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On the Flood Waves in a Prismatic Open Channel

By Tamotsu TAKAHASHI

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Abstract

The unsteady flows in a prismatic open channel are treated by the theoretical analysis of one-dimensional equations of motion and continuity. The author has shown that unsteady flows in a prismatic channel can be classified by an index λ, where λ has the value of the order of the ratio of water stage variation velocity to the vertical component of long wave celerity. The critical value of λ, above which the wave breaks into a bore at the wave front has obtained. Below the critical value, the bore fomation is prevented, and in the case where the value of λi is of the order of unity or larger the wave is propagated as "dynamic" and Saint-Venant's equation must be modified for the effects of vertical acceleration, and if $\lambda i \ll 1$ the wave is "kinematic."

The author has obtained the second approximation of the kinematic wave in cases where $\lambda \ll 1$ and $\lambda i \ll 1$ and has compared the results with those obtained directly by the computer.

Then the flood characteristics, such as the variation of wave profile, propagation speed and stage-discharge relationship, are discussed in connection with the appropriate theoretical considerations and experiments.

1. Introduction

A hydraulic analysis of flood waves must find the simultaneous solution of the one-dimensional system of equations of continuity and energy, but these equations are non-linear and it is difficult to solve them analytically without any abbreviations. So the theoretical analyses which have been conducted since the last century have omitted the secondary important terms in the equations and linearized them so as to solve the equations with some assumptions. For example, Seddon[1] adapted the equation of uniform flow to the dynamical equation and derived the so-called Kleitz-Seddon's law. This daring abbreviation is most suggestive for predicting the behavior of flood flow that propagates itself through a prismatic channel. Lighthill and Whitham[2] have pointed out that the wave phenomena characterized by the characteristic theory of the first order partial differential equation as it is derived by Seddon explains flood flows well and have named this type of wave "kinematic", distinguishing it from the "dynamic wave" which is characterized by the second order wave equation. The equation for uniform flow omits the terms of local derivative, convective and water surface slope, but the surface slope varies considerably during the period of inundation compared to the bed slope.

To take these surface slope effects into consideration, Forchheimer,[3] Hayami,[4] Yano[5] and etc. have adopted the equation of motion abbreviating the terms of

acceleration. The fundamental equation of flood which was derived using these equations of motion and continuity is a diffusion equation with convection, which can explain the attenuation of the peak depth with propagation. Among these theories, Hayami considered that flows in a river channel are transported at the same time by the average flow and the large scale turbulence produced by the irregularity of the channel. This theory considers the original character of flood flow to be diffusional, and it is different in this respect from others. The effects of the large scale turbulence can be included in the fundamental equation by an additional term to the diffusion coefficient, but now the relationship between the value of the additional diffusion coefficient and the channel characteristics is not clear, and it is presumed that the additional coefficient will be small in a prismatic channel.

The diffusion analogy of a flood wave is a prominent procedure for predicting the whole wave profile and it may be the only single method which enables a theoretical analysis of a flood wave in an irregular channel to be made.[5] But when we discuss a flood only in a uniform channel other analytical methods will be possible. Hayashi[1] has discussed the peak propagation considering that the curvature of the wave is important for propagation character, and has clarified the attenuation characteristics of the peak by a succesive approximation without neglecting the acceleration terms. Tanaka[6] has also proposed a new procedure which calculates the acceleration terms from the steady flow equation. By this method the whole wave profile can be routed and the peak attenuation is the same as Hayashi's.

In addition to the above mentioned theories, there is a method which solves the linearized equation by the small amplitude theory. These theories can also explain the qualitative characteristics of flood waves, but in discussing the quantitative characteristics the assumptions on which the theories are based includes a physical inconsistency.

Considering the above mentioned state of knowledge, few problems are left in analysing the so-called Saint-Venant's equation. But Saint-Venant's equation is derived from Navier-Stokes' equation by omitting the vertical and horizontal acceleration and assuming the flow to be unidirectional, so this equation can not stand for all flow phenomena even in a prismatic channel.

Therefore the author will first discuss the validity of Saint-Venant's equation for the unsteady open channel flow and try to classify the flow characteristics represented by this equation. Then he will try to obtain the solution for flood flow by succesive approximation and clarify the characteristcis of the flood wave that are still not yet clear using the appropriate theoretical considerations and experimental verifications.

2. Fundamental Equation of One-Dimensional Unsteady Prismatic Open Channel Flow and Introduction of a Parameter λ

Strictly speaking, unsteady open channel flows which are a kind of fluid motion have to be analyzed by using the system of three-dimensional equation of energy and continuity. But in the fields of hydraulics or river engineering it is preferable to clarify the macro behavior of flows in the average cross-

section, and usually the flow is treated by the one-dimensional method of analysis.

Recently Iwasa and the author[9] have derived a special system of equations for open channel flow in a rectangular channel through theoretical considerations upon the exact formulation of a one-dimensional system of equations of continuity, energy and linear momentum in the curvilinear orthogonal coordinate. According to this investigation, the newly derived systems of continuity and energy are as follows :—

$$\frac{\partial A}{\partial t}+\frac{\partial Q'}{\partial x}=0 \qquad (1)$$

$$\frac{\beta}{g}\frac{\partial V}{\partial t}-\frac{u_s^2}{2gAV}\frac{\partial A}{\partial t}+\frac{\alpha V}{g}\frac{\partial V}{\partial x}+\frac{\beta-\alpha}{2g}\frac{V}{A}\frac{\partial A}{\partial t}+\frac{1}{gAV}\int_0^B\left(\frac{u_s^2}{2}\right)\frac{\partial H}{\partial t}dz_s$$
$$+\cos\theta\frac{\partial H}{\partial x}=\sin\theta-\frac{\tau_b}{\rho gR}\frac{u_b}{V} \qquad (2)$$

$$\left.\begin{aligned}\beta&=\frac{1}{Q'}\int\left(\frac{u^2}{V^2}\right)udA\\ \alpha&=\frac{1}{A}\int\left(\frac{u}{V}\right)^2dA\end{aligned}\right\} \qquad (3)$$

where, A is the area of cross-section, Q' is the discharge, V is the cross-sectional mean velocity, u is the velocity and suffix s means surface and suffix b means bottom, B is the channel width, H is the depth, θ is the bed slope, τ_b is the shear on the bed, R is the hydraulic radius, g is the acceleration of gravity, ρ is the density of fluid, t is the time, x is the distance along the channel axis and z is the distance perpendicular to the x axis.

On the left hand side of Eq. (2), the second term comes from the local derivative term, and the fifth term comes from the convective term, and these are supplemental terms to the classical equation. If the surface velocity is the same as the average velocity and the lateral water stage variation is horizontal, the two terms are cancelled out by one other. But in the actual channel the surface velocity has a lateral distribution, and it is not the same as the average velocity ; and also the water stage variation in the lateral direction is not horizontal but has the upward convex curve at the rising stage and the concave one at the falling stage, so the supplemental terms have finite values.

