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Probability of Levee Breaks Due to Heavy Rainfalls in a River

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Abstract

Since Japan is small in area and has a large population, there are many intensively developed areas. To protect these areas against floods, levees are constructed to restrain the flood flows caused by heavy rainfalls. The results are often catastrophic when they are breached or overtopped. However, a levee break upstream in a river system results in reduction of the flow downstream, because flooding due to the levee break has a similar effect to diverting flood waters into a reservoir and impounding them.

This paper describes how to evaluate the probability of a levee break in the levee system of a river. Afetr examining critically the causes of flood losses, we show that this probability can be evaluated by applying the probability theory of multidimensions. A computational example, the case of two areas of protection by levees against floods is presented; and it is concluded that a levee break in the upstream area reduces the flood risk in the downstream area, and that the larger the correlation coefficient between the peak discharges of two flood hydrographs from the upper and lower sub-basins is, the less the effect of the reduction becomes.

1. Introduction

Most river disasters in Japan are caused by heavy rainfalls, during which a great deal of water and sediment flows through the stream channels. Several kinds of methods to reduce flood losses have been used, and, among others, the technique currently in the widest use is the construction of engineering works. Levees or embankments to restrain the flow, reservoirs to impound flood water, by-passes to carry water around towns, channel enlargement to reduce water stages, or a combination of several of these works are used.

Residential and production areas are protected by these engineering works against floods, arranged to reduce flood losses as much as possible; and when a flood of sufficient magnitude occurs which exceeds the design flood for the protection works, temporary evacuation of people and damageable goods is planned on the basis of an adequate flood forecast.

These methods of reduction of flood losses have different functions of protection against floods and are classified as follows:

1) control or regulation of floods,
2) direct protection against floods,
3) indirect protection against floods,

Flood control reservoirs, retention pools, diversion channels, and by-passes belong to the first classification. These functions are limited, because these have finite capacities for impounding flood water or for dividing flood flows. For a flood of magnitude less than the design flood, flood damages are prevented successfully, but it will be scarcely possible to prevent damage from floods exceeding this limitation. Levees or flood walls belong to the second classification. These cannot prevent damage from a flood exceeding their design capacities and resulting in a levee break or overtopping. Such damages far exceed what would occur under natural conditions. Therefore, the success of levees in preventing flood losses is intermittent. Reasonable arrangements of residential and production areas, and temporary evacuation belong to the third classification. These have the function not of thoroughly preventing flood damages but of reducing losses of people and goods as much as possible.

Most flood disasters in Japan will occur 1) when a flood exceeds the design flood for flood control system; 2) when a flood which exceeds the capacity of the stream channel flows down, and enlarges; 3) when living and production areas are arranged unsatisfactorily and evacuation is too late. One or more of these protection techniques is ready in each area, where it is desired, in the river basin. Generally, floods flow only downstream, except in special cases such as a back-water region. The downstream condition has little influence on the upstream flow. All the protection works in the upstream areas more or less affect the downstream flood flow. It is possible, therefore, by getting all the information on natural and engineering conditions from the headwaters to the area under consideration, and by analyzing their influences on the flow pattern at that area, to evaluate accurately the protection effect of the engineering works in the area. Moreover, it is obvious that, in an area which is directly protected by a levee, evaluating whether or not the levee will break down due to flood flows will give the conditions for determining the occurrence of catastrophes.

The causes of the breaking of earth-fill levees, which most levees in Japan are, are said to be overtopping of water, scouring by flow, and seepage of water. Levees constructed carefully so as to avoid scouring and seepage will break down only by overtopping. Overtopping occurs when the peak discharge of a flood at the location exceeds the conveyance capacity of the channel. Evaluating the probability of a levee break or the occurrence of catastrophic damage to an area in a river basin is equivalent to finding out the probability with which the peak discharges of flood hydrographs affected by the basin conditions upstream from the location exceed the conveyance capacity of the stream channel.

2. Change of Peak Discharge during Flood

Peak discharges flowing down a channel reach in a river system are influenced by conditions of rainfall, physical states of the river basin, improvement works in the stream channels, hydraulic works for flood control and so on in areas above than the channel reach, except in special cases as when the channel reach is located at a reservoir entrance or in a rivermouth.

Rainfall, which is in a way input of runoff process, and control of which is an important problem in the field of meteorology, cannot help being treated as a purely natural phenomenon for the present.

Physical states of vegetation, geology, geomorphology and so on in a river basin will have a potent influence on the runoff process of rainfall, so a large-scale transformation of land use, or a change of drainage areas in upstream areas will have a considerable influence on the states of flood runoff in downstream areas.

