On the Earthquake Response of Structural Systems Considering the Interaction Effects of the Ground

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Abstract

In earthquake engineering, it is important to estimate reasonably the effects of the ground characteristics on the earthquake responses of above-ground structures. In this paper, the ground characteristics in an elastic range are represented as the dynamic ground compliance of a foundation on an elastic half-space that means a force-displacement transfer function of an elastic ground-foundation system. Supposing bilinear hysteretic restoring force characteristics in an above-ground structure and an elasto-plastic boundary layer underneath the foundation, the nonlinear transient responses of a ground-structure system subjected to horizontal ground acceleration excitations are analyzed in a wide parameter range. The random time functions obtained through a noise generator are used for this response analysis as ground acceleration excitations. As a result of the present study, it is pointed out that an interaction effect of the ground may act advantageously on a structural earthquake response in usual cases, while a disadvantageous effect should be considered when large plastic behaviours of structures are anticipated during earthquakes.

1. Introduction

To make clear the requirements of structural safety to earthquake excitations, and to establish the most reasonable method of designing an antiseismic structure, some investigators have recognized the importance of earthquake response analyses in which the elasto-plastic responses of a structure are analyzed dynamically in a wide range of parameters with respect to both earthquake excitations and structural systems; and they have studied such structural response characteristics in detail and from various viewpoints. In these response analyses, the supposition of a mathematical model of a ground-structure system so as to simulate pertinently the dynamic characteristics of an actual system is especially important except for the suppositions of earthquake excitations and of measures of the structural safety to earthquakes.

The dynamic characteristics of middle or lower height structures, which are more rigid than tall buildings, are much affected by the properties of the soil ground under the structures[1)~3)]. To estimate reasonably the earthquake response characteristics of such a structure, the mathematical model must be constructed as a dynamically interacting system consisting of the structure and ground. So, we have studied first the dynamic characteristics of the ground-foundation system, and have investigated the earthquake response characteristics of a ground-structure system connecting an above-ground structure with the soil ground taking into account the dynamic characteristics of the ground-foundation system. That is, the displacement of a rectangular foundation on an elastic half-space excited by a harmonic force has been analytically and numerically evaluated[4),5)] in terms of the dynamic ground compliance; this latter is a complex-valued function of the excitation frequency, the density and elastic constants of the ground, and the length-width ratio of the rectangular foundation. For the convenience of the non-stationary earthquake response analysis, the transfer characteristics of the ground-foundation system in swaying motion to an applied horizontal force has been simulated[6)] by a transfer function of rational function type. Then, an idealized structural model taking into account the ground-structure interaction was obtained by combining an above-ground main structure of a elasto-plastic, lumped system with the ground-foundation system represented by the abovementioned transfer function. In addition, it was supposed that the foundation lies in contact with a massless, thin boundary layer having elasto-plastic restoring force characteristics on the elastic ground, since the ground neighbouring to a foundation would become excessively inelastic during intense earthquakes.

The elasto-plastic structural responses obtained by applying a class of random ground excitations to this structural model have been discussed in detail in preceding papers[7)~10)]. The present paper is another of these serial studies on earthquake response analyses of a ground-structure interaction system and is mainly concerned with the elasto-plastic response characteristics of an above-ground main structure in relation to the fundamental natural frequency ratio of a main structure to a ground-foundation system, which means an index of the degree of interaction effect of the ground on a structure.

2. Ground-Structure Interaction System

The dynamic response of an elastic half-space excited by a harmonic force distributed in a finite domain on the surface has been studied by many investigators. In most cases, however, the exciting force was distributed in a circular domain because of the primary interest of the problem and of the fineness of the mathematical treatment. But the foundations of actual buildings are often rectangular. Applying the multiple Fourier transform to the problem in which a harmonic excitation force is distributed in a rectangular domain, one of the authors derived the analytical solution of the dynamic ground compliance for the case of vertical excitation[4)] and then extended this solution to cover a horizontal and rotational excitations[5)]. The dynamic ground compliance is defined as the ratio of the complex amplitude of displacement of a foundation on an elastic medium to that of the harmonic force acting on the foundation, and means the transfer characteristics of the displacement of the ground-foundation system to the excitation force.

The dynamic ground compliance is expressed analytically, in an integral representation, as a complex-valued function of the excitation frequency, the density

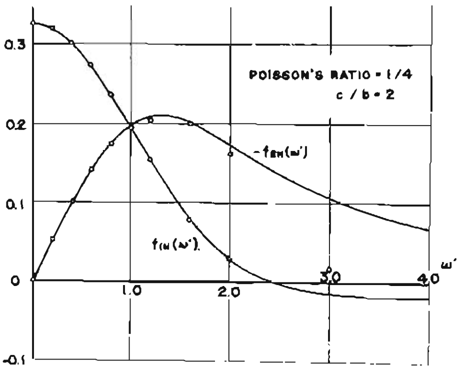

Fig. 1. Dynamic ground compliance for horizontal translation.

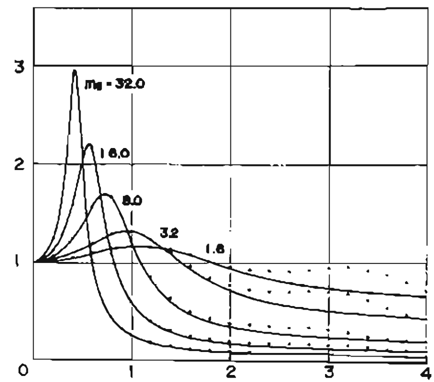

Fig. 2. Frequency response of the ground-foundation system.

and elastic constants of the ground, and the shape and stress distribution parameters of the foundation. It can be evaluated by a numerical integration of an improper, infinite integral in its analytical expression. Then, the numerical values of the dimensionless dynamic ground compliances of a rectangular foundation for the various excitation patterns are presented as the complex-valued functions of a dimensionless frequency variable, containing the length-width ratio of that foundation and Poisson's ratio of the ground as the parameters[4),5)]. In a typical case of horizontal excitation, where the length-width ratio (that is, the ratio of the length of a foundation in the excitation direction to its length in the perpendicular direction) is 1/2, and Poisson's ratio of the ground is 1/4, the exact values of the dimensionless dynamic ground compliance $f_{1H}(\omega')+jf_{2H}(\omega')$ are shown as the open circles in Fig. 1.

For the convenience of earthquake response analyses of a structural system taking into account the dynamic ground compliance, an approximate expression of the dynamic ground compliance, which is satisfied with the condition of physical realizability and also consistent with the transient response computation, is given by the following expressions:

$$\kappa_H(s')=\frac{c}{b}\frac{s'^2+c_1s'+c_0}{d_1s'+d_0}\simeq\frac{c}{b}\frac{1}{f_{1H}(\omega')+jf_{2H}(\omega')} \tag{1}$$

$$c_0=3.249,\ c_1=3.093,\ d_0=1.057 \text{ and } d_1=0.1573$$

where

$$s'=j\omega',\quad \omega'=b\sqrt{\frac{\rho}{\mu}}\,\Omega \text{ and } j=\sqrt{-1} \tag{2}$$

In the above, Ω is the angular frequency; ρ and μ are the density and shear modulus of the ground; b and c denote the half length of the rectangular foundation in the excitation direction and that in the other direction, respectively; ω' is the dimensionless angular frequency; and $f_{1H}(\omega')$ and $f_{2H}(\omega')$ are the real part and the

Table 1. Dynamic properties of the ground-foundation system.

m_g	Fundamental natural frequency, ${}_1\omega'_g$	Equivalent critical damping ratio, ${}_1h_{eq,g}$
1.6	1.1066	0.6182
3.2	1.0101	0.4926
8.0	0.7682	0.3357
16.0	0.5800	0.2445
32.0	0.4240	0.1740

imaginary part of the dimensionless dynamic ground compliance, respectively. The real and imaginary parts of this approximate dimensionless ground compliance are also shown in Fig. 1 as the solid lines, and the fitting may seem to be suitable at least in the range, $0 \leqq \omega' \leqq 2$.