Careful experiments made in a laboratory flume reveal that the order effects of additional terms are ordinarily smaller than those of the local derivative term, $(1/g)(\partial V/\partial t)$, but when the channel width is very small, or the side walls have a rough surface, the additional terms have the value of the order comparable to the local derivative term.

This fact suggests that attention must be paid to the fulfillment of the classical equation. The classical equation represents the flow in the prismatic channel well only when the flow is unidirectional and the acceleration to the vertical and lateral direction can be neglected and the channel width is wide enough to neglect the three-dimensional distribution of velocity.

When the last assumption is satisfied, the coefficients α and β will be unity and the classical system of equations will be as follows :—

$$\frac{1}{g}\frac{\partial V}{\partial t}+\frac{V}{g}\frac{\partial V}{\partial x}+\frac{\partial H}{\partial x}=i-\frac{n^2V^2}{H^{4/3}} \qquad (4)$$

$$\frac{\partial A}{\partial t}+\frac{\partial Q'}{\partial x}=0 \qquad (5)$$

where i is the channel slope and n is Manning's roughness.

Eq. (4) is sometimes called Saint-Venant's equation and we will discuss the physical behavior of this system of equations in the following, considering the above mentioned restrictions.

On analyzing the flood wave in a prismatic channel, Hayashi[7] has introduced a non-dimensional parameter $\sigma(=\sqrt{-\ddot{F}(o)/gi}$, where $\ddot{F}(o)$ is the curvature of the peak of the $H\sim t$ curve), which characterizes the flow, and he obtained the successive solution of the peak attenuation with propagation in the case where $\sigma\ll 1$. This procedure explains the characteristics of flood peak attenuation well, but we can not calculate the value of σ at each point except at the gaging station because this procedure does not predict the whole wave profile, so we can not actually know the peak attenuation by this method.

On the other hand, the author considers that the speed of the water stage variation is most important for the mechanism of wave propagation and deformation because the curvature of the hydrographs is similar when the average speed of the water stage variation is about the same.

Now, we will rewrite Eqs. (4) and (5), taking the unit width discharge to be Q, as follows :—

$$i-\left(1-\frac{Q^2}{gH^3}\right)\frac{\partial H}{\partial x}=\frac{1}{gH}\frac{\partial Q}{\partial t}+\frac{2Q}{gH^2}\frac{\partial Q}{\partial x}+\frac{n^2Q^2}{H^{10/3}} \qquad (6)$$

$$\frac{\partial H}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad (7)$$

We will take the linear transformation as shown below :—

$$\begin{aligned} H&=H_m h\\ Q&=\sqrt{gH^3{}_m}\,q\\ t&=\left(\frac{H_m}{H_m-H_0}\right)T_D\tau\\ x&=\sqrt{gH_m{}^3}\left(\frac{1}{H_m-H_0}\right)T_D\,\xi \end{aligned} \qquad (8)$$

where, h, q, τ, ξ are the non-dimensional quantities of H, Q, t, x respectively and T_D is the duration time of inundation at the upstream end. These notations are shown schematically in Fig. 1.

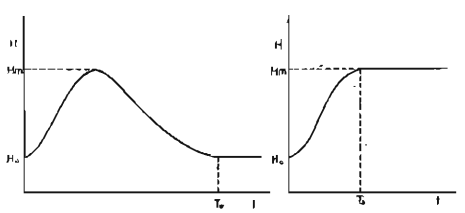

Fig. 1 Schematic diagram of $H\sim t$ curve at $x=0$.

The results of transformations are :—

$$\left(1-\frac{1}{F_r{}^2}\frac{q^2}{h^{10/3}}\right)=\lambda\left\{\left(1-\frac{q^2}{h^3}\right)\frac{\partial h}{\partial\xi}+\frac{1}{h}\frac{\partial q}{\partial\tau}+\frac{2q}{h^2}\frac{\partial q}{\partial\xi}\right\} \qquad (9)$$

and

$$\frac{\partial h}{\partial\tau}+\frac{\partial q}{\partial\xi}=0 \qquad (10)$$

where,

$$F_r{}^2=H_m{}^{1/3}i/n^2g \qquad (11)$$

is the square of the Froude number corresponding to the maximum depth and

$$\lambda = \frac{H_m - H_0}{i\sqrt{gH_m}\ T_D} \quad \cdots\cdots (12)$$

is the newly introduced non-dimensional parameter.

As is evident from Fig. 1, $(H_m - H_0)/T_D$ has the value of the order of the speed of the water stage variation and $i\sqrt{gH_m}$ is the vertical component of long wave celerity corresponding to the maximum depth. Therefore the ratio of the two values λ will control the physical characteristics of the unsteady flow, while for a quasi-steady flow λ will be much smaller than unity and for a bore λ will be much larger than unity. In view of this point, we will try to investigate the behavior of Saint-Venant's equation with respect to the value of λ.

If the value of λ is great enough, a shock will be made at the wave front. We will find this critical condition for bore formation according to the method of Lighthill and Whitham.[2]

Taking the transformations corresponding to the downstream propagating wave front as

$$\left.\begin{aligned} X &= \tau \\ Y &= (v_0 + c_0)\tau - \xi \end{aligned}\right\} \quad \cdots\cdots (13)$$

to Eqs. (9) and (10), we obtain

$$\left(1 - \frac{1}{F_r^2}\frac{v^2}{h^{1/3}}\right) = \lambda\left\{\left(-1 - \frac{v^2}{h} + \frac{vv_0 + vc_0}{h}\right)\frac{\partial h}{\partial Y} + \frac{v}{h}\frac{\partial h}{\partial X} + \frac{\partial v}{\partial X} + (v_0 + c_0 - 2v)\frac{\partial v}{\partial Y}\right\} \quad \cdots\cdots (14)$$

and

$$\frac{\partial h}{\partial X} + (v_0 + c_0)\frac{\partial h}{\partial Y} - v\frac{\partial h}{\partial Y} - h\frac{\partial v}{\partial Y} = 0 \quad \cdots\cdots (15)$$

where,

$$v_0 = \frac{H_0^{2/3}\ i^{1/2}}{n\sqrt{gH_m}} \quad \cdots\cdots (16)$$

$$c_0 = \sqrt{\frac{H_0}{H_m}} \quad \cdots\cdots (17)$$

Here the expansions are

$$\left.\begin{aligned} v &= v_0 + v_1(X)Y + v_2(X)Y^2 + \cdots \\ h &= h_0 + h_1(X)Y + h_2(X)Y^2 + \cdots \end{aligned}\right\} \quad \cdots\cdots (18)$$

and considering that the equations are satisfied individually to the zero and first order, we obtain from the zero order

$$h_1 = v_1 c_0 \quad \cdots\cdots (19)$$

and from the first order

$$\frac{\partial h_1}{\partial \tau} = \frac{1}{h_0^{1/2}}\left\{\frac{3}{2}h_1^2 - \frac{1}{\lambda F_r^2}\frac{f_0}{h_0^{1/3}}\left(1 - \frac{2}{3}f_0\right)h_1\right\} \quad \cdots\cdots (20)$$

where f_0 means the Froude number for the base flow. The positive value of $dh_1/d\tau$ means bore formation and this condition will be satisfied when

$$h_1(\tau) > \frac{2}{3\lambda F_r^2}\frac{f_0}{h_0^{1/3}}\left(1 - \frac{2}{3}f_0\right) \quad \cdots\cdots (21)$$