Concerning the improvement works of river channels, the most interesting case is when there exist levees in the upstream region and their break and flooding are possible. The flooding in the upstream region will reduce the peak discharge in the downstream region; but if the upstream levees are rebuilt higher than before, the risk of levee break in the downstream area will increase.

Existence of hydraulic structures for flood control will have converse results. Reservoirs constructed for flood control in the upstream regin will reduce the risk of levee breaks in the downstream region.

It follows from all this that until the influences of natural and artificial conditions in the upper regions are examined synthetically and systematically, we cannot estimate the peak discharge of flood runoff passing through a channel reach, and cannot evaluate the risk probability of levee break and flooding in a region.

Now, changes of flood runoff which are caused by changes of the physical state of a river basin may be estimated from our hydrological knowledge concerning runoff processes, moreover, the fact that a heavier rainfall brings a more powerful flood still holds in spite of changes in the physical conditions of a river basin. Therefore, converting the occurrence probability of heavy rainfall into that of flood is relatively easy, and so the influences of changes of the physical conditions of a river basin upon the risk probability of levee break can be easily estimated also.

How the existence of levees or flood control reservoirs in upstream regions can influence the probability of levee breaks in downstream regions, given the present physical state of a river basin, is examined below.

3. Occurrence Probability of Levee Break

The exceeding probability of peak discharges of floods passing through a channel reach in a river system is usually evaluated by applying logarithmic normal distribution, extreme value distribution, etc. to the peak discharges of floods.

However, in actual river basins in Japan there exist many regions to be protected against floods which are usually defended by flood walls or levees. Thus, the peak discharge value of a flood passing through a channel reach must be determined not only from the conditions of rainfall and the physical states of the river basin under consideration but also from the conditions of levee break and flooding in the upper region.

(1) A simple case having two protection regions against floods

Let us consider the river system shown in Fig. 1. There is one protection region, A, along the upstream channel reach, and another protection region, B, along the downstream channel reach. These regions are both defended by levees, and there is one interbasin between them. In the figure, the circled I and II represent the drainage basin above region A and the inter-basin between the regions A and B, respectively. Q_1 and Q_3 show the values of peak flood discharges passing respectively through the channel

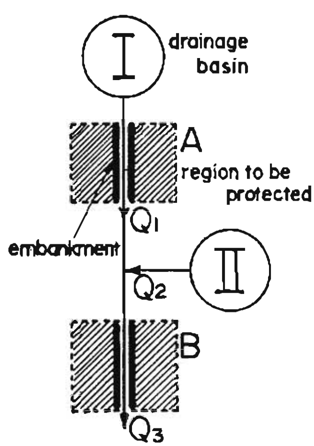


Fig. 1 Schematic model of river basin having two regions protected by embankments and one interbasin between them.

reaches in the region A and in the region B, without levee break and flooding; and Q_2 shows the peak discharge value of a flood from the inter-basin II, where there exists no levee to break down.

In this case, there are four simple possibilities
1) no levee break occurs.
2) a levee break occurs only at region B.
3) a levee break occurs at both regions A and B.
4) a levee break occurs only at region A.

Occurrence probabilities of these separate simple possibilities or of combinations of them can be evaluated by the following method.

Let us suppose that the conveyance capacities of the channel reaches in the regions A and B are QB_1 and QB_3, respectively, that the confluent coefficient of Q_2 to Q_1 is c, and that the peak discharge flowing down from region A at a levee break in region A is $k \cdot QB_1$. Then, the conditions under which the four elementary events mentioned above occur individually can be represented by the following equations:

1) no levee break occurs;

$$Q_1 \leq QB_1 \text{ and } Q_1 + c \cdot Q_2 \leq QB_3 \quad (1)$$

2) a levee break occurs only at B;

$$Q_1 \leq QB_1 \text{ and } Q_1 + c \cdot Q_2 > QB_3 \quad (2)$$

3) a levee break occurs at both A and B;

$$Q_1 > QB_1 \text{ and } k \cdot QB_1 + c \cdot Q_2 > QB_3 \quad (3)$$

4) a levee break occurs only at A;

$$Q_1 > QB_1 \text{ and } k \cdot QB_1 + c \cdot Q_2 \leq QB_3 \quad (4)$$

These conditions are illustrated on the $Q_1 - Q_2$ plane as c=const. and k=const. (<1) in Fig. 2, where the domains D_1, D_2, D_3 and D_4 correspond to the equations (1), (2), (3) and (4), respectively.

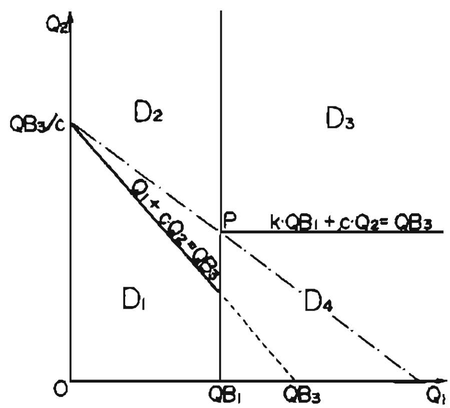


Fig. 2 Illustration diagram of the domains corresponding to the condition equations (1), (2), (3) and (4), for c=const. and k=const. (<1).