Fig. 2 shows the displacement amplification factor of the substructure system, which consists of a foundation mass and the elastic ground. In this figure, the parameter m_g is defined as the mass ratio of a foundation mass M_0 to the reference mass of the elastic ground as follows:

$$m_g = \frac{M_0}{\rho b^3} \tag{3}$$

The increase of m_g makes the fundamental natural frequency of the system be smaller and makes the damping effect due to the energy radiation into the elastic ground decrease. In Table 1, the values of the fundamental natural frequency ${}_1\omega'_g$ and of the equivalent critical damping ratio associated with the fundamental mode ${}_1h_{eq,g}$ of the substructure system are shown. These values are evaluated from either of the smallest, complex conjugate roots ${}_1s'$ and ${}_1s'^*$ of the characteristic equation of the substructure system,

$$m_g s'^2 + \kappa_H(s') = 0 \tag{4}$$

as follows:

$${}_1\omega'_g = |Im\ {}_1s'| = |Im\ {}_1s'^*| \tag{5}$$

$${}_1h_{eq,g} = \frac{-Re\,{}_1s'}{|{}_1s'|} = \frac{-Re\,{}_1s'^*}{|{}_1s'^*|} \tag{6}$$

An approximate expression for the dynamic characteristics of a rectangular foundation on an elastic half-space may be obtained as in eq. (1); however the perfectly elastic behaviour of the ground may be too idealized, especially in the cases of soft soil-ground and of intense earthquakes. In such cases, the behaviour of the soil ground surrounding the foundation would be elasto-plastic rather than elastic because of stress or strain concentration. Hence, a massless, thin boundary layer of soil-ground whose restoring force characteristics are bilinear hysteretic is supposed between the foundation and the elastic ground. Because of the physical property of this boundary layer, its initial rigidity is supposed to be very large and its elastic limit displacement to be extremely small.

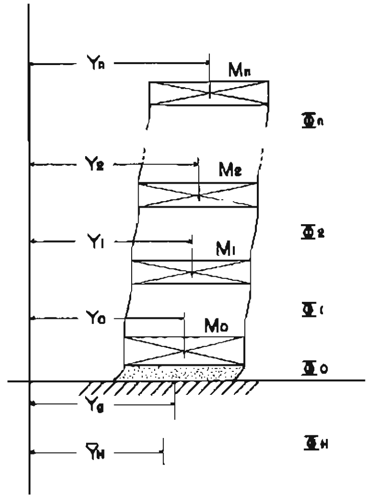

Fig. 3. Model of the ground-structure system.

In most of this paper, both the initial rigidity and the elastic limit strength of such a boundary layer are assumed to be infinitely large. This assumption means that this boundary layer is, in effect, neglected; in other words, the main structure is considered to be directly connected with the elastic ground. However, for purposes of comparison, the earthquake responses of a ground-structure system with a boundary layer which has a finite initial rigidity and a finite elastic limit strength are also shown partially.

On the other hand, the dynamic characteristics of a main structure are represented by a one-dimensional lumped elasto-plastic system of shear type, which is considered as an appropriate model of an actual building structure so far as a general survey of response characteristics is concerned. Since the primary interest of such a survey is the ultimate anti-seismic safety of a main structure during intense earthquakes, the restoring force characteristics of the main structure are also assumed to be of the bilinear hysteretic type which is actually expected for usual ductile building structures.

The above-mentioned ground-structure interaction system shows the damping effects caused by both the energy radiation from the foundation into the elastic ground and the energy dissipation due to the hysteresis loops of the restoring force characteristics. The internal viscous damping in both the main structure and the substructure is neglected, since the damping mechanisms which are significant and definite in the ground-structure system are precisely those of the energy radiation and the hysteretic dissipation.

3. Fundamental Equations of Motion

The mathematical model of the system considered is shown schematically in Fig. 3. The Laplace-transformed, dimensionless fundamental equations with respect to moving coordinate are expressed as follows:

$$\begin{aligned} &m_i s^2 u_i + g_i(u_i - u_{i-1}) - g_{i+1}(u_{i+1} - u_i) = -m_i f + \sigma_i \\ &\qquad\qquad i = 0, 1, 2, \ldots, n \\ &\kappa_g \kappa_H(qs) u_g - g_0(u_0 - u_g) = \sigma_g \end{aligned} \tag{7}$$

where s is the complex parameter of the Laplace transform associated with a dimensionless time τ, which will be defined by using the reference values of the above-ground structure as in the first equation of eqs. (8); and σ_i and σ_g denote inhomogeneous terms due to the initial conditions. In this paper, all the terms σ_i's and σ_g are taken to be zero, assuming that the system initially at rest is suddenly subjected to an earthquake excitation having zero initial velocity.

The independent and dependent variables and the inhomogeneous terms in the Laplace-transformed, dimensionless fundamental equations with respect to s and the associated dimensionless fundamental equations with respect to τ are related to each other and to the original physical quantities as follows:

$$s \sim \tau = \sqrt{\frac{\bar{K}}{\bar{M}}}\,T, \quad u_i = U_i - F_H \subset \eta_i = \frac{Y_i - \bar{Y}_H}{\bar{\Delta}}, \quad U_i \subset \frac{Y_i}{\bar{\Delta}} \qquad (8)$$

$$i = g,\ 0,\ 1,\ 2,\ \ldots,\ n$$

and

$$F_H \subset \frac{\bar{Y}_H}{\bar{\Delta}}, \quad f = s^2 F_H \subset \alpha\alpha(\tau)$$

$$\alpha = \frac{A\bar{M}}{\bar{K}\bar{\Delta}} = \frac{A\bar{M}}{\bar{B}}, \quad \alpha(\tau) = \frac{d^2}{dT^2}\left(\frac{\bar{Y}_H}{A}\right)_{T=\sqrt{\bar{M}/\bar{K}}\,\tau} \qquad (9)$$

where: T is time; $Y_i (i=1, 2, \ldots n)$; Y_0 and Y_g are the displacement with respect to the absolute coordinate of the *i-th* mass in the main structure, that of the foundation mass and that of the elastic ground, respectively; $\bar{Y}_H$ is the displacement of an earthquake excitation with respect to the same absolute coordinate; A is the maximum amplitude of acceleration of the earthquake excitation; $\bar{M}$, $\bar{K}$, $\bar{\Delta}$ and $\bar{B}$ are, respectively, the reference values of mass, stiffness, displacement and strength of the main structure.

The correspondence of the transformed, dimensionless elasto-plastic restoring force characteristics in eq. (7) and the original characteristics as shown in Fig. 4 is given by

$$g_i(u_i - u_{i-1}) \subset \kappa_i \varphi_i(\eta_i - \eta_{i-1};\ \delta_i,\ r_i,\ \beta_i)$$
$$= \frac{1}{\bar{B}} \Phi_i (Y_i - Y_{i-1};\ K_{i1},\ K_{i2},\ \Delta_i) \qquad (10)$$

in which

$$\kappa_i = \frac{K_{i1}}{\bar{K}}, \quad r_i = \frac{K_{i2}}{K_{i1}}, \quad \delta_i = \frac{\Delta_i}{\bar{\Delta}}, \quad \kappa_i \delta_i = \frac{K_{i1}\Delta_i}{\bar{K}\bar{\Delta}} = \frac{B_i}{\bar{B}} = \beta_i \qquad (11)$$

$$i = 0,\ 1,\ 2,\ \ldots,\ n$$

where: Φ_i and Φ_0 are the restoring force characteristics of the *i-th* story of the main structure and of the boundary layer; K_{i1} and K_{i2} are respectively the stiffnesses of the first and the second branch of Φ_i; Δ_i and B_i denote the displacement and strength at the elastic limit of Φ_i, respectively. Similarly, the masses M_i are expressed in the following dimensionless form:

$$m_i = \frac{M_i}{\bar{M}} \tag{12}$$

$$i = 0,\ 1,\ 2,\ \dots,\ n$$

The equivalent stiffness κ_g of the ground-foundation system in the last equation of eqs. (7) is defined by the following equation:

$$\kappa_g = \frac{\mu b}{\bar{K}} \tag{13}$$

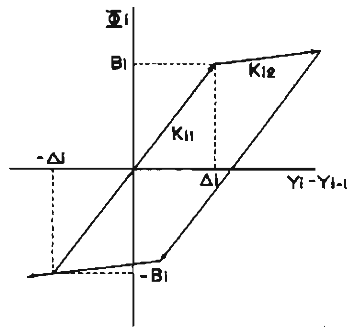

Fig. 4. Bilinear restoring force characteristics.