Now, considering the monoclinal wave and defining the angle between the wave front surface and the base flow surface to be θ,

$$\theta=\left(-\frac{\partial H}{\partial x}\right)_{front}=-\frac{H_m-H_0}{\sqrt{gH_m}\ T_D}\left(\frac{\partial h}{\partial \xi}\right)_{\tau=0}=\lambda i h_1(\tau) \quad \cdots\cdots (22)$$

then inequality (21) will be

$$1>\frac{2i}{3\theta F_r^2}\frac{f_0}{h_0^{1/3}}\left(1-\frac{2}{3}f_0\right) \quad \cdots\cdots (23)$$

On the other hand

$$\theta\approx\frac{\left(\frac{\partial H}{\partial t}\right)_{front}}{V_0+\sqrt{gH_0}} \quad \cdots\cdots (24)$$

then inequality (23) can be rewritten as

$$\left(\frac{\partial H}{\partial t}\right)_{front}>\frac{2}{3}n^2g^{3/2}H_0^{1/3}(1+F_0)\left(1-\frac{2}{3}F_0\right)F_0 \quad \cdots\cdots (25)$$

If we write

$$H_m=\alpha H_0 \quad \cdots\cdots (26)$$

inequality (25) can be approximately transformed as

$$\lambda>\frac{2}{3}\left(\frac{1}{\alpha}\right)^{1/2}\frac{1}{F_0}(1+F_0)\left(1-\frac{2}{3}F_0\right) \quad \cdots\cdots (27)$$

This indicates the condition for bore formation at the wave front, where F_0 is the Froude number of the base flow and is written as

$$F_0=\frac{i^{1/2}\ H_0^{1/3}}{g^{1/2}\ n} \quad \cdots\cdots (28)$$

Using the relation

$$\lambda=\frac{n^3g}{i^{3/2}\ T_D}\alpha^{1/2}\left(1-\frac{1}{\alpha}\right)F_0^3 \quad \cdots\cdots (29)$$

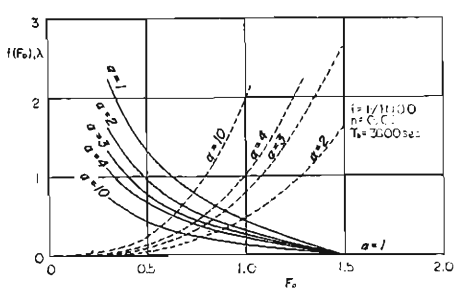

Fig. 2 (1) Critical value of λ and F_0 for bore formation

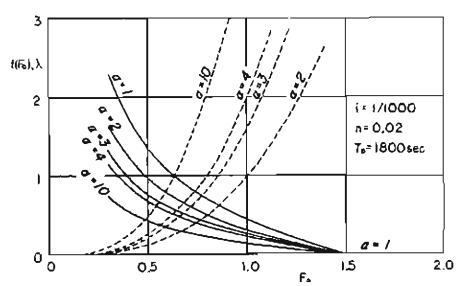

Fig. 2 (2) Critical value of λ and F_0 for bore formation.

we can obtain the critical values of F_0 and λ above which the bore will be formed with propagation on a graph like Fig. 2. In the figure the full line shows the border line of inequality (27) and the broken line shows Eq. (29), so the cross point of each pair of curves indicates the critical value of λ and F_0, and if λ is greater than this value at the time when this critical base flow Froude number

is maintained the bore will be formed, and if F_0 is greater than this value at the time when this critical value of λ is maintained the bore will again be formed. The full line corresponding to $\alpha=1$ coincides with the F_0 axis. This case corresponds to steady uniform flow and if $F_0>3/2$ and some small perturbation have added to the flow, the flow will be unstable. The critical value of F_0 for resistance laws of the Chézy type is 2. These are the most simple conditions for instability of steady uniform flow, which Keulegan and Patterson[10] have introduced.

On the other hand, when inequality (27) is not satisfied, $dh_1/d\tau$ is smaller than zero and bore formation is prevented. Then Eq. (20) will be solved as

$$h_1(\tau)=\frac{\frac{2}{3\lambda F_r^2 h_0^{1/3}}h_1(0)e^{-B\tau}f_0\left(1-\frac{2}{3}f_0\right)}{\frac{2f_0}{3\lambda F_r^2 h_0^{1/3}}\left(1-\frac{2}{3}f_0\right)-h_1(0)(1-e^{-B\tau})} \quad \cdots\cdots(30)$$

where

$$B=\frac{1}{2\lambda F_r^2 h_0^{5/3}}f_0\left(1-\frac{2}{3}f_0\right) \quad \cdots\cdots(31)$$

From this solution we can find that the decay of the wave front, which propagates itself with the characteristic velocity, is exponential and considering that we are discussing the case where

$$h_1(\tau)<\frac{2}{3\lambda F_r^2}\frac{f_0}{h_0^{1/3}}\left(1-\frac{2}{3}f_0\right) \quad \cdots\cdots(32)$$

the decay of the wave front will be approximately

$$h_1(\tau)\approx h_1(0)e^{-B\tau} \quad \cdots\cdots(33)$$

If we think about the wave tip of mono-declining type which propagates upstream with upward characteristic velocity, it will be clarified by a similar procedure that the wave tip always decays exponentially as

$$h_1(\tau)\approx h_1(0)e^{-B'\tau} \quad \cdots\cdots(34)$$

where

$$B'=\frac{1}{2\lambda F_r^2 h_0^{5/3}}f_0\left(1+\frac{2}{3}f_0\right) \quad \cdots\cdots(35)$$

From the above mentioned discussion, it is clear that the parameter λ classifies the physical behavior of Saint-Venant's equation, so we will proceed to analyze the equations in each region of λ values in the next chapter.

3. Analysis of One-Dimensional Unsteady Flow in a Prismatic Open Channel by Use of the Parameter λ.

1) When λ is much greater than 1 and the wave is dynamic.

When the value of λ is greater than 1, we are no longer generally justified in neglecting the vertical acceleration, but in the case of the bore, where there is a transition from one base flow depth to another, it may be discussed by Saint-Venant's equation. This corresponds to the case of a comparatively large base flow Froude number which is derived from the critical condition in Fig. 2.

The friction term, which is on the left hand side of Eq. (9) has the value of 1, so that when λ is much greater than 1, the whole of the left hand side of

Eq. (9) can be safely neglected.

