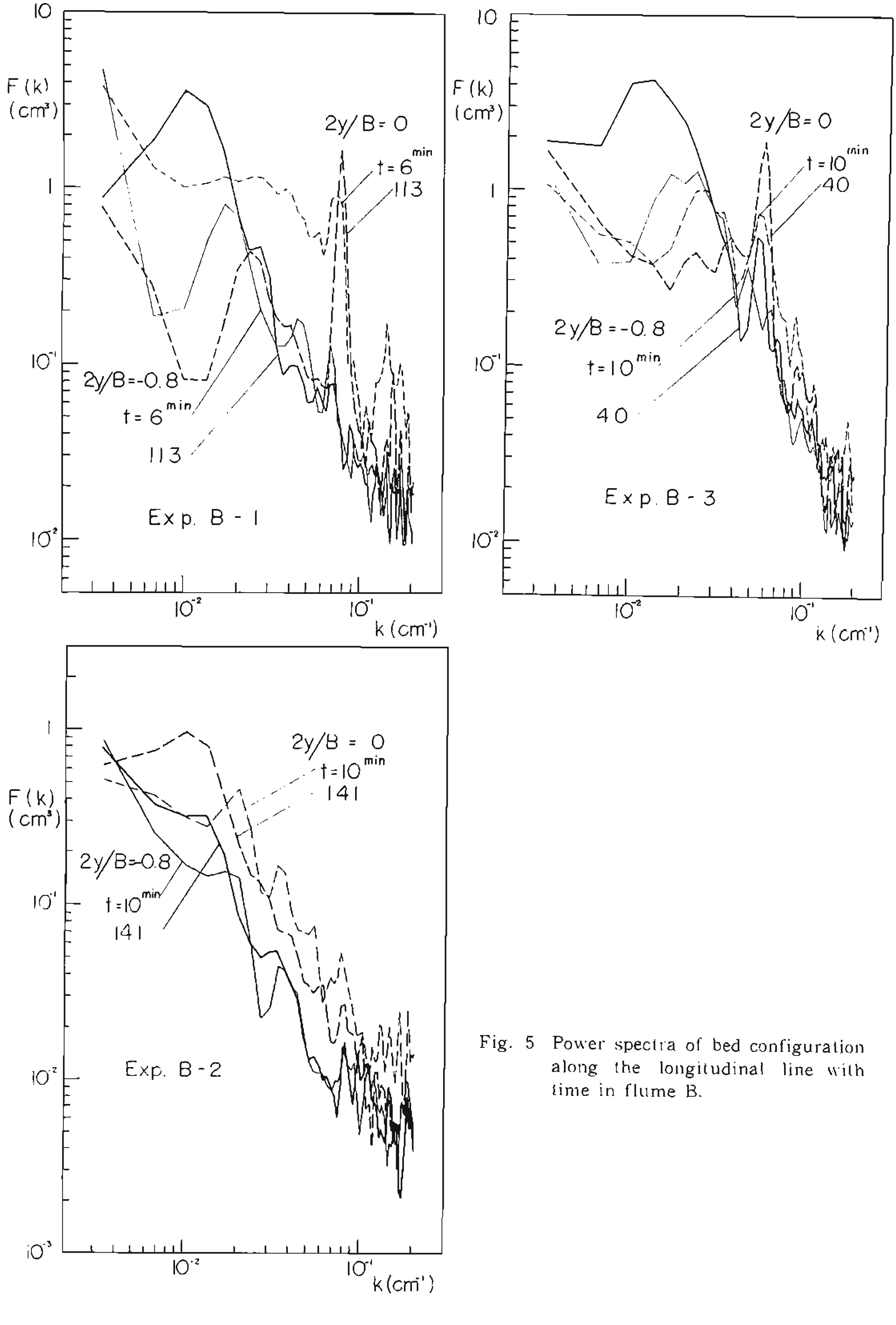


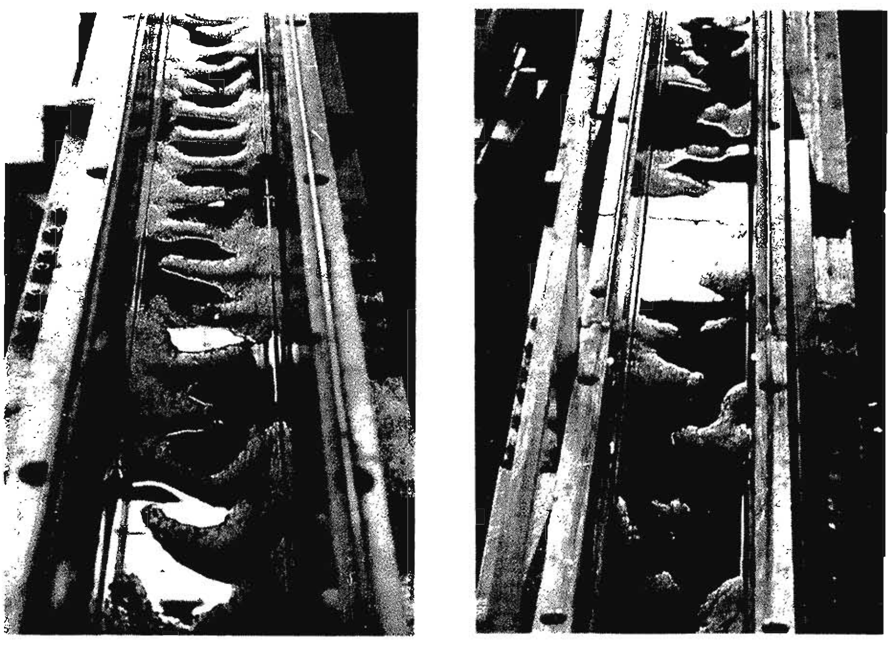
Fig. 5 Power spectra of bed configuration along the longitudinal line with time in flume B.

The value of non-dimensional wave number kB, at which the spectrum density $kF(k)$ is maximum, is almost constant, and the $kB \fallingdotseq 0.25$ except Exp. A-4. This subrange may be shown to be classified as alternating bars from the observation of the bed configuration in Exp. A-1, A-3 or the phase lag of the cross spectral analysis. And it is also seen that it takes much time to develop the bed configuration in this region.

(2) *Flume — B*

The statistical structure of the bed configuration in flume B was almost similar to that obtained in flume A.