The occurrence probabilities for these individual simple events or for combinations of them, therefore, can be obtained by integrating the density function of the joint probability of Q_1 and Q_2, denoted as $p(Q_1, Q_2)$, in the domains corresponding to the events under consideration. For example, the probability of a levee break in region B, which is denoted P_{23}, can be evaluated by the following equation, because this event is the combined event of events 2) and 3).

$$P_{23}=\iint_{D_2+D_3} p(Q_1, Q_2)\,dQ_1 dQ_2 \qquad (5)$$

Fig. 2 shows that increment of QB_1, leaving QB_3=const., enlarges the domain (D_2+D_3), because the point P moves along the chain line with the variation of QB_1, and results in an increase of the value of P_{23}, and shows that the domain (D_3+D_4), accordingly the value of the probability of levee break P_{34} at the region A does not vary with variation of QB_3 when QB_1=const. This means that except for a high dam various kinds of flood prevention works in the downstream regions have no influence on flood figures in the upstream regions.

(2) General case

Actually, there are various arrangements of flood walls or levees to protect living and production areas against floods. For example, Fig. 3 shows one protection region

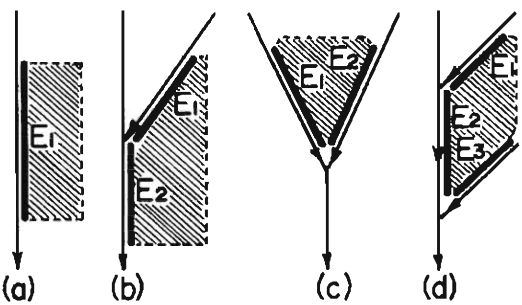


Fig. 3 Illustration of various protection forms by embankments or levees for the residential and production areas in a river basin.

in various situations in a river system. Case (a) assumes that the region is protected by only one levee, E_1, along one channel reach, so the risk probability of the region can be evaluated by one-dimensional analysis if there is no protection region upstream. Case (b) or (c) assumes that the region is protected by two levees, E_1 and E_2, along two different channel reaches, so the region will be damaged by a break in either E_1 or E_2, or by breaks in both, and the risk probability of the region can be evaluated by two-dimensional analysis if there is no protection region upstream. Case (d) assumes that the region is protected by three levees, E_1, E_2 and E_3, along three different channel reaches, so the region will be damaged by a breakdown of any one of them, or any two of them, or all of them (the number of these distinguishable elementary events is counted up as follows; $\binom{3}{1}+\binom{3}{2}+\binom{3}{3}=7$), and the risk probability of the region must be evaluated by three-dimensional analysis even if there exists no protection region upstream.

Generally, an actual river system is composed of subsystems such as the above, and it is necessary, therefore, in evaluating the risk probability of a protection region to examine systematically the possibilities of a break in each levee existing in the region and those of regions above the location under consideration, as shown in (1) of this section. If there exist m protection regions against floods in a river system and they are protected by n levees (of course, $n \geq m$), it will be clear that the number N of such elementary events as shown in (1) (except for the case of no levee break) is generally given by the following equation;

$$N=\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\cdots+\binom{n}{n-1}+\binom{n}{n}=2^n-1 \tag{6}$$

Thus, N increases by geometrical progression with n, and so it is very difficult to give such general expressions as equations (1), (2), (3) and (4) or (5). However, such expressions will be possible when an actual river system is concretely studied. It seems, therefore, that this problem must be discussed as a case study as well as a general study of water resources development systems.

4. Computational Example

As a river system becomes more complicated, it will become more difficult to discuss the general properties of river disasters. In this section, a computational example is shown for a river system as shown in Fig. 1, where the problems under consideration are clearly involved in spite of the simple case.

(1) Occurrence probability of flood

Generally, when the physical properties of a river basin are given, the hydrograph of the runoff from the river basin caused by heavy rainfall can be obtained reliably. Therefore, either rainfall or the discharge caused by it may be used as a sample in evaluating the occurrence probability of flood. But, strictly speaking, the physical conditions of a river basin will vary with time, so data on river discharges must include the effects due to such variations. Therefore, it will be better to obtain the occurrence probability of peak discharge after transforming rainfall (hyetograph) into discharge (hydrograph) by the use of the present physical conditions of the river basin than to evaluate the observed data of discharges directly.