Then the transformed, dimensionless restoring force characteristics of the elastic ground are related to the original quantity as follows:

$$\kappa_H(s')u_g = \kappa_H(qs)u_g \subset \frac{1}{\mu b \bar{\Delta}} \Phi_H(Y_g - \bar{Y}_H) \tag{14}$$

where Φ_H denotes the restoring force characteristics of the elastic ground represented by the dynamic ground compliance, and q is the coefficient connecting the dimensionless variables associated with the elastic ground, which are attached with $'$, and those concerning the main structure such that

$$s' = qs,\ \omega' = q\omega \text{ and } \tau' = \frac{\tau}{q} \tag{15}$$

in which

$$q = b\sqrt{\frac{\rho}{\mu}}\sqrt{\frac{\bar{K}}{\bar{M}}} \tag{16}$$

4. System Parameters

As the basic quantities representing the vibrational characteristics of a ground-structure interaction system, the natural frequency ${}_\nu\omega$ of the ν-*th* mode of the relevant elastic system and the corresponding equivalent critical damping ratio ${}_\nu h_{eq}$ are determined by solving the characteristic equation of the system. From eq. (7), the dimensionless characteristic equation of the ground-structure system is obtained as follows:

$$\Delta_c(s) = \det\,[W(s)] = \det \begin{pmatrix} \kappa_g\kappa_H(qs) + \kappa_0 & -\kappa_0 & \cdots & 0 \\ -\kappa_0 & m_0 s^2 + \kappa_0 + \kappa_1 & -\kappa_1 & \vdots \\ \vdots & -\kappa_1 & & \vdots \\ \vdots & & & -\kappa_n \\ 0 & \cdots & -\kappa_n & m_n s^2 + \kappa_n \end{pmatrix} = 0 \tag{17}$$

From the complex eigen-value ${}_\nu s$, which is the ν-*th* smallest complex root of the above equation, the natural frequency ${}_\nu\omega$ and the equivalent critical damping ratio ${}_\nu h_{eq}$ associated with the ν-*th* natural vibration of the system are calculated by the following equations:

$$ {}_\nu\omega = |Im_\nu s| \text{ and } {}_\nu h_{eq} = -\frac{Re_\nu s}{|_\nu s|} \tag{18}$$

Similarly, from the dimensionless characteristic equation of the main structure with a rigid base, the ν-*th* natural frequency ${}_\nu\omega_s$ of the associated elastic system is obtained. It is noted that the corresponding critical damping ratio ${}_\nu h_s$ is zero since no damping mechanism is considered in the elastic main structure.

The natural frequency ratio of an elastic ground-structure system to the relevant main structure, defined by

$$\frac{{}_\nu\omega}{{}_\nu\omega_s} = \frac{|Im_\nu s|}{|Im_\nu s_s|} \tag{19}$$

shows the degree of interaction effect of the ground-foundation system. In particular, for the fundamental natural vibration, which is considered as the most significant vibrational characteristic of the elastic structural system, the fundamental natural frequency ratio ${}_1\omega/{}_1\omega_s$ is always less than unity. And, with the elongation of the fundamental natural period of the ground-structure system due to the interaction effect of the ground-foundation system, the corresponding critical damping ratio ${}_1h_{eq}$ caused by the energy radiation into the ground increases monotonously.

To construct the ground-structure system by connecting the substructure, which consists of an elastic ground and a foundation mass, with the main structure, which has a rigid base, the parameter λ, which is defined as the ratio of the fundamental natural frequency of the main structure ${}_1\Omega_s$ to that of the substructure ${}_1\Omega_g$ is introduced.

$$\lambda = \frac{{}_1\Omega_s}{{}_1\Omega_g} = \frac{{}_1\omega_s}{{}_1\omega_g} = \frac{{}_1\omega'_s}{{}_1\omega'_g} \tag{20}$$

This quantity means the degree of interaction effect of a ground-foundation system on a main structure.

Accordingly, the parameter q, which is the ratio of reference value of frequency of a substructure to that of a main structure, is expressed as

$$q = \lambda \cdot \frac{{}_1\omega'_g}{{}_1\omega_s} \tag{21}$$

By using the above equation some interrelations among the various system parameters are obtained as follows:

$$m_0 = m_g q^2 \kappa_g = m_g \lambda^2 \kappa_g \left(\frac{{}_1\omega'_g}{{}_1\omega_s}\right)^2 \tag{22}$$

In this paper, the mass ratio of a foundation to a main structure m_0, the equivalent mass ratio of a foundation to an elastic ground m_g and the fundamental natural frequency ratio of a main structure to a substructure λ are selected as the independent system parameters. Hence, the equivalent stiffness ratio of a substructure to a main structure κ_g and the parameter q become dependent system parameters.

To find the predominant fundamental vibration characteristics of a main structure, and to find explicitly the interaction effect of a ground-foundation system on a main structure, it is supposed that the main structure is a single-degree-of-freedom system. And, the values of the physical quantities of the main structure are chosen as the relevant reference values of the ground-structure system. Consequently, the values of the dimensionless parameters of the main structure are given by

$$m_1=\kappa_1=\delta_1=\beta_1=1$$

And the rigidity ratio in the bilinear hysteretic characteristics of the main structure is supposed thus:

$$r_1=0.1$$

On the other hand, the bilinear hysteretic characteristics of the boundary layer are assumed as follows:

$$\kappa_0=40,\ \delta_0=0.025,\ \beta_0=1 \text{ and } r_0=0.05$$

As for the ground-foundation system, several combinations of numerical values of the relevant parameters are supposed, in contrast with a simplified idealization of the main structure, as follows:

$$m_0=0.2 \text{ and } 0.8$$
$$m_g=1.6 \text{ and } 32.0$$

Considering the dynamic characteristics of actual ground-structure systems, the value of the parameter λ is varied in the range from 0 to 2.

In Fig. 5, solid line curves represent the relation between ${}_1\omega/{}_1\omega_s$ and ${}_1h_{eq}$ associated with the fundamental natural vibration of a ground-structure interaction system, eliminating the parameter λ and selecting m_0 and m_g as a set of explicit parameters. When the implicit parameter λ increases, a change of the basic vibrational properties of the ground-structure systems is also shown graphically as a significant

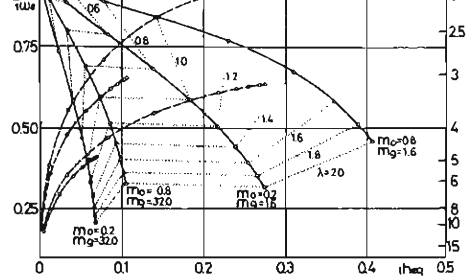

Fig. 5. Basic dynamic properties of the ground-structure system.

increase of the equivalent critical damping ratio ${}_1h_{eq}$ especially for a large m_0 or a small m_g, and as a remarkable decrease of the natural frequency ratio ${}_1\omega/{}_1\omega_s$ particularly for a small m_0 or a large m_g.