Fig. 5 (a)~(c) show the temporal variation of the spectra along the various longitudinal lines. The range of $0.1 < kB < 0.5$ in Exp. B-1 and B-2 corresponds to the regime of the alternating bars. The power spectral density shows the maximum at the $kB \fallingdotseq 0.25$ same as in the case of flume A with a different width. The equilibrium sub-range of the spectrum with a constant slope is noticed in



(a) (b)

Photo. 2 Bed configuration to up stream in flume B.
(a) After 113 min. in Exp. B-1.
(b) After 40 min. in Exp. B-3.

the high wave number region.

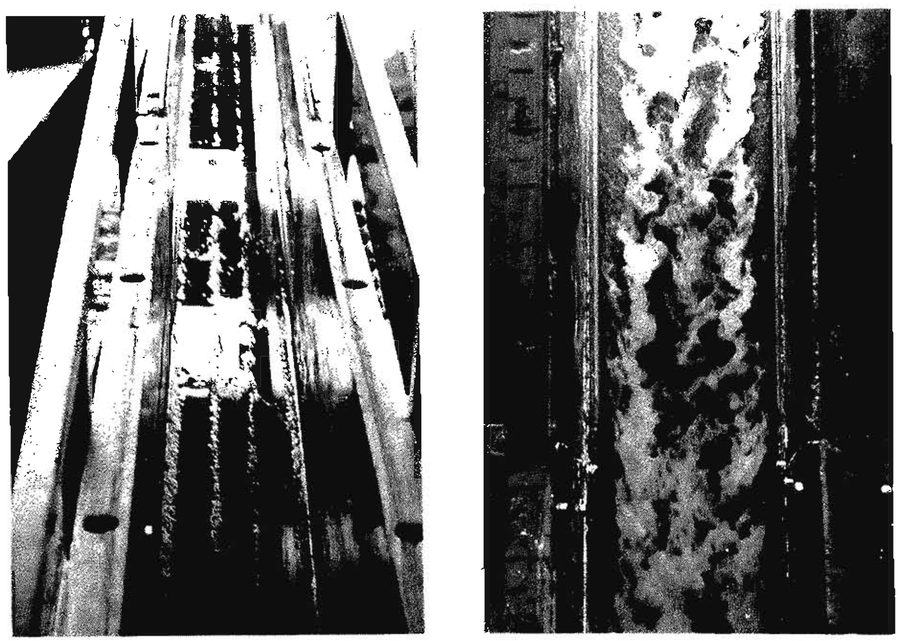
The dominant peak of the power spectral density seen in the epuilibrium subrange is a fluctuation component with dominant water waves of which the non-dimensional wave number *kh* is equal to 0.2～0.3.