However, even rainfall data are not always complete, and it is very difficult to treat hyetographs, which can be transformed into hydrographs, as random samples. So, here, the occurrence probability of peak discharges of floods is conveniently evaluated by using the average relations between daily rainfalls and peak discharges after estimating the occurrence probability of maximum daily rainfall in a year, because the aim of this computation is to investigate how the risk probability of the protected regions varies with levee breaks in the upper regions.

The computations are carried out for a river system obtained by modifying an actual river basin in Central Japan; and the hydrological data are taken from those of that river basin.

The results of statistical analyses of the hydrological data are as follows:

1) The distribution functions of occurrence probabilities of maximum daily rainfalls in a year, R_1 and R_2, which correspond to those of the basins I and II, respectively, conform to the logarithmic normal distribution, and the normalized variables x_1 and x_2 are expressed as follows:

$$x_1=3.2436 \log_{10}\{(R_1-12)/87\} \tag{7}$$

$$x_2 = 3.0193 \log_{10}\{(R_2 - 30)/102\} \tag{8}$$

2) The relations between peak discharge and daily rainfall are given as follows:

$$Q_1 = 7.3R_1 - 620 \quad (\text{for } R_1 \geq 100) \tag{9}$$
$$= 1.1R_1 \quad (\text{for } 0 \leq R_1 < 100)$$

$$Q_2 = 3.3R_2 - 305 \quad (\text{for } R_2 \geq 100) \tag{10}$$
$$= 02.5R_2 \quad (\text{for } 0 \leq R_2 < 100)$$

where Q_1 and Q_2 are in m^3/sec and R_1 and R_2 are in mm.

3) The density function of joint probability of the normalized variables x_1 and x_2 is generally expressed in the following equation;

$$p(x_1, x_2) = \exp\{-(x_1^2 - 2\rho x_1 x_2 + x_2^2)/2(1-\rho^2)\}/2\pi\sqrt{1-\rho^2} \tag{11}$$

where ρ is the correlation coefficient of x_1 and x_2. If ρ is given, the values of the joint density function of R_1 and R_2 or of Q_1 and Q_2 are obtained from the equations (7), (8), (9), (10) and (11). An example of this is shown in Fig. 4 for $\rho = 0.5$.

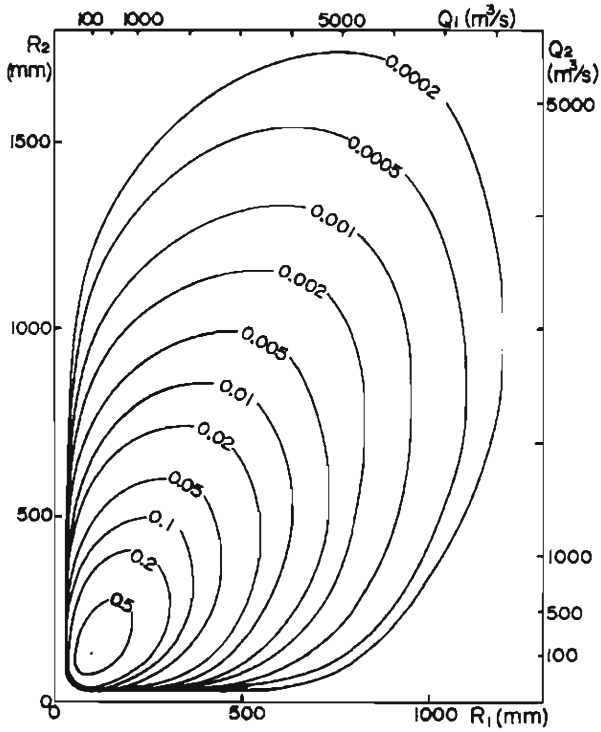


Fig. 4 The equi-probability curves outside of which the point(R_1, R_2) or (Q_1,Q_2) exists, obtained from the equations (7), (8), (9), (10) and (11) for the case of $\rho = 0.5$.

(2) Values of c and k

The value of the confluent coefficient c will vary with each rainfall, and this variation will be especially remarkable when the directions of movement of rain storms differ very much from each other. Even if the peak discharges from both sub-basins join simultaneously at the confluent point, the peak discharge in the downstream reach usually becomes less than their sum because of the effects of back-water or channel storge. Then, strictly speaking, the confluent coefficient c must also be treated as a random variable. However, it is assumed here for simplicity that $c=1$, because the main object of this investigation is how the risk probability of the downstream region B varies by levee breaks in the upstream region A. It is clear that this assumption will not affect the general value of our considerations.

The value of k will be decided by the circumstances around the point of levee break. As the conditions of levee break are now given as the equations (2), (3) and (4), the maximum discharge flowing down the channel reach of the upstream region A is the discharge just before the levee break, which is equal to the conveyance capacity of the reach QB_1, and the discharge after the levee break must become less than QB_1. Therefore, the assumption that $k=1$ will be not very erroneous.