5. Earthquake Excitation

In usual earthquake response analyses of structures, the waveforms of the destructive earthquake accelerograms recorded at any given building structure have been used as the patterns of earthquake acceleration excitations only by scaling thier amplitude to an appropriate level. Strictly speaking, an accelerogram recorded by a accelerometer is considered to be a specific structural response affected by the dynamic characteristics of the structure and of the ground. Therefore, from the standpoint of the anti-seismic design of a structure for future earthquakes, the white noise process or, more preferably, a stochastic process having a spectral characteristic, which is consistent with the wave-transfer characteristics of the site of the structure, might be adopted for the earthquake acceleration excitations[11].

Since the present study is intended to discuss mainly the interaction effects of subsoil ground on the earthquake response of above-ground main structures, it may be suitable that the earthquake acceleration excitations are supposed to belong to a band-limited white noise process. As in the foregoing studies[7~10], twelve member functions from such a process are used in this paper as a group of earthquake excitations. The frequency ratio of the upper bound to the lower one is taken as 50 : 1, and the duration time is assumed to be 30 times the period corresponding to the upper bound frequency, which may be pertinent so long as an intense phase of earthquakes is concerned. And each member function is normalized to have the same mean square value.

The average spectral density of the above-mentioned twelve earthquake acceleration excitations fluctuates somewhat about a constant value. Instead of showing the average spectral density directly, the average pseudo-velocity response spectrum is shown in Fig. 6, since the closed relation between the average spectral density and the average pseudo-response spectrum has been known[12]. The latter is considered as a kind of spectral representation of an input, which is convenient for the response analysis of the relevant linear system. In Fig. 6, the ordinate shows the average pseudo-velocity response spectra for the 1 g maximum acceleration of the earthquake excitations; the abscissa shows the ratio ν of the natural frequency of a single-degree-of-freedom, linear vibrational system ${}_1\Omega_s$ to the upper bound of frequency of the band-limited white noise process Ω_u; and the parameter h

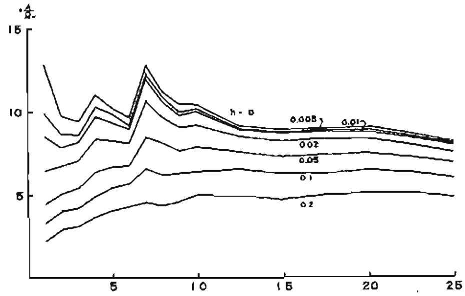

Fig. 6. Average pseudo-velocity response spectra for band-limited white excitations.

denotes the critical damping ratio of the system. Generally, the response spectra seem to be of a considerably smoothed variation because of the averaging effect of the transfer characteristics of the system on the spectral fluctuation of the excitations. However, in the case of the sharp filtering characteristics of a system with a small critical damping ratio, the spectral fluctuation of the excitations may appear directly in the response spectra. This is shown in the fact that the response spectrum in the case of no damping becomes uneven in the range $0 \leqq \nu \leqq 12$.

The inhomogeneous term f in the fundamental equation (7) is given by the dimensionless earthquake acceleration excitation $\alpha\alpha(\tau)$ in eq. (9) where the intensity parameter α and the waveform function $\alpha(\tau)$ represent respectively the dimensionless maximum amplitude and the dimensionless waveform, which is normalized such that the maximum value of an earthquake acceleration excitation is unity. The relation between the intensity parameter α and the so-called base shear coefficient $\bar{s}$ of a main structure is shown in the following equation for the case where the structure is idealized as a single-degree-of-freedom system:

$$\alpha = \frac{A\overline{M}}{\overline{B}} = \frac{A/g}{\bar{s}} \tag{23}$$

The minimum value of the intensity parameter α of the twelve earthquake acceleration excitations is adopted for the value of the intensity parameter of the whole group of earthquake excitations considered. The numerical value of such an intensity parameter α is taken from 0.4 to 2.0 so as to vary structural responses from the elastic to the elasto-plastic range.

The dimensionless frequency parameter ν of earthquake excitations, which relates the spectral characteristics of the earthquake excitations to those of a structural system, is defined by the following equation:

$$\nu = \frac{\Omega_u}{{}_1\Omega_s} = \frac{\omega_u}{{}_1\omega_s} = \frac{\omega'_u}{{}_1\omega'_s} \tag{24}$$

In this paper, two cases are considered regarding this frequency parameter: the case where $\nu = 2$; and the case where $\lambda\nu = 2$. In the former case, the fundamental natural frequency of the main structure ${}_1\Omega_s$ is fixed near the center of the frequency band of the excitations; and the fundamental frequency of the substructure ${}_1\Omega_g$ becomes smaller as the frequency ratio λ defined by eq. (20) increases. In the latter case, the frequency parameter ν is combined with the parameter λ so that:

$$\lambda\nu = \frac{\Omega_u}{{}_1\Omega_g} = \frac{\omega_u}{{}_1\omega_g} = \frac{\omega'_u}{{}_1\omega'_g} = 2 \tag{25}$$

Hence the fundamental frequency of the substructure ${}_1\Omega_g$ is fixed near the center of the frequency band of the excitations, and the fundamental natural frequency of the main structure ${}_1\Omega_s$ is shifted from a small value to a large one as the parameter λ increases:

Now, in a similar fashion to that of the parameter ν, another dimensionless frequency parameter of excitations ν' can be defined by using the fundamental natural frequency of the ground-structure interaction system ${}_1\Omega$ instead of the fundamental natural frequency of the main structure ${}_1\Omega_s$ in eq. (24) as follows:

$$\nu' = \frac{\Omega_u}{{}_1\Omega} = \frac{\omega_u}{{}_1\omega} = \frac{\omega'_u}{{}_1\omega'} \tag{26}$$

The dependence of the parameter ν' on the parameters λ, m_0 and m_g, in the above-mentioned two cases, is shown in Fig. 5. The right-side ordinate ν' in the figure is to be referred to the solid lines in the former case and to the broken lines in the latter case. Though the equivalent damping ratio ${}_1h_{eq}$ and the frequency ratio ${}_1\Omega/{}_1\Omega_s$ are equal in both cases for the given values of λ, m_0 and m_g, the frequency ratio ν' in eq. (26) increases infinitely with λ in the case where ν=constant, and decreases from infinity and tends to a finite value as λ increases from zero to infinity in the case where $\lambda\nu$=constant. For the given excitations, the ground-structure system at the limiting condition $\lambda=\infty$ reduces to a system consisting of a rigid main structure and a substructure with a finite rigidity, namely, to a ground-foundation system having an equivalent mass ratio in eq. (3) equal to $m_g\,(1+1/m_0)$ in the latter case, and to a system which consists of a main structure with a finite rigidity and a substructure with zero rigidity in the former case.

6. Response Analysis and Its Results

In this paper, attention is focused only on maximum relative displacement, which may be considered the most important response in estimating the aseismic safety of a structural system. The dimensionless expressions of the maximum relative displacement of each part of the ground-structure system are written as follows:

$$\begin{aligned}
\gamma_{r1} &= |Y_1 - Y_0|_{\max}/\bar{\Delta} && \text{for a main structure}\\
\gamma_{r0} &= |Y_0 - Y_g|_{\max}/\bar{\Delta} && \text{for a boundary layer}\\
\gamma_{rg} &= |Y_g - \bar{Y}_H|_{\max}/\bar{\Delta} && \text{for an elastic ground with a boundary layer}\\
\gamma_{rg} &= |Y_0 - \bar{Y}_H|_{\max}/\bar{\Delta} && \text{for an elastic ground without a boundary layer}
\end{aligned} \tag{27}$$

Since $\bar{\Delta}$ represents the elastic limit displacement of the main structure, γ_{r1} means the maximum ductility ratio of the main structure. For several sets of numerical values of the parameters previously described, transient response analyses to the twelve member functions of earthquake excitations are made by using an electronic analog computer, and the ensemble average $E\gamma_{ri}$, $i=1, 0, g$, and the standard deviation $\sqrt{V\gamma_{ri}}$ of the maximum relative displacement response to the excitation group are evaluated.