The bed configuration in Exp. B-1, B-3 as shown in the Photo. 2(a), 2(b) is a regime in which dunes and alternating bars coexist, and the dominant disturbance due to water waves comes to coexist in the dune regime when the Froude number is nearly one.

The spectrum of the bed configuration in Exp. B-2 has an equilibrium subrange for the broad wave number region of $kh > 0.07$.

The bed configuration was observed as a regime in which the alternating bar did not appear and in which two-dimensional dunes and streaking coexist.

On the bed of Exp. B-4 near the critical tractive force for the mean diameter of



(a) (b)

Photo. 3 Longitudinal streakings in Exp. B-4.
(a) Bed configuration to up stream after 2 hr.
(b) Surface flow pattern with aluminum powder to up stream.

sand, two streakings formed with finer sediment converging and miniature two-dimensional dunes in three unit flumes divided by two streakings appear as shown in Photo. 3 (a).

The flow pattern of the aluminum powder on the water surface is shown as in Photo. 3(b). This shows the existence of vortices with a longitudinal axis having an interval equal to depth as has been pointed out by Kinnoshita[9].

From these results, the structures of bed configuration may be summarized as follows,

(1) The bed configuration in the range of $0.1<kh$ is considered to be dunes or ripples. The statistical structure in this region constiutes an equilibrium subrange which has a similarity not affected by external conditions. When the Froude number is nearly equal to 1. the dominant fluctuation component accompanied with water surface waves of the non-dimensional wave number $kh \doteq 0.2\sim 0.3$ comes to coexist in the dune regime.

(2) In the range of $0.1<kB<0.5$, three-dimensional alternating bars develop.

(3) Longitudinal streakings due to the vortex with a longituditudinal axis can be formed on the bed.

(4) Three types of bed configuration mentioned above can coexist.

The structure of bed configuration is similar in many points to that of the turbulence in open channels obtained by Y. Ishihara and Yokoshi[10], who have pointed out that the turbulence structure of flow may be characterized by three different scales : width of channel B, horizontally, water depth h vertically, and furthermore that the smallest eddies or Kolmogorov microscale, and the largest eddies of the horizontal turbulence are of the order of 10 times the width of the channel longitudinally, while on the other hand the length of the largest eddies of the vertical turbulence is about 10 times the depth of flow longitudinally. From these results, dunes or ripples and alternating bars may correspond to the vertical turbulent field and horizontal turbulent field respectivelly.

In order to further clarify this point, the authors will discuss the relation between the bed configuration and the turbulence structure measured on the movable bed.

3. Experiments for the Turbulence on Rigid and Movable Beds

3.1 Measurment procedures

Turbulent fluctuations of velocity on rigid and movable beds were measured at various stages during the process of bed from growth with a miniature propeller-type current meter which consists of a four-bladed propoller 2.5 cm in diameter. The pulses generated by the current meter were recoreded continuously by a pen-writing oscillograph with a speed of 5~10cm/sec.

The recorded pulses were counted continuously at 0.2sec intervals for flume A and 0.1sec for flume-B because the time constant of the instrument was evaluated to be less than 0.1sec for the range of velocity experimented.

The recording duration time was decided to be 200 sec for flume A and 80 sec for flume B so that the data might contain about ten of the largest disturbances of which the scale may be of the order of ten times of the width.

In flume A, the turbulent fluctations of velocity on movable beds were measured at two points of $2y/B = \pm 0.57$, 2 cm under the water surface in a section

98 m from the upsteam end at the various stages during the growing process of bed forms.

In flume B, the turbulence on the rigid bed was measured at several points in a section of 15 m from the upstream end. These results were analyzed in order to discuss the relation between the structure of the bed configuration and the turbulence structure.

3.2 Analytical results

(1) *The turbulence structure on rigid beds*

Fig. 6 is an example of power spectral density measured in flume B. The inertia subrange of the "$-\frac{5}{3}$ power law" is recognized in the high frequency region.