Hence, Eq. (9) will be

$$\left(1-\frac{q^2}{h^3}\right)\frac{\partial h}{\partial \xi}+\frac{1}{h}\frac{\partial q}{\partial \tau}+\frac{2q}{h^2}\frac{\partial q}{\partial \xi}=0 \quad \cdots\cdots (36)$$

If we consider the case where the wave is of infinite amplitude, Eq. (36) will be linearized as

$$\left(1-\frac{q_0^2}{h_0^3}\right)\frac{\partial^2 h}{\partial \xi^2}-\frac{1}{h_0}\frac{\partial^2 h}{\partial \tau^2}-\frac{2q_0}{h_0^2}\frac{\partial^2 h}{\partial \xi \partial \tau}=0 \quad \cdots\cdots (37)$$

The solution of this equation under the boundary conditions

$$\left.\begin{array}{ll} h(0,\tau)=1 & \text{for } \tau>0 \\ \quad\;\;=0 & \text{for } \tau\leqq 0 \\ h(\xi,0)=0 & \\ h_\tau(\xi,0)=0 & \end{array}\right\} \quad \cdots\cdots (38)$$

is obtained on the characteristic velocity

$$\omega=u_0+\sqrt{h_0} \quad \cdots\cdots (39)$$

as

$$\left.\begin{array}{ll} h(\xi,\tau)=1 & \text{for } \tau>0 \\ \quad\;\;=0 & \text{for } \tau\leqq 0 \end{array}\right\} \quad \cdots\cdots (40)$$

This result shows an ideal bore of unit step which propagates itself with the velocity of

$$\Omega=V_0+\sqrt{gH_0} \quad \cdots\cdots (41)$$

For the finite amplitude bore, Saint-Venant[1] has treated under the assumption that the velocity is a single valued function of depth and the result is

$$\Omega=V_0+3\sqrt{gH}-2\sqrt{gH_0} \quad \cdots\cdots (42)$$

2) *When λ is much smaller than 1 and the wave is kinematic.*

If we consider a wave whose height is much greater than the depth of the base flow and where the value of λ is much smaller than 1, it is clear from the discussion in 2. that shock does not occur at the wave front, and the wave front which propagates itself with the velocity of a long wave decays exponentially with distance and the main part of the wave propagates as kinematic. This is the case generally called flood wave or flood flow.

We can obtain the solution of the fundamental Eqs. (9) and (10) by the expansion of h and q as

$$\left.\begin{array}{l} h=h_0(\xi,\tau)+h_1(\xi,\tau)\lambda+h_2(\xi,\tau)\lambda^2+\cdots \\ q=q_0(\xi,\tau)+q_1(\xi,\tau)\lambda+q_2(\xi,\tau)\lambda^2+\cdots \end{array}\right\} \quad \cdots\cdots (43)$$

The initial and boundary conditions are

$$\left.\begin{array}{lll} \tau=0 & , & h=h_0(\xi)=\text{const} \\ \xi=0 & , & h=h_0(\tau)=f(\tau) \\ \xi=0 & , & h_1=h_2=\cdots=0 \end{array}\right\} \quad \cdots\cdots (44)$$

We substitute the expansions (43) in Eq. (9) and obtain an equation from the zero order term of λ as

$$1-\frac{1}{F_r^2}\frac{q_0^2}{h_0^{19/3}}=0 \quad \cdots\cdots (45)$$

Hence,

$$q_0 = F_r h_0^{5/3} \quad \cdots\cdots (46)$$

Substitution of Eq. (46) into Eq. (10) will be

$$\left(\frac{\partial}{\partial \tau} + \frac{5}{3} F_r h_0^{2/3} \frac{\partial}{\partial \xi}\right) h_0 = 0 \quad \cdots\cdots (47)$$

and the general solution of (47) which satisfies the conditions (44) will be

$$h_0 = f\left(\tau - \frac{\xi}{\frac{5}{3} F_r h_0^{2/3}}\right) \quad \cdots\cdots (48)$$

This solution shows that on the characteristics

$$\omega = \frac{\partial \xi}{\partial \tau} = \frac{5}{3} F_r h_0^{2/3} \quad \cdots\cdots (49)$$

h_0 will be held constant. This means a kinematic wave of velocity ω which propagates without attenuation.

Next, from the first order term of λ we obtain

$$\frac{1}{F_r^2} \frac{q_0^2}{h_0^{10/3}} \left(\frac{10}{3} \frac{h_1}{h_0} - \frac{2q_1}{q_0}\right) = \left(1 - \frac{q_0^2}{h_0^3}\right) \frac{\partial h}{\partial \xi} + \frac{1}{h_0} \frac{\partial q_0}{\partial \xi} + \frac{2q_0}{h_0^2} \frac{\partial q_0}{\partial \xi} \quad \cdots\cdots (50)$$

considering Eq. (47), it is derived from Eq. (50) that

$$q_1 = \frac{5}{3} F_r h_0^{2/3} h_1 - \frac{1}{2} F_r h_0^{5/3} \frac{\partial h_0}{\partial \xi} \left(1 - \frac{4}{9} F_r^2 h_0^{1/3}\right) \quad \cdots\cdots (51)$$

and substituting this into the equation of continuity

$$\frac{\partial h_1}{\partial \tau} + \frac{\partial q_1}{\partial \xi} = 0 \quad \cdots\cdots (52)$$

we obtain the equation

$$\frac{\partial h_1}{\partial \tau} + \frac{5}{3} F_r h_0^{2/3} \frac{\partial h_1}{\partial \xi} = \frac{F_r}{h_0^{1/3}} \left[\left(\frac{\partial h_0}{\partial \xi}\right)^2 \left(\frac{5}{6} h_0 - \frac{4}{9} F_r^2 h_0^{4/3}\right) + \left(\frac{\partial^2 h_0}{\partial \xi^2}\right) \left(\frac{h_0^2}{2} - \frac{2}{9} F_r^2 h_0^{7/3}\right) - \frac{10}{9} h_1 \frac{\partial h_0}{\partial \xi}\right] \quad \cdots\cdots (53)$$

The solution of Eq. (53) will be obtained on the characteristics

$$\frac{d\xi}{d\tau} = \omega_0 = \frac{5}{3} F_r h_0^{1/3} \quad \cdots\cdots (54)$$

considering Eq. (48) as

$$h_1 = \frac{9}{10} \frac{f^2}{\dot{f}(2\xi - 5F_r f^{5/3})} \left[2\xi\left(3 - \frac{14}{9} F_r^2 f^{1/3}\right) \dot{f}^3 + 5F_r f^{5/3}\left(\frac{9}{2} + \frac{4}{9} F_r^2 f^{1/3}\right) \dot{f}^2 - F_r f^{5/3}\left(\frac{3}{2} - \frac{2}{3} F_r^2 f^{1/3}\right) \ddot{f}\right] \left[1 - \exp\left(\frac{2\xi}{2\xi \dot{f} - 5F_r f^{5/3}}\right)\right] \quad \cdots\cdots (55)$$

And the second approximation of depth will be

$$h = h_0 + h_1 \lambda \quad \cdots\cdots (56)$$

Hence the second approximation of q will be

$$q = q_0 + \left\{\frac{5}{3} F_r f^{2/3} h_1 - \frac{1}{2} F_r f^{5/3} \frac{3f\dot{f}}{2\xi \dot{f} - 5F_r f^{5/3}} \left(1 - \frac{4}{9} F_r^2 f^{1/3}\right)\right\} \lambda \quad \cdots\cdots (57)$$

The third approximation will be obtained in the same manner considering the

second order of λ in the expansions (43) by neglecting small terms, but for the ordinary flood in a river channel, the second approximation will be a pretty good approximation.