Figs. 7(a) to (d) present the basic response diagrams of the maximum relative displacement of the main structure. The solid lines in each figure show the response curves which correspond to the system without a boundary layer in the case where $\nu=2$. From these curves, it is found that a linear or slightly nonlinear response, which corresponds to the curve for a small α, decreases with λ mainly because of

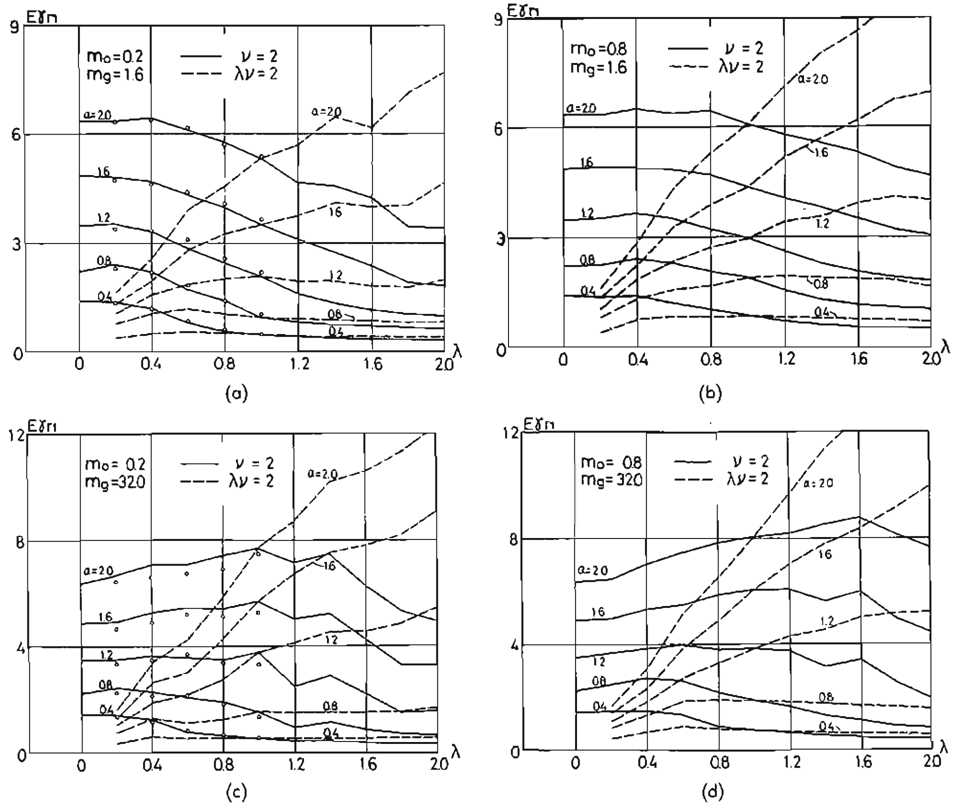

Fig. 7. Basic response diagram of a main structure, (a) $m_0=0.2$ and $m_g=1.6$, (b) $m_0=0.8$ and $m_g=1.6$, (c) $m_0=0.2$ and $m_g=32.0$, (d) $m_0=0.8$ and $m_g=32.0$.

the increase of the equivalent damping due to energy radiation into the ground and the decrease of the effective duration-time of excitation which is defined as the ratio of the duration-time of excitation to the equivalent fundamental period of the system. As regards the linear ground-structure system and the earthquake excitations considered, the equivalent duration-time of excitation is expressed as $30/\nu'$.

When plastic behaviour in the main structure becomes predominant for a large α, the relative displacement response does not always decrease with λ. Especially in the case where $m_g=32.0$, there even exsists a domain in which responses increase with λ. The abovementioned interaction effect of the substructure, which makes the elastic responses of the main structure decrease with λ, is weakened by the plastic yielding of the main structure, particularly in an intermediate range of λ. Consequently, the relative displacement response of the main structure in a plastic range is not suppressed by the interaction effect of the substructure, particularly in the case of large m_g and m_0, as compared with the considerable interaction effect on the decrease of elastic response of the main structure.

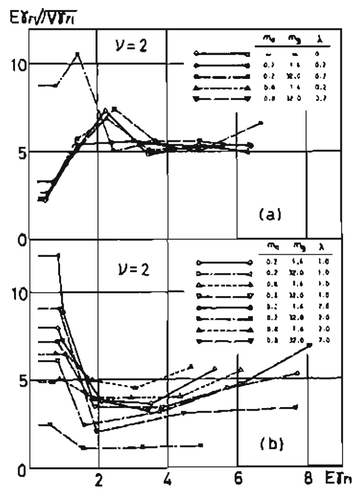

Fig. 8. Variation of $E\gamma_{r1}/\sqrt{V\gamma_{r1}}$ with $E\gamma_{r1}$.

Though elasto-plastic responses seem, in general, to vary smoothly with λ, a local fluctuation of the response curve with respect to λ becomes large as plastic behaviour increases with α. Especially in the case where $m_g=32.0$, a considerable fluctuation appears in the range of large λ and for an intermediate level of α because the effective duration time of excitation is small for a large λ and the equivalent damping due to energy radiation into the ground is comparatively small for a large m_g. It may be considered that the fluctuation of the response curve with respect to λ is caused by the presence of a spectral fluctuation of excitations and is apt to appear in the elasto-plastic response in the case of strong non-stationarity and weak damping characteristics.

The broken lines in Figs. 7 (a) to (d) indicate the response curves which correspond to a system without a boundary layer, when $\lambda\nu=2$. The maximum ductility ratio response of a linear or slightly nonlinear main structure is almost constant over a wide range of λ, since the effect of any increase of the effective duration-time of excitation with λ is canceled by the effect of an increase of the equivalent damping due to energy radiation into the ground, as shown in Fig. 5. As the plastic behaviour of the main structure becomes predominant for a large α, the maximum ductility ratio response increases remarkably with λ having the tendency described by the inequality $\partial^2 E\gamma_{r1}/\partial\alpha\partial\lambda>0$. That is, compared to the elastic limit deformation, the plastic yielding increases extremely with λ for a large α, because the elastic limit deformation of the main structure decreases proportionally to $1/\lambda^2$; and the damping effect due to energy radiation into the ground decreases with the elongation of the equivalent fundamental period of the nonlinear main structure for a large α.

The small open circles plotted in Figs. 7 (a) and (c) represent the responses of the main structure in terms of a ground-structure system with the boundary layer whose restoring force characteristics are described in the foregoing section. They are almost identical to the corresponding responses obtained without considering the boundary layer, since the significant dynamic characteristics of the ground-structure interaction system are scarcely altered by the plastic yielding of any boundary layer having a considerably large stiffness in the second branch of its restoring force characteristics. However, careful observation of these figures shows that the response of the main structure becomes smaller due to the hysteretic energy dissipation in the boundary layer for a small λ and for a large m_g.

Fig. 8 represents a variation of $E\gamma_{rl}/\sqrt{V\gamma_{rl}}$ with $E\gamma_{rl}$ for an interpretation of a trend of scattering of the responses of the main structure over its ensemble when $\nu=2$ and the system without the boundary layer is considered. In the cases in which $\lambda=0$ and 0.2 shown in Fig. 8 (a), the value of $E\gamma_{rl}/\sqrt{V\gamma_{rl}}$ increases remarkably in the range $E\gamma_{rl}=1$ to 2, where the response changes from an elastic range into a plastic range. This means that the damping effect due to the hysteretic energy dissipation in the main structure makes the scattering of its responses decrease. Since the value of $E\gamma_{rl}/\sqrt{V\gamma_{rl}}$ is almost constant over the range $E\gamma_{rl}>2$, perhaps the effect due to hysteretic energy dissipation on the suppression of scattering of the responses can not be anticipated from an unexpectedly low level of the ductility ratio response even for a small λ for which the damping effect due to energy radiation into the ground is not expected.