The non-dimensional frequency of the largest disturbance, where the power spectral density $nF(n)$ shows the maximum, takes the following value :

$$nB/U = 0.25$$

This corresponds to waves having a length over ten times the flowing depth according to the assumption of frozen turbulence. Therefore, this region may correspond to the horizontal turbulent field. It is very interesting that the wave numder of the alternating bars in the flume B $kB = 0.25$ concides with the one of the largest wave length of velocity fluctuations measured on the rigid bed in the same flume.

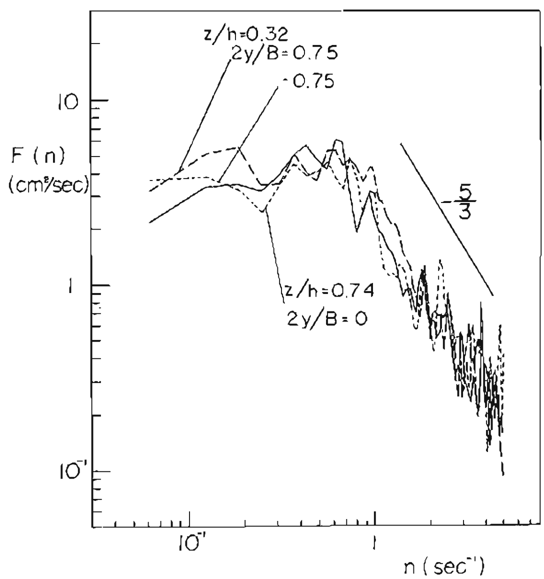


Fig. 6 Energy spectra of the longitudinal velocity on the steel bed in flume B, U=70.6cm/s, h=4.75cm, i = 1/200.

The structure of turbulence in a shear flow hae been investigated in various fields such as in rivers, in the atmosphere and in a tidal current. According to these results, the power spectral density shows the maximum value at $nz/U \doteqdot 0.1 \sim 0.25$, in which z is the height from the boundary. Therefore, the wave number for the disturbance of the largest wave length in a shear flow with free surface may be considered to take the following value :

$$nh/U \doteqdot 0.1 \sim 0.25$$

This wave number concides with that of the largest dunes and ripples in the above turbulent field.

As seen above, the statistical structure of the bed configuration has many similar points to that of turbulence.