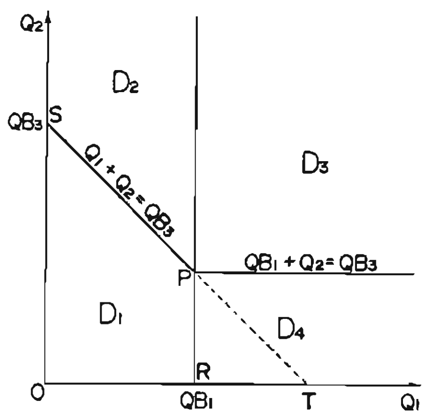


Fig. 5 Illustration diagram of the domains corresponding to the condition equations (1), (2), (3) and (4) for $c=1$ and $k=1$.

Under the assumptions that $c=1$ and $k=1$, Fig. 2 showing the domains corresponding to the condition equations (1), (2), (3) and (4) on the Q_1-Q_2 plane is changed into the figure shown in Fig. 5.

(3) Results of Computations and Discussions

A. The general characteristics

a. the case of $0 \leq \rho < 1$

In Fig. 5, let us consider the case where QB_3 is fixed and QB_1 varies. When QB_1 is less than QB_3, that is, the point R exists on the left of the point T, the domain (D_2+D_3) increases with increment of QB_1. But, when QB_1 is equal to or greater than QB_3, it is clear from the figure that increment of QB_1 causes no variation of the domain (D_2+D_3). This means that the risk probability P_{23} of the region B, whose value is obtained by integrating the density function $p(Q_1, Q_2)$ in the domain (D_2+D_3), increases according as QB_1 increases to become equal to QB_3, but that it is constant when QB_1 is greater than QB_3, and its constant value is equal to the one for $QB_1=QB_3$, that is, for the case where the points R, T and P come together in the one point.

b. the case of $\rho=1$

In the special case of the correlation coefficient $\rho=1$ all the random variables are on one curve in the Q_1-Q_2 plane. This means that the following relation holds:

$$Q_2=F(Q_1) \tag{12}$$

where $F(Q_1)$ is the monotone increasing one-valued function of Q_1.

Now, if the point P in Fig. 5 exists on the upper side of this curve, P_{23} is calculated by the following expression.

$$P_{23}=\int_{a_1}^{\infty}p_1(Q_1)\cdot dQ_1=\int_{a_2}^{\infty}p_2(Q_2)\cdot dQ_2 \qquad (13)$$

where $p_1(Q_1)$ and $p_2(Q_2)$ are the probability density functions of the marginal distributions of Q_1 and Q_2, respectively, and a_1 and a_2 are the coordinates of the intersection point of the curve $Q_2=F(Q_1)$ and the straight line $Q_2=QB_3-QB_1$ (for $Q_1\geq QB_1$). In this case, if QB_3 is fixed and QB_1 increases, P_{23} increases correspondingly.

On the other hand, when the point P exists on the lower side of the curve $Q_2=F(Q_1)$, P_{23} is calculated by the following expression.

$$P_{23}=\int_{b_1}^{\infty}p_1(Q_1)\cdot dQ_1=\int_{b_2}^{\infty}p_2(Q_2)\cdot dQ_2 \qquad (14)$$