On the other hand, for a large λ, as shown in Fig. 8 (b), the value of $E\gamma_{rl}/\sqrt{V\gamma_{rl}}$ in the range $E\gamma_{rl}=1$ to 2 decreases considerably. In this case, a scattering of the response may be produced by an abrupt occurrence of a plastic yielding caused by the excitation having a rather smaller effective duration-time and a comparatively large α. However, the hysteretic damping of the main structure seems to be only slightly effective to suppress the scattering of its response in the range $E\gamma_{rl}>2$.

The maximum relative displacement response of the elastic ground $E\gamma_{rg}$ varies strongly with λ, for its equivalent stiffness κ_g is proportional to $1/\lambda^2$, as indicated in eq. (22). Then, as the basic response diagram of the elastic ground, the product of the maximum relative displacement by the static stiffness of the ground-foundation system $\kappa_g\kappa_H(0)$, which means the maximum shear force of the elastic ground, is presented in Figs. 9 (a) to (d) for evaluating the interaction effect on the response characteristics of the elastic ground. In the case of a ground-structure interaction system without a boundary layer, the second and third equations in the fundamental equation (7) are reduced to the following equation by taking account of the relation $u_g=u_0$:

$$m_0s^2u_0+\kappa_g\kappa_H(qs)u_0-g_l(u_l-u_0)=-m_0f \tag{28}$$

The maximum shear force of the elastic ground may have a close relation to that of the main structure, though the inertia force of the foundation mass somewhat influences this relation, as shown in eq. (28). Consequently, for a linear ground-structure system the maximum shear force response of the elastic ground $\kappa_g\kappa_H(0)E\gamma_{rg}$ has, with respect to the parameter λ, similar characteristics to the maximum relative displacement response of the main structure $E\gamma_{rl}$. The maximum relative displacement response of the nonlinear main structure has a general trend represented by a remarkable increase when α becomes large, as shown in Figs. 7 (a) to (d), while the maximum shear force response increases slowly as the parameter α increases because of a degraded stiffness in the plastic region. Hence, the responses of the elastic ground shown in Figs. 9 (a) to (d) are considerably suppressed for large

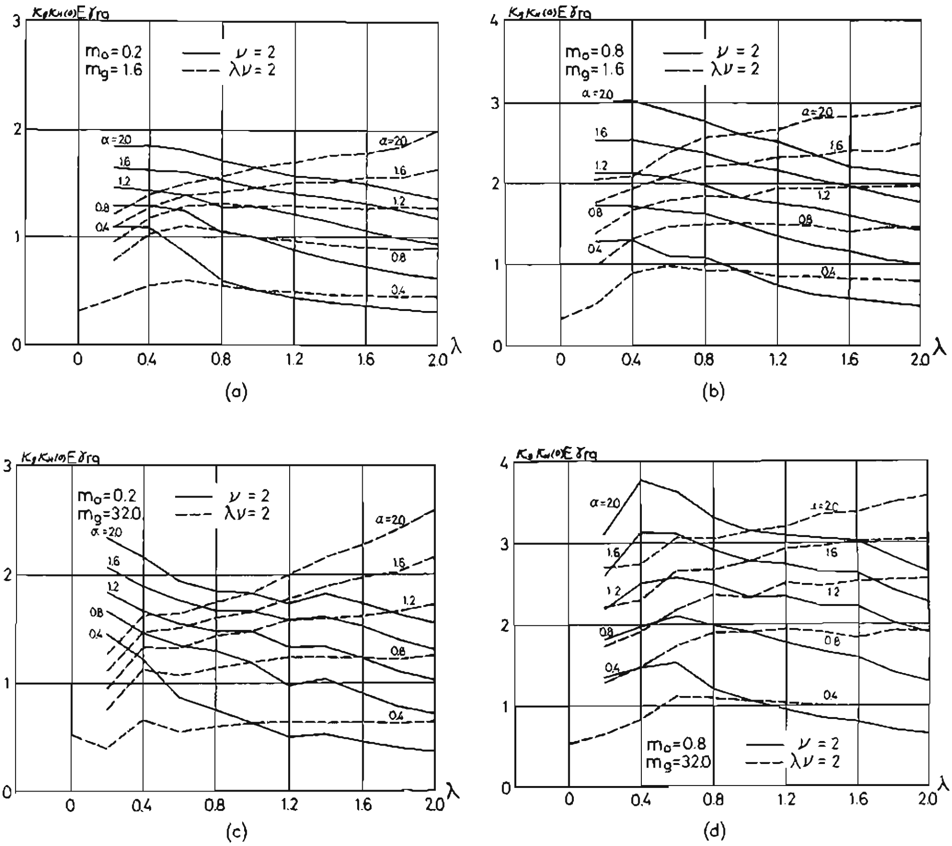

Fig. 9. Basic response diagram of an elastic ground, (a) $m_0=0.2$ and $m_g=1.6$, (b) $m_0=0.8$ and $m_g=1.6$, (c) $m_0=0.2$ and $m_g=32.0$, (d) $m_0=0.8$ and $m_g=32.0$.

values of α. In these figures, the solid and broken lines correspond respectively to the cases where $\nu=2$ and $\lambda\nu=2$, as in Figs. 7 (a) to (d).

7. Discussions

As described in the preceding section, the response characteristics of the main structure in a ground-structure system vary considerably with the degree of non-linearity. To evaluate the interaction effect of the substructure on the non-linear response of the main structure in relation to the earthquake-resistant design of structures, the dimensionless intensity parameter of the earthquake excitations α, which is the ratio of the dimensionless maximum acceleration of excitation A/g to the so-called base shear coefficient $\bar{s}$, is calculated as a function of the response level $E\gamma_{r1}$ and the parameter λ from the basic response diagrams shown in Figs. 7 (a) to (d). The results are presented in graphic form.

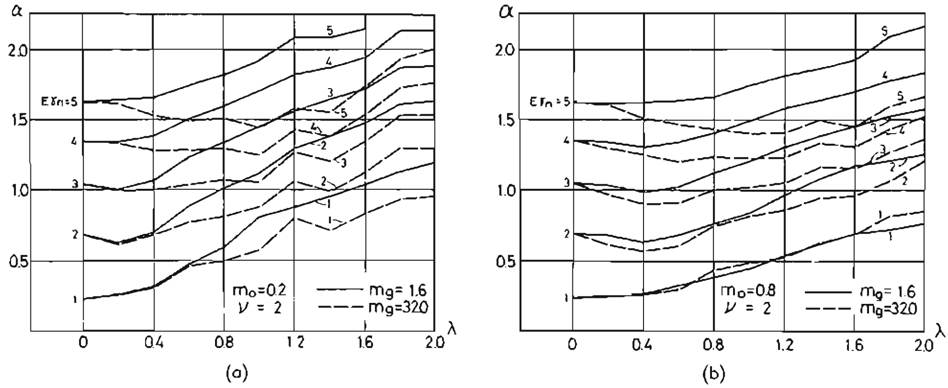

Fig. 10. Dimensionless intensity parameter α for the response level $E\gamma_{rl}$, (a) $\nu=2$ and $m_0=0.2$, (b) $\nu=2$ and $m_0=0.8$.