(2) *The turbulence structure on movable beds.*

Temporal variation of the power spectral density of turbulence measured on the

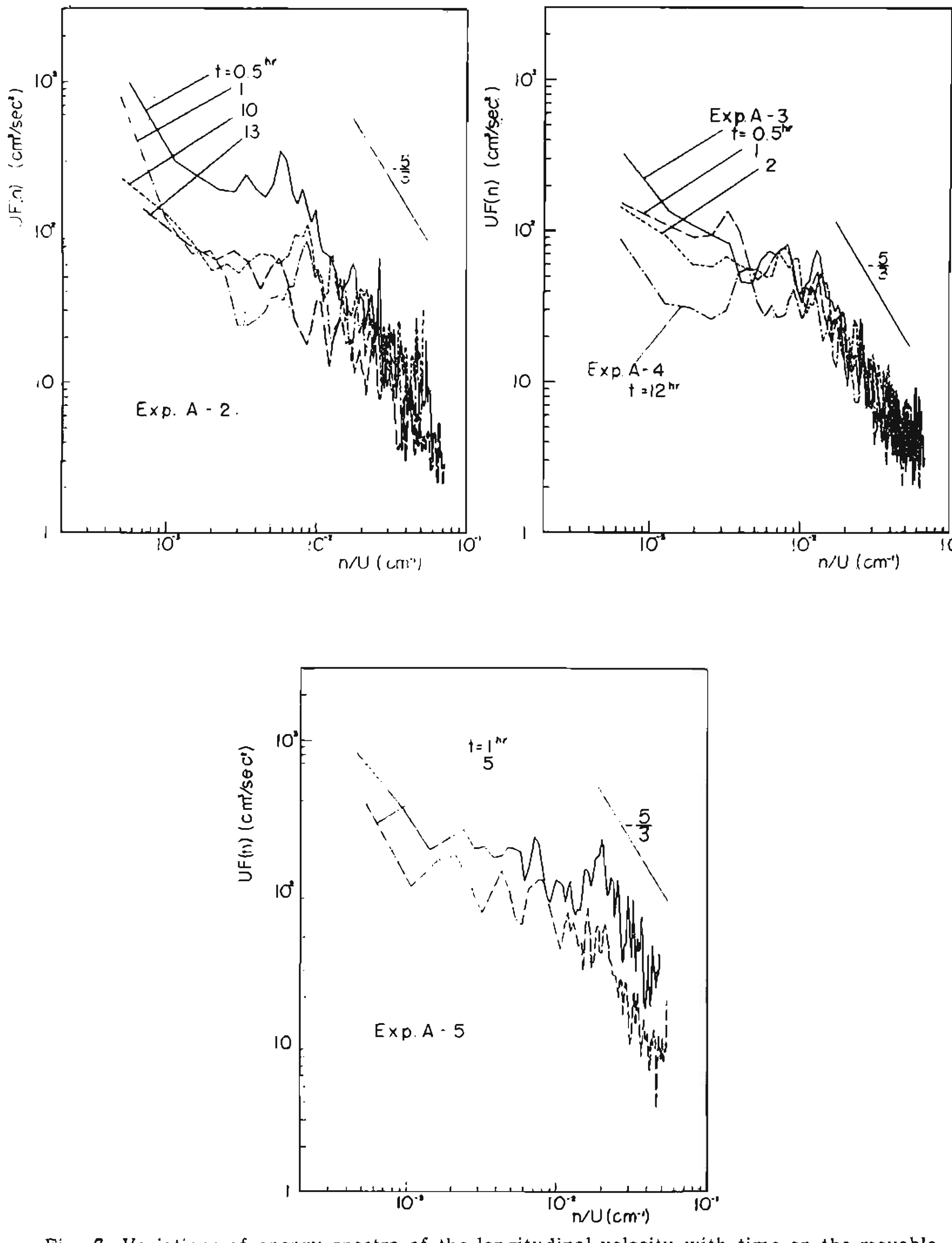


Fig. 7 Variations of energy spectra of the longitudinal velocity with time on the movable bed in flume A, at $2y/B=-0.57, z=(h-2)$cm.

movable bed is shown in Fig. 7(a)～(c). It was observed that the micro-scale of the bed configuration was of the order of depth and the moving velocity was less than 20 cm/min in the experiment. The turbulence characteristics for the lower frequency of $n<0.1$ can not be discussed from the data because the bed variation during the measurement affects the lower frequency component.

The fact that the power spectral density of turbulence at an initial stage of the movable bed takes the maximum value at $nB/U \fallingdotseq 0.3$ coincides with the results on rigid bed.

The power spectral density in the range of $0.1<nB/U<0.5$ rapidly decreases with time as shown in Fig. 7(a).

This range corresponds to that of the alternating bars which develop with time as shown in Fig. 3(b).

It is very interesting that the power spectral density of bed configurations increases with time relatively as that of turbulence decreases at the same range of frequency. It may be interpreted that the energy of turbulence is absorbed into the developing process of bed configuration.

The change of turbulence structure for the high frequency region on movable beds was not so obious as that in the low frequency region.

This may be due to the fact that the measuring point was not near the bed but near the water surface. Therefore, discussion in detail about this point is not available here. From the comparison of the spectrum of Exp. A-3, 4, which has a rather broad inertial subrange of the "$-5/3$ power law," and those in other cases, the structure of turbulence may be assumed to be changed even in the high frequency region by bed variation. Measurements of turbulence near the bed are needed to clarify this point.

The fact that the turbulence structure is changed by the movable bed can be supposed from a similar tendency in a spectrum of water surface fluctuation on a movable bed by Fukuoka[11].

From the above results, it may be rather obious that close correlation exists between the variation of the structure of turbulence and the channel bed process. In order to clarify the above point furthermore, three-dimensional components of the turbulent fluctuation on a movable bed should be measured, because the vertical component w' has a close connection with two-dimensional dunes or ripples and the lateral component v' may be responsible for three-dimensional alternating bars.

4. Mechanics of the Movable Bed Process

It has been made clear that the developing process of bed configuration is closely connected to the turbulent structure. Although a decisive conclusion about the mechacics of the movable bed process should be avoided at the present stage of insufficient knowledge about turbulent structure and the mechanics of sediment transportation, it may be assumed as follows, from the expermental results mentioned in chapters 2. and 3.