At the peak of the flood wave $\dot{f}$ equals zero and the discussion above can not be adopted for the peak.

At the peak

$$\left.\begin{aligned} &f=1 \\ &\dot{f}=0 \\ &\frac{\partial^2 h_0}{\partial \xi^2} = \frac{9}{25F_r^2}\ddot{f}_{\text{peak}} \end{aligned}\right\} \quad \cdots\cdots (58)$$

and on the characteristics

$$\frac{d\xi}{d\tau} = \frac{5}{3}F_r \quad \cdots\cdots (59)$$

$$h_1 - \int_0^\tau \frac{9}{50}\frac{1}{F_r}\left(1-\frac{4}{9}F_r^2\right)\dot{f}_{\text{peak}}D\tau = \text{const} \quad \cdots\cdots (60)$$

The solution of Eq. (60) which satisfies the conditions (58) is

$$h_1 = \frac{27}{250}\left(\frac{1}{F_r^2}-\frac{4}{9}\right)\ddot{f}_{\text{peak}}\,\xi \quad \cdots\cdots (61)$$

Then the second approximation of the peak is

$$h = 1 + \frac{27}{250}\lambda\left(\frac{1}{F_r^2}-\frac{4}{9}\right)\ddot{f}_{\text{peak}}\xi \quad \cdots\cdots (62)$$

Putting back to the original dimensions,

$$H = H_m\left\{1+\frac{27}{250}\frac{\ddot{F}_{\text{peak}}}{igH_m}\left(\frac{1}{F_r^2}-\frac{4}{9}\right)x\right\} \quad \cdots\cdots (63)$$

is derived. This result is the same as those of Hayashi and Tanaka. The discharge at this time will be

$$q = F_r\left\{1+\frac{9}{50}\lambda\left(\frac{1}{F_r^2}-\frac{4}{9}\right)\ddot{f}\,\xi\right\} \quad \cdots\cdots (64)$$

This will be written by the original dimensions

$$Q = \frac{1}{n}H_m^{5/3}\,i^{1/2}\left\{1+\frac{9}{50}\left(\frac{1}{F_r^2}-\frac{4}{9}\right)\frac{\ddot{F}_{\text{peak}}}{gH_m i}x\right\} \quad \cdots\cdots (65)$$

Putting Q_m corresponding to the uniform flow discharge of which the depth is H_m

$$\frac{Q_m-Q}{Q_m}\Big/\frac{H_m-H}{H_m} = \frac{5}{3} \quad \cdots\cdots (66)$$

will be derived from the Eqs. (63) and (65).

This relation shows that Kleitz-Seddon's law can be adopted for the second approximation.

To show the applicability of this approximation to the actual flood, we will calculate the case where the boundary condition at the upstream end is

$$\left.\begin{aligned} F(t) &= (H_m-H_0)\sin\frac{\pi}{T_D}t + H_0, \quad 0\leq t\leq T_D \\ &= H_0 \qquad\qquad\qquad\qquad\quad , \quad t > T_D \end{aligned}\right\} \quad \cdots\cdots (67)$$

and H_m equals 10m, H_0 equals 2m, n equals 0.03m$^{-1/3}$sec, i equals 0.001 and T_D equals 4 hours. In this case λ equals 0.056 and is much smaller than unity.

The non-dimensional expression of (67) is

$$\left.\begin{aligned} f(\tau) &= \frac{4}{5}\sin\frac{5\pi}{4}\tau+\frac{1}{5}\ , \quad 0\leqq\tau\leqq\frac{4}{5} \\ &= \frac{1}{5} \qquad\qquad\qquad\ , \quad \tau>\frac{4}{5} \end{aligned}\right\} \cdots\cdots (68)$$

and calculation can be carried out on the $(\tau\sim\xi)$-plane along the characteristics

$$s=\tau-\frac{\xi}{\frac{5}{3}F_r\{f(s)\}^{3/5}} \cdots\cdots (69)$$

Fig. 3 shows the results of calculation on a one kilometer pitch and compares these with the results obtained by use of Stoker's[12] method with KDC-II electronic computer. The errors around the wave front and tip are comparatively large because the wave has the property of a dynamic wave there, but except for the wave front and tip the errors are not so large and it proves that the procedure shown above is a good approximation of flood flow.

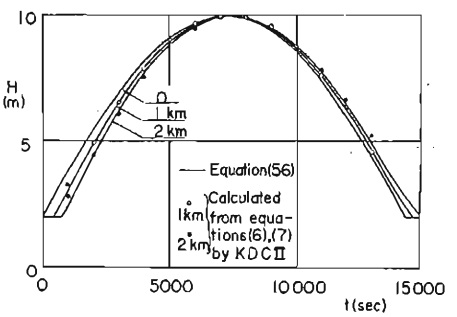

Fig. 3 Comparison of results between theoretical and direct computations by KDC II.

3) When λ is of the order of 1 or larger.

When the value of λ is large, the effect of the vertical acceleration must be considered. The equation of motion in which the vertical acceleration is taken into account is derived by Iwasa.[13] This equation can be written in the non-dimensional form neglecting the small terms as

$$\begin{aligned}\left(1-\frac{1}{F_r^2}\frac{q^2}{h^{19/5}}\right) &= \lambda\left\{\left(1-\frac{q^2}{h^3}\right)\frac{\partial h}{\partial\xi}+\frac{1}{h}\frac{\partial q}{\partial\tau}+\frac{2q}{h^2}\frac{\partial q}{\partial\xi}\right. \\ &\left.+\frac{q^2}{3h}(\lambda i)^2\frac{\partial^3 h}{\partial\xi^3}+\frac{2q}{3}(\lambda i)^2\frac{\partial^3 h}{\partial\xi^2\partial\tau}+\frac{h}{3}(\lambda i)^2\frac{\partial^3 h}{\partial\xi\partial\tau^2}\right\} \cdots\cdots (70)\end{aligned}$$

In this equation it is known that when λi is much smaller than 1, the 4th, 5th and 6th terms are negligible in comparison to the other terms and the effect of the vertical acceleration is small.

In this case, in place of expansions (43) the following expansions will be adopted to obtain the approximate solution with the similar method. Here the expansions are

$$\left.\begin{aligned} h &= h_0(\xi,\tau)+h_1(\xi,\tau)\lambda i+h_2(\xi,\tau)\lambda^2 i^2+\cdots \\ q &= q_0(\xi,\tau)+q_1(\xi,\tau)\lambda i+q_2(\xi,\tau)\lambda^2 i^2+\cdots \end{aligned}\right\} \cdots\cdots (71)$$

and the second approximation except for the peak is

$$h=h_0+\frac{9}{10}\frac{K}{\frac{\partial h_0}{\partial\xi}}\left\{1-\exp\left(-\frac{2}{3}\frac{\frac{\partial h_0}{\partial\xi}}{h_0 i}\right)\xi\right\}\lambda i \cdots\cdots (72)$$

where

$$K=\left(\frac{\partial h_0}{\partial \xi}\right)^2\left(\frac{5}{6}h_0-\frac{4}{9}F_r^2h_0^{5/3}\right)+\left(\frac{\partial^2 h_0}{\partial \xi^2}\right)\left(\frac{h_0^2}{2}-\frac{2}{9}F_r^2h_0^{7/3}\right)\cdots\cdots(73)$$

and h_0 is shown in Eq. (63).