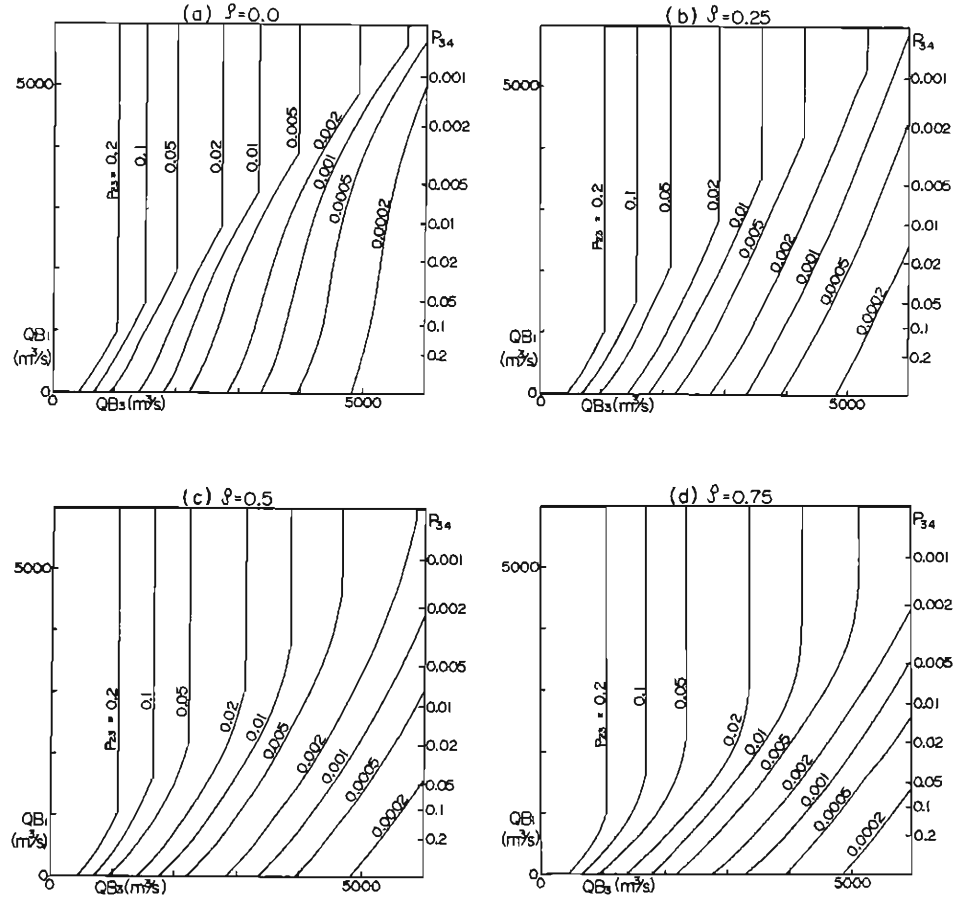
where b_1 and b_2 are the coordinates of the intersection point of the curve $Q_2=F(Q_1)$ and the straight line $Q_1+Q_2=QB_3$ (for $0\leq Q_1\leq QB_1$). In this case, if QB_3 is fixed, P_{23} does not vary even with a change of QB_1. Then the constant value of P_{23} is equal to the value for $a_1=b_1$, being identical with $a_2=b_2$, that is, for the case where the point P exists on the curve $Q_2=F(Q_1)$.

There is another characteristic. Again, in the case where the point P exists on the upper side of the curve $Q_2=F(Q_1)$, if QB_1 and QB_3 increase (or decrease) by the same value, respectively, the value of P_{23} does not vary because the straight line $Q_2=QB_3-QB_1$ does not move. This means that a certain constant value of P_{23} is plotted on a straight line with the inclination angle of 45° on the QB_1-QB_3 plane while the point P exists on the upper side of the curve $Q_2=F(Q_1)$.

B. The relations between QB_1, QB_3 and P_{23}

The computational results of the relations between QB_1 and QB_3 are shown in Fig. 6 using P_{23} as a parameter, where (a), (b), (c), (d) and (e) correspond to the cases of $\rho=0.0$, 0.25, 0.5, 0.75 and 1.0, respectively; and P_{34} represents the risk probability of the region A in Fig. 1 which is equal to the probability that Q_1 exceeds QB_1. These figures show clearly the characteristics mentioned in A., as well as the following interesting matters.

1) In the domain in which the value of P_{23} varies with the increase of QB_1, that is, in the domain under the straight line $QB_1=QB_3$, except for the case of $\rho=1.0$, and in the domain under the curve $QB_3=F(QB_1)+QB_1$, in Fig. 6(e) for the case of $\rho=1.0$, the lines of P_{23}=const. rise towards the right, and the value of P_{23} for a certain value of QB_1 becomes less according as QB_3 becomes greater. This means that P_{23} increases with increment of QB_1 under a fixed QB_3. This is shown more clearly in Fig. 7-(a), (b), (c), where the relations between QB_1 and P_{23} are shown by using QB_3 as a parameter. The risk of levee break in the downstream region B increases with increment of the conveyance capacity of the channel reach in the upstream region A; and the risk probability of the region B becomes independent of the conveyance capacity of the channel reach in the region A when this capacity exceeds that of the region B except when $\rho=1.0$. This will be true also when new embankments are made in the upstream region where there was no levee formerly. Hence, for example, in building or heightening embankments in the upstream region, the effects upon the risk of levee break in the downstream region caused by increment of the conveyance capacity of the upstream channel reach must be taken account of.



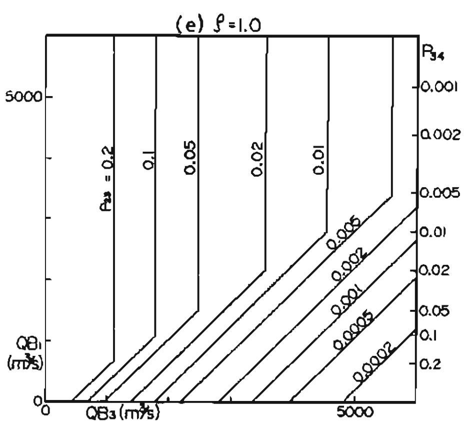


Fig. 6 Relations between QB_1 and QB_3 using P_{23} as a parameter for $\rho=0.0$, 0.25, 0.5, 0.75 and 1.0.