As shown in Figs. 10 (a) and (b), both of which correspond to the case where $\nu=2$, the value of α increases generally with λ for a small m_g. This means that the increase of λ advantegeously decreases the base shear coefficient of the main structure as long as the equivalent mass ratio m_g is comparatively small and the allowable ductility ratio is not too large. But when m_g is large the value of α does not always increase with λ, and does decrease, in general, for a comparatively large $E\gamma_{rl}$ and an intermediate range of λ. As shown in Figs. 10 (a) and (b), the minimum values of α exist in the range $\lambda=0.2$ to 1.0 for $m_0=0.2$, and $\lambda=0.4$ to 1.0 for $m_0=0.8$ corresponding to the response level $E\gamma_{rl}=2$ to 5, and the extremal value of λ varies from the lower to the upper side with the increase of $E\gamma_{rl}$. In any range of λ smaller than this extremal value, the interaction of the substructure has a disadvantageous effect on the nonlinear response of the main structure, though this effect may not be very serious.

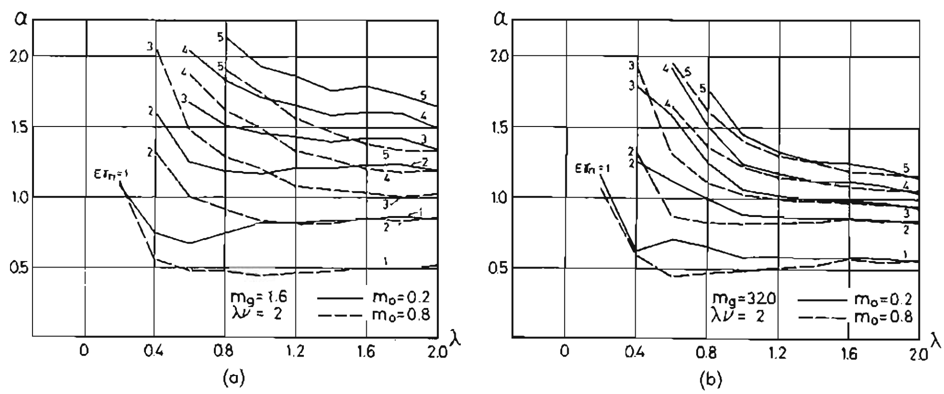

Fig. 11. Dimensionless intensity parameter α for the response level $E\gamma_{rl}$ (a) $\lambda\nu=2$ and $m_g=1.6$, (b) $\lambda\nu=2$ and $m_g=32.0$.

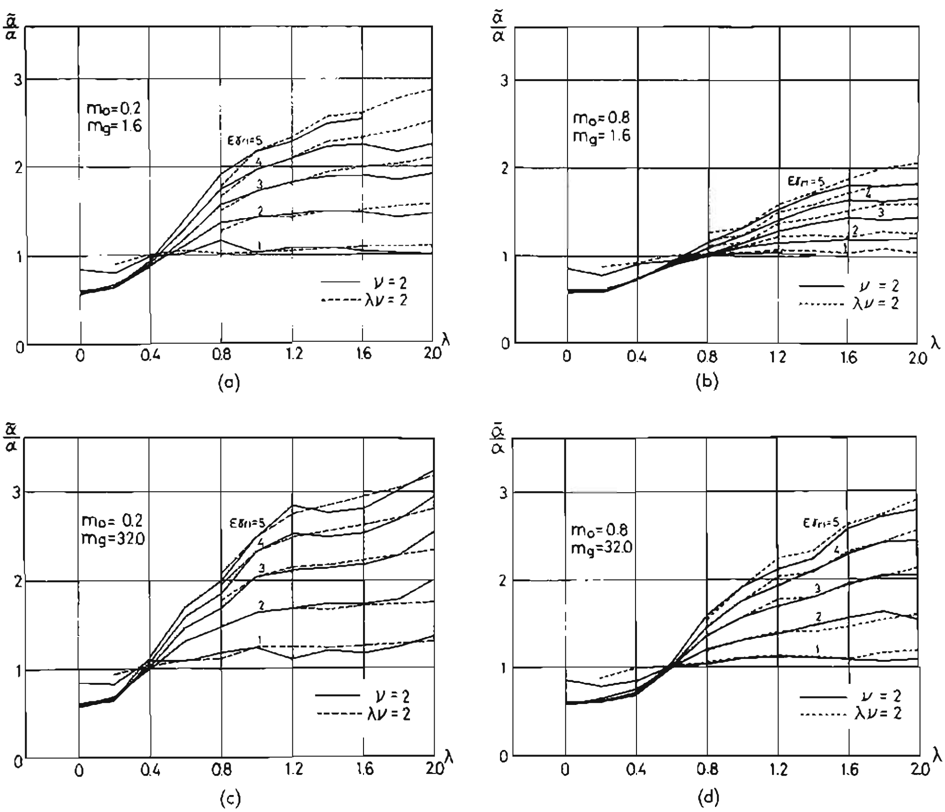

Fig. 12. Ratio of elasto-plastic response to elastic response of a main structure, (a) $m_O=0.2$ and $m_g=1.6$, (b) $m_O=0.8$ and $m_g=1.6$, (c) $m_O=0.8$ and $m_g=1.6$, (d) $m_O=0.8$ and $m_g=32.0$.

Similarly, the values of α corresponding to the response level $E\gamma_{rl}$ are shown in Figs. 11 (a) and (b) in the case where $\lambda\nu=2$. If the local fluctuation of α due to the spectral fluctuation of the excitations is neglected, the variation of α may be interpreted by an almost decreasing function of λ, except when $E\gamma_{rl}=1$. As previously mentioned, for the case where $\lambda\nu=2$, the fundamental natural frequency of the substructure is near the center of a frequency band of the excitations and the natural frequency of the main structure comes close to the upper bound frequency of excitations with the increase of λ. Consequently, as the parameter λ increases, the frequency band of the excitations shifts to the relatively lower range of the natural frequency of the ground-structure system. Since an increase of λ means to increase the rigidity of the main structure and to decrease elastic limit deformation rather than to increase the interaction effect of the substructure on the response of the main structure, the ductility ratio response of the main structure increases remarkably with λ for a large α; and the value of α corresponding to a comparatively large response level $E\gamma_{rl}$ decreases with λ, particularly if m_g or m_O is large.

From these figures it follows that in the case of a comparatively large allowable ductility ratio, the more rigid the main structure becomes as compared with the ground-foundation system, the larger base shear coefficient is necessary in the anti-seismic design of the main structure, especially for a large mass ratio m_g or m_0. Also, in the case of a comparatively small allowable ductility ratio, the base shear coefficient for the main structure increases as the fundamental natural frequency of the main structure becomes larger, but converges almost to a constant for any fundamental natural frequency of the main structure larger than half that of the ground-foundation system.

Figs. 12 (a) to (d) show the ratio of an imaginary dimensionless intensity of excitations $\tilde{\alpha}$, defined as the product of the response level $E\gamma_{r1}$ by the value of α corresponding to $E\gamma_{r1}=1$, to the dimensionless intensity of excitations α associated with the same response level $E\gamma_{r1}$ which is shown in Figs. 10 or 11. The ratio $\tilde{\alpha}/\alpha$ means also the ratio of the elasto-plastic relative displacement response of the main structure to the elastic one for a common intensity of excitations, that is, it is a magnification factor of the nonlinear response to the linear response. The degree of nonlinearity of the response is represented by the parameter $E\gamma_{r1}$ in the figures. The variation of $\tilde{\alpha}/\alpha$ with λ shows the local fluctuation due to a spectral fluctuation of the excitations used, and also shows a little difference between the two cases, $\nu=2$ and $\lambda\nu=2$, affected by the frequency relation of the system to the excitations. However, without regard to the values of the parameters m_g and m_0, there is a general tendency as follows:

In the case of a small λ, the values of $\tilde{\alpha}/\alpha$ are less than unity and show little difference for a wide range of the response level $E\gamma_{r1}=2$ to 5. On the contrary, in the case of a large λ, the ratio $\tilde{\alpha}/\alpha$ increases with $E\gamma_{r1}$. In other words, the value of $\partial E\gamma_{r1}/\partial\alpha$ seems to converge to a positive constant as α increases for the range of comparatively small λ where $\tilde{\alpha}/\alpha<1$ is valid, whereas for the range of comparatively large λ in which $\tilde{\alpha}/\alpha>1$ is valid, the value of $\partial E\gamma_{r1}/\partial\alpha$ increases with α. As regards the variation of $\tilde{\alpha}/\alpha$ with λ, it is noted that the ratio $\tilde{\alpha}/\alpha$ is an almost increasing function of λ, and the value of $\partial^2 E\gamma_{r1}/\partial\lambda^2$ is positive if $\tilde{\alpha}/\alpha<1$ but negative when $\tilde{\alpha}/\alpha>1$.