When the value in the parentheses of the exponential term in Eq. (72) is much smaller than 1, Eq. (72) is approximately the same as Eq. (55). For the peak Eq. (62) is established.

In a case where the value of λi is larger and the terms of the third order derivatives in Eq. (70) can not be neglected, Keulegan and Patterson's[14] method is applicable, and the case where the left hand side is neglected in Eq. (70) has been discussed by Iwasa.[13]

4. Characteristics of Flood Waves in a Prismatic Open Channel

1) Experiments for floods in a prismatic channel

(1) Test flume

As shown in Fig. 4, the experimental equipment consisted of the flume, the supporting screw jacks, the reservoirs upstream and downstream, the returning pipe, the head tank, the gaging tank, the flood wave generator and the water stage controller downstream, etc.

The flume 60cm wide, 60cm deep and 150m long is made of steel. Its bed is coated with cement mortar to eliminate the unevenness at the joints and is supported by the screw jacks at 6m intervals. By adjusting these jacks the inclination of the flume can be changed from zero to 1/150. A flood wave up to 100ℓ/sec is generated by the pneumatic automatic control which opens and closes the air valve inserted between the head tank and the gaging tank.

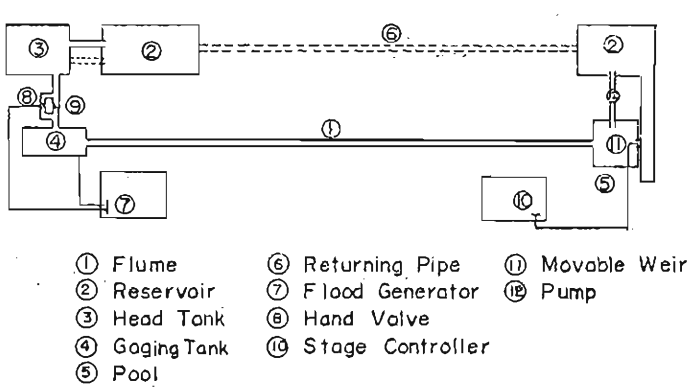

Fig. 4 Experimental flume.

(2) Experimental procedure

The kinds of experiment are shown in Table 1. The shapes of the $h\sim t$ curves on the steady base flow discharge were parabolic or triangular where h meant the wave height on the base flow discharge.

$h\sim t$ curves at all measuring stations were measured by the resistive wave meters and recorded by the electromagnetic oscillographs. The arrangement of ten measuring stations was decided in the manner of trial to remove the effects

Table 1 The kinds of experiment.

Run	Base discharge (ℓ/sec)	Max. Discharge (ℓ/sec)	Duration Time (sec)	λ
1	5	31.5	450	0.71
2	5	28.7	140	2.15
3	5	36.5	160	2.17
4	5	15.5	400	0.47
5	5	25.5	420	0.67
6	5	31.0	400	0.79
7	5	35.0	420	0.81
8	5	40.0	450	0.82
9	10	26.0	390	0.51
10	10	30.7	450	0.52
11	10	36.0	470	0.57
12	10	40.0	430	0.69

of flume joint irregularity and were set at points 18, 34, 50, 60, 65, 70, 88, 98, 118 and 137meters apart from the upsteam end.

The slope of the flume was set at 1/500 and the average Manning roughness was $n=0.0116m^{-1/3}$ sec.

The velocity was not measured.

The similarity of these hydrographs to actual river floods are described by the following relationships, [9]

$$\left.\begin{aligned} \sigma_x &= \varepsilon^{-1}\sigma_H \\ \sigma_v &= \sigma_H^{1/2} \\ \sigma_t &= \varepsilon^{-1}\ \sigma_H^{1/2} \\ \sigma_i &= \varepsilon \\ \sigma_n &= \varepsilon^{1/2}\sigma_R^{2/3}\sigma_H^{-1/2} \end{aligned}\right\} \quad \cdots\cdots (74)$$

where σ means the reduced scale of that represented by the suffix and ε means the skewness of the model. If we assume that the width of the channel 60cm corresponds to 100m and $\varepsilon=1$, the duration time of 5 minutes corresponds to 65 minutes and the discharge of 30ℓ/sec corresponds to $1089m^3$/sec. These results show that the hydrographs adopted in the experiments are very sharp ones compared with those which generally occur in an actual river. But they have been adopted to facilitate an understanding of the effects of the acceleration terms.

2) Experimental results and considerations.

(1) *On the transformation of the flood wave*

Fig. 5 shows an example of wave transformation with propagation. In the figure the depth above the base flow surface at an arbitrary time at each station is divided by the wave height there, so that the maximum value of the ordinate is 1, and the times of peak attainment at each mea-

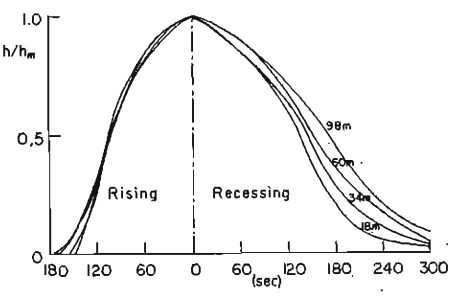

Fig. 5 Wave transformation with propagation (Run 1).

suring station are superposed one over another. By this manipulation the figure shows well the characteristics of transformation, such as the less deformability at the rising stage and the contrary at the recessing stage. the circumstaces are the same for the other runs. This means that the nonlinearity effects are important for the wave transformation.

These characteristics are explained qualitatively as follows :—
The fundamental Eqs. (6) and (7) will be transformed as

$$\left\{1-\frac{F^2}{(1-\kappa)}\phi\right\}\frac{\partial H}{\partial t}+V\left\{\frac{5}{3}-\frac{F^2}{(1-\kappa)}\left(3-\frac{3}{2\alpha_1}-\frac{\alpha_2}{2}\right)\phi\right\}\frac{\partial H}{\partial x}$$
$$=\frac{Q}{2(1-\kappa)i}\{1-(1+\alpha_3\alpha_4-2\alpha_4)F^2\}\frac{\partial^2 H}{\partial x^2} \quad \cdots\cdots (75)$$

where

$$\begin{aligned}
&\kappa=\{\alpha_2-1/\alpha_1-(2-1/\alpha_1)F^2\}\phi\\
&\phi=(1/iV)(\partial H/\partial t)\\
&F^2=Q^2/gH^3\\
&\alpha_1 V=-(\partial H/\partial t)/(\partial H/\partial x)\\
&\alpha_2 V=(\partial Q/\partial t)/(\partial H/\partial t)\\
&\alpha_3 V=(\partial^2 H/\partial t^2)/(\partial^2 Q/\partial x^2)\\
&\alpha_4 V=(\partial^2 Q/\partial x^2)/(\partial^2 H/\partial x^2)
\end{aligned} \quad \cdots\cdots (76)$$