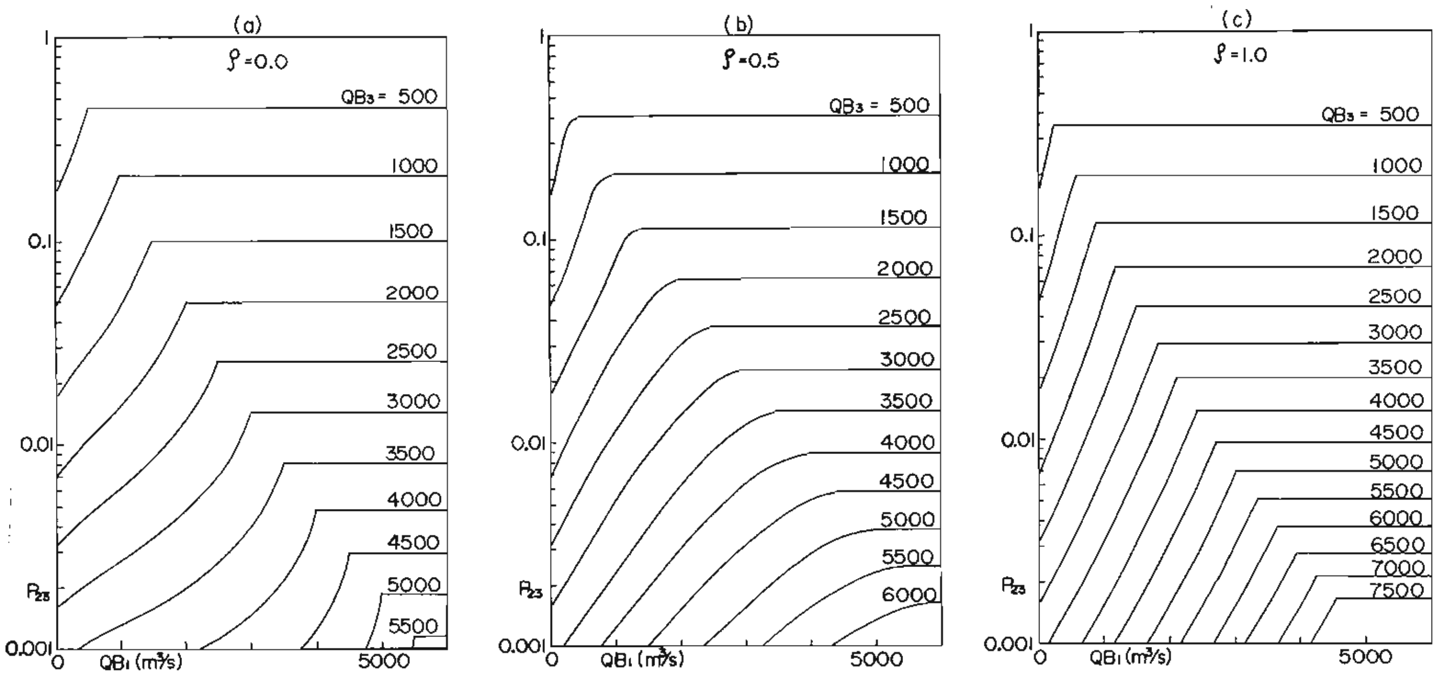


Fig. 7 Relations between QB_1 and P_{23} using QB_3 as a parameter for $\rho=0.0$, 0.5, and 1.0.

2) Secondly, the figures, (a), (b), (c), (d) and (e) of Fig. 6 are superposed and it is shown only for $P_{23}=0.1$, 0.01 and 0.001 in Fig. 8. This figure shows that the lines of P_{23}=const. move towards the right situation according as ρ becomes greater. This means that the risk probability of the downstream region B increases according as ρ becomes greater in the range of $\rho \geq 0$ for a combination of QB_1 and QB_3.

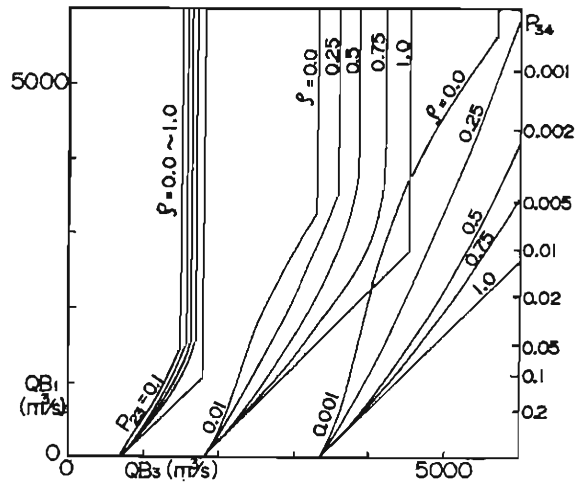


Fig. 8 Superposed figure of Fig. 6 (a), (b), (c), (d) and (e) for $P_{23}=0.1$, 0.01 and 0.001.