It is noticeable that the value of λ satisfying the critical condition $\tilde{\alpha}/\alpha=1$ is almost independent of the response level $E\gamma_{r1}$, and is determined in a rather narrow range depending on the parameters m_0 and m_g. As shown in Figs. 12 (a) to (d), this value becomes large for a larger m_0 and for a smaller m_g; and this tendency is similar to the trend that as m_0 increases and m_g decreases, the interaction effect of the substructure on the vibrational properties of the linear ground-structure system varies from the larger elongation of the fundamental natural period with a smaller critical damping ratio to the smaller elongation with a larger damping.

The abovementioned variation of $\tilde{\alpha}/\alpha$ with λ and $E\gamma_{r1}$ may be explained by the fact that a substructure makes the spectral characteristics of excitations to a main structure change according to the dynamic interaction between the substructure and the main structure. When λ is small, the filtering characteristics of a linear or slightly nonlinear ground-structure system may be comparatively sharp because of the small critical damping ratio due to the energy radiation into the ground; hence the hysteretic damping in a nonlinear main structure is effective to suppress its nonlinear relative displacement response as compared with the linear response, in spite of the inverse effect of the degrading stiffness in the plastic range on the nonlinear response. As λ increases, however, the filtering characteristics of the linear ground-structure system become broader because of the increase of the critical damping ratio. Then, the linear displacement response of the main structure is considerably restrained as compared with the nonlinear response, because the large nonlinearity of the main structure may reduce the effective value of λ due to the decrease of the equivalent fundamental frequency of the main structure. In the case of a weak interaction, in which $\tilde{\alpha}/\alpha<1$, the value of $\tilde{\alpha}/\alpha$ is irrelevant to the response levels $E\gamma_{r1}=2$ to 5 since both the energy transmitted to the system and the hysteretic energy dissipation in the main structure are approximately proportional to the intensity parameter of the excitations. On the other hand, in the case of a strong interaction, in which $\tilde{\alpha}/\alpha>1$, the interaction effect may vary depending on the response level $E\gamma_{r1}$ and the energy transmitted to the nonlinear system may increase as compared with that in a linear system, since the filtering characteristics of the nonlinear ground-structure system become sharp due to the decrease of the effective value of λ with the increase of $E\gamma_{r1}$. And, in the range of a relatively large λ, the linear response of the main structure is large for a large m_0, whereas the nonlinear response becomes large for a large m_g. Hence, the ratio $\tilde{\alpha}/\alpha$ in the range of the strong interaction is small for a large m_0 and a small m_g.

8. Concluding Remarks

This paper discusses a ground-structure system consisting of a main structure with elasto-plastic restoring force characteristics and a ground-foundation system having dynamic ground compliance; this paper also discusses an earthquake response analysis of the ground-structure interaction system subjected to the acceleration excitations represented by a band-limited white noise process. From this analysis, the following conclusions are obtained:

1. As the interaction of a substructure increases, the elastic or weakly inelastic displacement response of a main structure to a moderately intense earthquake excitation decreases, in general, due to the increase of energy radiation into the ground.
2. However, the strongly elasto-plastic displacement response of a main structure to a severely intense earthquake excitation does not always become small as compared with the response of the non-interacting main structure because of an amplifying effect from the substructure.
3. In a non-interacting or a weakly interacting system, the damping effect caused by hysteretic dissipated energy in a main structure suppresses effectively the elasto-plastic displacement response of that main structure.

4. In a strongly interacting system the elasto-plastic displacement response of the main structure becomes remarkably large as compared with the elastic response, since the filtering characteristics of the ground-structure system become sharp according to the increase of the response level of the main structure.
5. From the viewpoint of the anti-seismic design of structures, the interaction effect of a ground-foundation system may act advantageously on the earthquake response of a main structure as long as a comparatively small value of the allowable ductility ratio is used. However, when a relatively large allowable ductility ratio is assumed, the interaction effect is not always advantageous on the anti-seismic safety of a main structure.

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References

1) Idriss, I. M. and H. B. Seed: Seismic Response of Horizontal Soil Layers, Proc. of A.S.C.E., Vol. 94, No. SM4, July, 1968, pp.1003–1031.
2) Parmelee, R. A., D. S. Perelman, S. L. Lee and L. M. Keer: Seismic Response of Structure-Foundation Systems, Proc. of A.S.C.E., Vol. 94, No. EM6, December, 1968, pp.1295–1315.
3) Parmelee, R. A., D. S. Perelman and S. L. Lee: Seismic Response of Multiple-Story Structures on Flexible Foundations, Bull. of S.S.A., Vol. 59, No. 3, June, 1969, pp.1061–1070.
4) Thomson, W. T. and T. Kobori: Dynamical Compliance of Rectangular Foundations on an Elastic Half-Space, Jour. of Appl. Mech., Vol. 30, No. 4, 1963, pp.579–584.
5) Kobori, T., R. Minai, T. Suzuki and K. Kusakabe: Dynamical Ground Compliance of Rectangular Foundation on a Semi-Infinite Elastic Medium, Part 1, Annuals, Disas. Prev. Res. Inst., Kyoto Univ., No. 10A, March, 1967, pp.283–314.
6) Kobori, T., R. Minai and Y. Inoue: Ground Compliance of Rectangular Foundation and its Simulation, Annuals, Disas. Prev. Res. Inst., Kyoto Univ., No. 7, March, 1964, pp. 164–178.
7) Kobori, T., R. Minai and Y. Inoue: Earthquake Response of the Structure Considering the Effect of Ground Compliance, 2nd Report, Annuals, Disas. Prev. Res. Inst., Kyoto Univ., No. 8, March, 1965, pp. 193–218.
8) Kobori, T., R. Minai, Y. Inoue and T. Kamada: Earthquake Response of a Structure with a Rectangular Foundation, Proc. of the 16th Japan Nat. Cong. for Appl. Mech., 1966, pp. 307–315.
9) Kobori, T., R. Minai, Y. Inoue and T. Kamada: The Effect of Ground Compliance on Earthquake Response of Elasto-Plastic Structure, Proc. of the 2nd Japan Earthq. Engng. Symp., 1966, pp. 267–272.
10) Kobori, T., R. Minai and Y. Inoue: On Earthquake Response of Elasto-Plastic Structure Considering Ground Characteristics, Proc. of the 4th W.C.E.E., Chile, 1969, A-6, pp. 117–132.
11) Kobori, T. and Minai, R.: One-Dimensional Wave-Transfer Functions of the Linear Visco-Elastic Multi-Layered Half-Space, Bull. of the Disas. Prev. Res. Inst., Kyoto Univ., Vol. 18, Part 4, March, 1969, pp.27–64.
12) Kobori, T. and Minai, R.: Response Spectra of Quasi-Stationary Random Excitations, Bull. of the Disas. Prev. Res. Inst., Kyoto Univ., Vol. 17, Part 3, March, 1968, pp.45–94.