As discussed in the last chapter, except at the beginning of the rising stage, the characteristics of the kinematic wave are prominent and we can put $\alpha_1=\alpha_2=\alpha_3=\alpha_4=5/3$ as the first approximation and then,

$$\left\{1-\frac{F^2}{(1-\kappa)}\phi\right\}\frac{\partial H}{\partial t}+V\left\{\frac{5}{3}-\frac{19}{15}\frac{F^2}{(1-\kappa)}\phi\right\}\frac{\partial H}{\partial x}$$
$$=\frac{Q}{2(1-\kappa)i}\left(1-\frac{4}{9}F^2\right)\frac{\partial^2 H}{\partial x^2} \quad \cdots\cdots (77)$$

Dividing both sides of Eq. (77) by $\partial H/\partial x$ and remembering that $\alpha_1 V$ represents the propagation speed of the same depth, we obtain the second approximation of the propagation speed of the same depth ω neglecting the small terms, such as κ and $F^2\phi$, as follows :—

$$\omega\approx\frac{5}{3}V\left\{1+\frac{9}{50}\frac{nH^{1/6}}{i^{3/2}}\left(1-\frac{4}{9}F^2\right)\frac{\dfrac{\partial^2 H}{\partial t^2}}{\dfrac{\partial H}{\partial t}}\right\} \quad \cdots\cdots (78)$$

In Eq. (78), (5/3) V shows the kinematic wave and the socond term in the parentheses is its modification. The first term of the right hand side acts to make the wave acute and if $\partial^2 H/\partial t^2<0$ when $\partial H/\partial t>0$, the second term acts to flatten the wave. Accordingly, the curvature and the slope of the tangent line of the $h\sim t$ curve controls the transformation characters and the flood propagates itself in a channel like unifomly translation wave at the rising stage as the result of rapid deformation, if it had not the pattern of a uniform translation wave at the upstream end.

Taking Run 1 as an example, we calculate Eq. (78) at station at 18m as in Table 2. This reult reveals that the celerity of each of the three stages is almost the same and that the wave deforms little with propagation.

At the stage maximum, Eq. (78) diverges to infinity and this fact explains

Table 2. Calculation of Eq. (78) (Run 1, Station at 18m).

h (cm)	0.5	3.0	5.0
H (cm)	3.1	5.6	7.6
$\partial H/\partial t$ (cm/sec)	3.8×10^{-3}	6.1×10^{-3}	3.0×10^{-2}
$\partial^2 H/\partial t^2$ (cm/sec^2)	1.6×10^{-3}	-5.7×10^{-4}	-7.3×10^{-4}
$5V/3$ (cm/sec)	63.5	94.0	115.0
ω (cm/sec)	79	88	95

the mechanism of peak attenuation. The peak attenuation can be calculated along the characteristic curve on which $\partial H/\partial t$ equals zero and the result is Eq. (63) in the last chapter. In Fig. 6, we compare the experimental results and the calculation results from Eq. (63). Considering the accuracy in graphcally obtaining the value of $\partial^2 H/\partial t^2$ from the $h\sim t$ curve and the irregularity of the channel at the welding joints, it can be said that the calculation has sufficient accuracy.

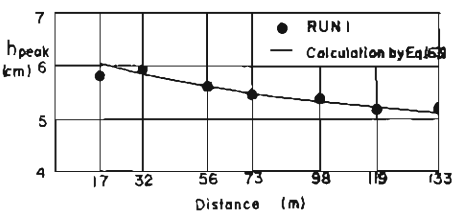

Fig. 6 Attenuation of stage maximum (Run 1).

(2) *On the propagation of peak and front*

The validity of applying Kleitz-Seddon's law to the propagation of stage maximum has been discussed in the last chapter. To be confident of these results,

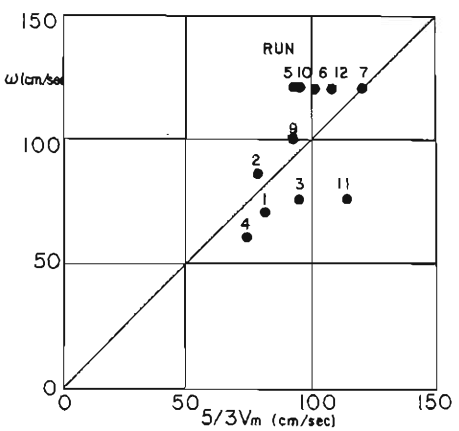

Fig. 7 Validity of Kleitz-Seddon's law to the peak propagation.

we have measured the propagation speed of the peak among the stations at 70m and 88m and have compared these with $(5/3)V_m$ as in Fig. 7. The experimental results are largely scattered around the theoretical results but

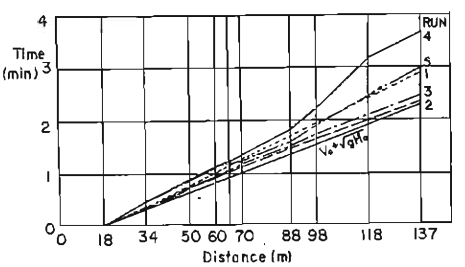

Fig. 8 Propagation of front (Base flow discharge 5ℓ/sec).

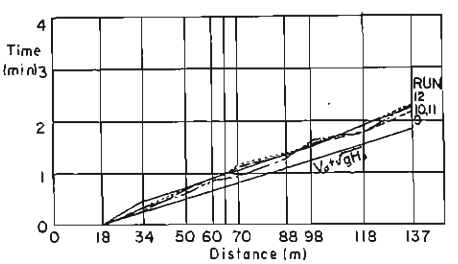

Fig. 9 Propagation of front (Base flow discharge 10ℓ/sec).

it can be considered that propagation of the stage maximum follows Kleitz-Seddon's law.

On the other hand, the arrival time of the front at each station is shown in Figs. 8 and 9, in which Fig. 8 shows the cases where the base flow discharge are 5ℓ/sec and Fig. 9 shows the cases where the base flow discharge is 10ℓ/sec. Here the front does not mean the theoretical one but the actual one which can be observed by the wave meters.

From Fig. 8 we can see that the speed of propagation of the wave front is larger in the run which has a larger value of λ and in Fig. 9, which shows the results for the cases of about the same value of λ, it is difficult to find the difference of speed. This fact reflects the effects of acceleration on wave front celerity and the qualitative explanation is as follows :—

We discussed how the wave front which propagates itself with long wave celerity decays exponentially as shown in Eq. (33). Here, we put back the dimensions of exponent B as

$$B=\frac{1}{2\lambda F_r^2}\left(\frac{H_m}{H_0}\right)^{5/6} F_0\left(1-\frac{2}{3}F_0\right) \quad \cdots\cdots (79)$$

So the larger the base flow depth or the rate of water stage variation, the smaller the value of B, becomes and the decay of the front will be smaller and the propagation speed will get nearer to the celerity of the long wave.