3) The reason has already been given at A. why P_{23} does not vary even with variation of QB_1 for a fixed QB_3 in the domain above the straight line $QB_1=QB_3$ for the case of $0 \leq \rho < 1$, or above the curve $QB_3=F(QB_1)+QB_1$ for the case of $\rho=1$ in Fig. 6. From that it is apparent that the constant value of P_{23} for a fixed QB_3 is equal to that of P_{23} for the same QB_3 in the case of $QB_1=\infty$, that is, when there are such high embankments in the upstream region that breakdown will never occur, or when there is no levee at all in the upstream region.

When the risk probability of any region to be defended against floods in a river system is evaluated individually, supposing that there will be no levee break anywhere above that region, the result will be right if there is no levee in the upper regions, but it will be overstimation if there is a levee to break down. For example, in the present model of river system, if the risk probability of the upper region A is 0.02 corresponding to the case of $QB_1=2150$ m^3/sec, and that of the lower region B (evaluated ignoring levee break in the region A) is 0.01 corresponding to the case of $QB_3=3850$ m^3/sec for the case of $\rho=0.5$, the true risk probability of the region B is estimated as 0.0041 by reading Fig. 6 or Fig. 7.

4) It is posible, moreover, from Fig. 6 or Fig. 7 to evaluate the effects of a reservoir constructed for flood control upstream from region A. If such a reservoir is designed to decrease the peak discharge Q_1 from the basin I by the amount of QD, it is identical under the assumptions mentioned previously with the case where QB_1 and QB_3 increase together by the same value QD even without the reservoir. Therefore, the evaluation of the contribution of such a reservoir to the risk probability of the lower

region B can be easily made by reading Fig. 6 or Fig. 7. For example, if QB_1 and QB_3 have the same values as in the example mentioned above, and QD is 500 m^3/sec, the risk probability of the region B is estimated as 0.0030 for the case of $\rho=0.5$.

Generally, it is shown in Fig. 6 and Fig. 8 that if both points (QB_1, QB_3) and (QB_1+QD, QB_3+QD) exist in the domain under the curve $QB_3=F(QB_1)+QB_1$ the effects of such a reservoir become less according as ρ is greater; and that if $\rho=1$, such a reservoir will cause no variation of the risk probability of the lower region B.

5) Lastly, the most remarkable conclusion is that by such figures as Fig. 6 and Fig. 7 the risk probabilities of regions to be defended from floods can be reasonably evaluated by regarding the whole river basin as a system. It will be possible, therefore, to make such a river plan rationally, for example, a plan of embankments or of reservoirs, so that all regions have same risk probability, or that they have various risk probabilities corresponding to the importance of each region individually.

5. Conclusions

In this paper, the conditions of occurrence of river disasters are discussed first, then a general method is presented which can correctly evaluate the risk probability of each region in a river system where there are many regions to be protected by levees against floods, by the use of the theory of multi-dimensional probability.

The following results are obtained for the most basic case by computations and analyses:

1) The right evaluation of risk probabilities of regions along rivers is possible by using the method presented, and only by such an evaluation will it be possible to make a rational plan of levees, reservoirs, retention pools, channels enlargement and so on.

2) The increment of the correlation coefficient ρ between rainfalls of the upper and lower basins has a danger effect on the risk probability of the downstream region B.

3) Increasing the conveyance capacity of the upstream channel reach increases the risk probability of the downstream region.

4) But, if the conveyance capacity of the upstream region exceeds that of the downstream region (i. e., if $QB_1 \geq QB_3$), increasing the former has no effect on the risk probability of the downstream region.

5) When the risk probability of a region is evaluated individually ignoring levee breaks upstream from it, that evaluation will be an overestimation.

These results are very intersting and include many useful suggestions for a general river plan in spite of having been obtained for a simple and basic river system under many conditioning assumptions. However, many problems, such as: multidimensionality, decision problems of distribution functions of probability of hydrological random variables, correlation coefficients, confluent coefficients of flood flows and so on, will occur in applying this method to actual river systems.

In any case, a river has the general characteristic that the influence of upstream states is cumulative and is propagated downstream step by step. Therefore, we believe that such characteristics must be fully considered in planning flood protection or in evaluating the present risk probabilities of river regions, and we are happy if this paper will give planners of river works any help.

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