It is well known that the formation and dissipation of eddies in a flow constitute such an equilibrium process that the energy supplied continuously to the largest eddies, whose size is of the order of the characteristic dimension of the flow, from the mean flow by the cause of hydrodynamical instability is transfered to the eddies of smaller size and lastly dissipated by viscosity to heat in the smallest eddies. During this process the eddies are convected downstream by the flow on the rigid bed.

On the other hand, concerning the turbulence on the movable bed, a part of the energy of the eddies may be used for the development of bed configuration having the same wave length as the eddies. As the bed configuration develops, the eddies are not convected downstream by the mean flow, but are fixed in space. In other words, the energy of the eddies is directly dissipated in developing the bed forms. This may explain the experimental fact that the turbulence energy of the flow measured at a fixed point on the movable bed decreases with time relatively in the range of frequency as the bed forms develop. The bed configuration may be considered to maintain the development until a statistical equilibrium is attained between the energy supplied to the largest eddies from the mean flow and the energy dissipated in keeping the bed forms. Such a process may determine a statistical character of the spectrum of movable bed configuration. Although the turbulence structure of flow changes with the development of bed forms as mentioned above, which frequency components of bed configuration will come to be dominant can be assumed from the knowledge of the turbulence structure on the rigid bed, because the bed configuration mostly develops at the frequency at which the energy power density of turbulence is dominant in an initial stage similar to that of the rigid bed. Different scales, characterized by width of channel horizontally and water depth vertically, are considered for the dominant frequency. Bars and, dunes and ripples correspond to the horizontal and vertical turbulent fields rspectively. Therefore, the dominant wave length of each component of the bed configuration may be assumed from that of each turbulence field.

5. Concluding Remarks

The structure of movable bed configuration in a lower flow regime has been investigated using statistical analysis. The temporal and spatial variation of the spectrum of the movable bed configuration were discussed from the data measured along longitudinal lines located at various distances from the side wall at three or four stages of the developing process up to the equilibrium state of the bed forms in two flumes with different widths. Also, the turbulence of the fluctuations of velocity was measured in the flumes with rigid and movable beds during the developing process of the bed forms using a miniature propeller-type current meter.

From these data, it has been shown that the structure of bed configuration and also its developing process are closely connected to the turbulence structure. The structure of bed configuration has many similar points to the turbulence structure, which has been pointed out to be characterized by different scales : width of channel horizontally, water depth vertically and the smallest eddies. Bars may correspond to the horizontal turbulent field and dunes and ripples to the vertical one. The dominant wave length of each component of the bed configuration may be assumed from that of each turbulent field.

The turbulence energy of flow measured at a fixed point on movable beds decreases with time relatively in the range of frequency as the bed forms increase. This fact may be interpreted as showing that the energy of the eddies is directly dissipated in developing the bed forms.

References

1) Simons D. B. and Richardson E. V. : Forms of bed roughness in alluvial channels, Proc. of ASCE, HY 3, May, 1961.
2) Kennedy J. F. : The mechanics of dunes and antidunes in erodible-bed channels, J. Fluid Mech. 16, 1961.
3) Hayashi T. : The formation of dunes and antidunes on the erodible bed of an open channel, Proc. of ASCE, 1969.
4) Reynolds A. J. : Waves on the erodible bed of open channel. J. Fluid Mech. 22, 1965.
5) Raudkivi A. J. : Bed forms in alluvial channels, J. Fluid Mech. 26, 1966.
6) Nordin C. F. and Algert J. H. : Spectral analysis on sand waves. Proc. of ASCE, HY 5, Sept., 1966.
7) Ashida K. and Tanaka Y. : Experimental study on sand waves. Disaster Prevention Research Institute Annuals, Kyoto University, No. 10, 1967.
8) Hino M. : Equilibrium-range spectra of sand waves formed by flowing water, J. Fluid Mech. 34, 1968.
9) Kinoshita R. : An analysis of the movement of flood waters by aerial photography, concerning characterstics of turbulence and surface flow, Photographic surveying. 6, 1967.
10) Yokoshi S. : The structure of river turbulence, Bulletin of the Disaster Prevention Research Instiute of Kyoto University, Vol. 17, part 2, 1967.
11) Fukuoka S. : Generation, development and spectrum of sand waves. Dep. Civil Eng., Tokyo Inst. Tech. Rept. No. 4, 1968.