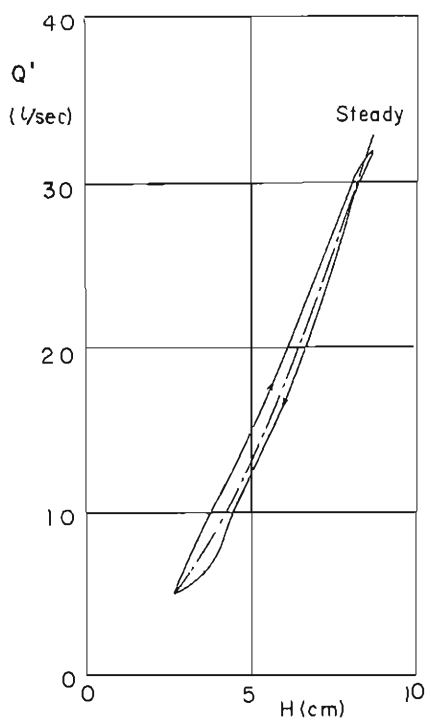

Fig. 10 Depth~discharge relationship (Run 1, Station at 18m).

(3) *On the relationship between depth and discharge*

In the experiments, it was difficult to observe the discharge at each station directly. Hence, the discharge was routed from the upstream hydrograph using the storage equation between two measuring stations. An example of depth~discharge relationship thus calculated is shown in Fig. 10. At the rising stage, the dischrge is larger than the steady flow for the same depth and at the recessing stage it is less than the steady flow, so that the relationship draws a loop-line. This is the effect of acceleration, and the loop is nearer at the recessing than at the rising stage to the steady stage~discharge relation curve because the acceleration is smaller at the recessing stage. In any case, the loop is not so greatly separated from the curve for steady flow and this fact will indicate that the acceleration terms can be neglected in the case of a prismatic channel flood.

In fact, the equation of motion can be written as

$$\frac{\partial H}{\partial x}=i\frac{1-\left(\frac{H_0}{H}\right)^2\left(\frac{R_0}{R}\right)^{4/3}+2\left(\frac{H_c}{H}\right)^3\frac{\frac{\partial H}{\partial t}}{Vi}}{1-\left(\frac{H_c}{H}\right)^3}-\frac{1}{gAi}\frac{\partial Q'}{\partial t} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (80)$$

and the order of the terms is shown in Table 3 for Run 1 at the station at 17m. In the equation suffix 0 means for uniform flow and c means for critical flow.

Table 3 Comparison of order of terms in flood equation (Run 1).

	①	②	③	④	⑤	⑥	⑦	⑧
t	H(cm)	$Q(\ell/s)$	$\frac{\frac{\partial H}{\partial t}}{Vi}$	$\left(\frac{H_0}{H}\right)^2\left(\frac{R_0}{R}\right)^{4/3}$	$\left(\frac{H_c}{H}\right)^3$	$2\left(\frac{H_c}{H}\right)^3\frac{\frac{\partial H}{\partial t}}{Vi}$	$\frac{1}{gAi}\frac{\partial Q}{\partial t}$	$\frac{1}{i}\frac{\partial H}{\partial x}$
3'00''	3.05	7.0	0.359	1.18	0.516	0.370	0.410	−1.070
3'30''	4.60	13.5	0.578	1.12	0.520	0.600	0.468	−0.025
4'00''	6.40	21.0	0.486	1.00	0.410	0.298	0.359	−0.104
4'30''	7.60	27.5	0.232	1.025	0.492	0.299	0.226	−0.043
5'00''	8.20	31.0	0.198	1.097	0.500	0.198	0.093	−0.016
5'30''	8.70	31.5	−0.062	0.892	0.428	−0.053	−0.020	0.130
6'00''	8.40	30.5	−0.116	0.940	0.444	−0.103	−0.076	0.060
6'30''	7.90	27.8	−0.183	0.928	0.440	−0.161	−0.135	0.082
7'00''	7.20	23.5	−0.253	0.908	0.410	−0.207	−0.231	0.196
7'30''	6.40	18.5	−0.363	0.777	0.360	−0.262	−0.293	0.387
8'00''	4.90	12.2	−0.530	0.792	0.366	−0.388	−0.412	0.366
8'30''	4.00	7.0	−0.150	0.561	0.230	−0.228	−0.281	0.634
9'00''	3.20	5.5	−0.306	0.615	0.265	−0.161	−0.093	0.430

From this table we can conclude that for the first approximation of the equation of motion

$$Q'=\frac{1}{n}BHR^{1/3}\sqrt{i-\frac{\partial H}{\partial x}(1-F^2)} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (81)$$

is satisfactory. And for a comparatively small Froude number F^2 can be neglected in comparison to 1, This confirms the equation of motion adopted in previous theories like diffusion analogy.

Discussions in the last section show that the water surface slope of a flood in a uniform channel can be approximately represented by

$$\frac{\partial H}{\partial x}=-\frac{1}{\omega}\frac{\partial H}{\partial t}$$

This equation with Eq. (81) proves the validity of the well known John's formula.

5. Conclusion

The author has studied the flood flow in a prismatic open channel theoretically and experimentally. The main results obtained are as follows :

1) The exact energy equation in the one-dimensional open channel flow must

be supplemented into the classical equation by additional terms concerning three-dimensional variations of transverse velocity and stage in the unsteady behavior, but for the ordinary flood in a river if the acceleration terms are small compared to the other terms in the classical equation, the additional terms are also small and can be neglected.

2) The characteristics of open channel unsteady flow can be cassified by the index λ, which has the order of ratio of stage variation to the vertical component of long wave celerity along the channel.

3) The critical condition to bore formation at the wave front is shown by Eq. (27) and for arbitrary bed slope and bed roughness the critical value of λ can be obtained graphically as in Fig. 2.

4) In the case where the λ value is much greater than 1, the results are the same as when the term of roughness slope is considered to be equal to the bed slope in the fundamental equation. In this case the wave is dynamic and if the Froude number is large it is often the bore.

5) In the case where λ is much smaller than 1, the main part of the wave propagates as a kinematic wave. This is the case often called flood wave and is analyzed by the successive method using the expansion of λ.

6) If the value of λi is much smaller than 1, a similar method of analysis as in the case where λ is much smaller than 1 is possible, and in this case the effect of vertical acceleration is not so evident.

7) Floods often propagate like uniform translation waves at the rising stage and these characteristics are explained as the results of the synthetic effects of the nonlinear kinematic wave and the curvature of the wave profile.

8) The attenuation of the peak with propagation is shown by Eq. (63) and this is the same as the result obtained by Hayashi and Tanaka.

9) The peak propagation approximately obeys to Kleitz-Seddon's law and the actual front propagates with about the same speed to the peak because the theoretical front which propagates with long wave celerity decays exponentially. But the larger the value of λ, the faster the propagation velocity of the front.

10) Stage~discharge relationship generally draws a loop line reflecting the effects of acceleration. But in the prismatic channel the acceleration terms are ordinarlly much smaller than the water surface term and the roughness loss term and can be safely neglected compared to these terms.

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